Quantum Vacuum & Cosmology on a Background of Zeta Functions

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Outline

- On Zeta Function Techniques
- Quadratic Affine Case: Chowla-Selberg and Extensions
- Traces and determinants
- Regularizing Quantum Vacuum Fluctuations
- Dark Energy vs Modified Gravity
- On the recent, impacting discovery by BICEP2
\[ \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{ns} \]

Diagram showing the behavior of \( \zeta(s) \) in the complex plane.
Basic strategies

- **Jacobi’s identity** for the $\theta$–function

  \[ \theta_3(z, \tau) := 1 + 2 \sum_{n=1}^{\infty} q^n \cos(2nz), \quad q := e^{i\pi\tau}, \quad \tau \in \mathbb{C} \]

  \[ \theta_3(z, \tau) = \frac{1}{\sqrt{-i\tau}} e^{z^2/i\pi\tau} \theta_3 \left( \frac{z}{\tau}, \frac{-1}{\tau} \right) \]

  equivalently:

  \[ \sum_{n=-\infty}^{\infty} e^{-(n+z)^2t} = \sqrt{\frac{\pi}{t}} \sum_{n=0}^{\infty} e^{-\frac{\pi^2n^2}{t}} \cos(2\pi nz), \quad z, t \in \mathbb{C}, \quad \text{Re} t > 0 \]

- Higher dimensions: **Poisson summ formula** (Riemann)

  \[ \sum_{\vec{n} \in \mathbb{Z}^p} f(\vec{n}) = \sum_{\vec{m} \in \mathbb{Z}^p} \tilde{f}(\vec{m}) \]

  $\tilde{f}$ Fourier transform

  [Gelbart + Miller, BAMS ’03, Iwaniec, Morgan, ICM ’06]

- **Truncated sums** $\longrightarrow$ asymptotic series
3: EXPLICIT CALCULATIONS

Epstein zeta function
\[ Z_E = \sum_{\vec{n} \in \mathbb{Z}^d} \varphi(\vec{n})^{-s} \]  
D quadratic form

Barnes zeta function
\[ Z_B = \sum_{\vec{n} \in \mathbb{N}^d} L(\vec{n})^{-s} \]  
L affine form  
(coeff's \( \in \mathbb{R} \))

Extension:

\[ Z_E \rightarrow \varphi + L \text{ affine} \]
\[ \sum_{\vec{n} \in \mathbb{N}^d} \rightarrow \sum_{\vec{n} \in \mathbb{Z}^d} \text{ (truncation)} \]

\[ Z_B \rightarrow Z_B'(0) \text{ (new formulas)} \]
\[ \sum'_{\vec{n} \in \mathbb{Z}^d} \text{ (by analytic cont.} \]

\[ \sum'_{\vec{n} \in \mathbb{Z}^d} \text{ (by analytic cont.} \]
Example of the ball:

- Operator

\[ (-\Delta + m^2) \]

on the \( D \)-dim ball \( B^D = \{ x \in \mathbb{R}^D; |x| \leq R \} \) with Dirichlet, Neumann or Robin BC.

- The zeta function

\[ \zeta(s) = \sum_{k} \lambda_k^{-s} \]

- Eigenvalues implicitly obtained from

\[ (-\Delta + m^2)\phi_k(x) = \lambda_k \phi_k(x) + BC \]

- In spherical coordinates:

\[ \phi_{l,m,n}(r, \Omega) = r^{1-\frac{D}{2}} J_{l+\frac{D-2}{2}}(w_{l,n}r) Y_{l+\frac{D}{2}}(\Omega) \]

\( J_{l+(D-2)/2} \) Bessel functions

\( Y_{l+D/2} \) hyperspherical harmonics

- Eigenvalues \( w_{l,n} (\geq 0) \) determined through BC

\[ J_{l+\frac{D-2}{2}}(w_{l,n}R) = 0, \quad \text{for Dirichlet BC} \]
\[ \frac{u}{R} J_{l+\frac{D-2}{2}}(w_{l,n} R) + w_{l,n} J'_{l+\frac{D-2}{2}}(w_{l,n} r) \mid_{r=R} = 0, \text{ for Robin BC} \]

- Now, \( \lambda_{l,n} = w_{l,n}^2 + m^2 \)

\[ \zeta(s) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} d_l(D) (w_{l,n}^2 + m^2)^{-s} \]

\( w_{l,n} (> 0) \) is defined as the \( n \)-th root of the \( l \)-th equation, \( d_l(D) = (2l + D - 2) \frac{(l+D-3)!}{l! (D-2)!} \)

- **Procedure:**

  - Contour integral on the complex plane

\[ \zeta(s) = \sum_{l=0}^{\infty} d_l(D) \int_{\gamma} \frac{dk}{2\pi i} \left( k^2 + m^2 \right)^{-s} \frac{\partial}{\partial k} \ln \Phi_{l+\frac{D-2}{2}}(kR) \]

\( \gamma \) runs counterclockwise and must enclose all the solutions [Ginzburg, Van Kampen, EE + I. Brevik]

- **Obtained:** [with Bordag, Kirsten, Leseduarte, Vassilievich,...]
  - Zeta functions
  - Determinants
  - Seeley [heat-kernel] coefficients
The Chowla-Selberg Formula (CS)


A. Selberg and S. Chowla, On Epstein’s Zeta function (I), Proc. Nat. Acad. Sci. 35 (1949) 371-74
1. This paper contains a short account of results whose detailed proofs will be published later.

We define the function $Z(s)$ by

$$Z(s) = \sum' (am^2 + bmn + cn^2)^{-s}$$

where $s = \sigma + it (\sigma \text{ and } t, \text{ real}), \sigma > 1$, and the summation is for all integers $m, n$ (each going from $-\infty$ to $+\infty$), while the dash indicates that $m = n = 0$ is excluded from the summation; further $a$ and $c$ are positive numbers while $b$ is real and subject to $4ac - b^2 = \Delta > 0$.

It is well known that the function $Z(s)$, defined for $\sigma > 1$ by (1), can be continued analytically over the whole $s$-plane, and satisfies a functional equation similar to the one satisfied by the Riemann Zeta Function. The function $Z(s)$, thus defined, is a meromorphic function with a simple pole at $s = 1$.

Deuring (Math. Ztschr., 37, 403–413 (1933)) obtained an important formula for $Z(s)$. Deuring's work led Heilbronn (Quart. J. Maths., Oxford, 5, 150 (1934)) to the proof of the following famous conjecture of Gauss on the class-number of binary quadratic forms with a negative fundamental discriminant: let $h(-\Delta)$ denote the number of classes of binary quadratic forms of negative fundamental discriminant $-\Delta = b^2 - 4ac$, then

$$h(-\Delta) \to \infty \quad \text{as} \quad \Delta \to \infty$$

(2)

Again using the ideas of Heilbronn and Deuring, Siegel proved that

$$h(-\Delta) > \Delta^{1/2 - \varepsilon} \quad [\Delta > \Delta_0(\varepsilon)]$$

(3)

which is a great advance on (2).

Our starting point is the formula:

$$Z(s) = 2\zeta(2s)a^{-s} + \frac{2^{2s}a^{-s-1}\sqrt{\pi}}{\Gamma(s)\Delta^{s-1/2}}(2s-1)\Gamma(s-1/2) + \mathcal{Q}(s)$$

(4)

where

$$\mathcal{Q}(s) = \frac{\pi^s\cdot 2^s + \zeta(1/2)}{a^{1/2}\Gamma(s)\Delta^{s-1/2}} \sum_{n=1}^{\infty} n^{s-1/2} \sigma_{1-2s}(n) \cos \left( \frac{n\pi b}{a} \right) \int_0^\infty \phi^{s-1/2} \exp \left\{ - \frac{\pi n \Delta^{1/2}}{2a} (\phi + \phi^{-1}) \right\} d\phi$$

(4)
The Chowla-Selberg Formula (CS)


On Epstein's Zeta-function

By Atle Selberg at Princeton (N. J.), and S. Chowla at State College (Pa.)

Introduction

This paper was written in the Spring of 1949, and a resumé appeared in the note: On Epstein's zeta Function (I), Proceedings of the National Academy of Sciences (U. S. A.), 35 (1949), 371—374.

Meanwhile, the following papers which have reference to the Proceedings paper, came to our attention:


§ 1.

We define the function $Z(s)$ by

$$
Z(s) = \sum' (am^2 + bmn + cn^2)^{-s}
$$

where $s = \sigma + it$ ($\sigma$ and $t$, real), $\sigma > 1$, and the summation is for all integers $m$, $n$ (each going from $-\infty$ to $+\infty$), while the dash indicates that $m = n = 0$ is excluded from the summation; further $a$ and $c$ are positive numbers while $b$ is real and subject to $4ac - b^2 = \Delta > 0$.

It is well-known that the function $Z(s)$, defined for $\sigma > 1$ by (1), can be continued analytically over the whole $s$-plane. The function $Z(s)$, thus defined, is a meromorphic function with a simple pole at $s = 1$.

In 1933, Deuring obtained an important formula for $Z(s)$. Deuring's work led Heilbronn to his proof of a famous conjecture of Gauss on the class number of binary quadratic forms with a negative fundamental discriminant. If $h(-\Delta)$ is the number of classes of binary quadratic forms of negative fundamental discriminant $-\Delta = b^2 - 4ac$, Gauss conjectured that

$$
h(-\Delta) \to \infty \quad \text{as} \quad \Delta \to \infty.
$$
Transforming this we get
\[
\sum_{j=1}^{n} \log A \left( \frac{b_j + i\sqrt{|d|}}{2a_j} \right) = 6 \left\{ \hbar \gamma + \log \prod_{j=1}^{n} a_j \right\} - \frac{3w}{\pi} \sqrt{|d|} \text{ } L'_d(1).
\]
Inserting here the value (obtained like (58))
\[
L'_d(1) = -\frac{\pi}{\sqrt{|d|}} \sum_{m=1}^{d} \left( \frac{d}{m} \right) \log \Gamma \left( \frac{m}{|d|} \right) + \frac{2\hbar \pi (\gamma + \log 2\pi)}{w \sqrt{|d|}}
\]
one gets, writing \( \tau_j = \frac{b_j + i\sqrt{|d|}}{2a_j} \),
\[
(2) \quad \prod_{j=1}^{n} A(\tau_j) = \frac{\prod_{j=1}^{n} a_j^{d_j}}{(2\pi |d|)^{\frac{1}{2}}} \left\{ \prod_{m=1}^{d} \Gamma \left( \frac{m}{|d|} \right) \right\}^{\frac{d}{m}}.
\]
Now let \( \tau = i \frac{K'}{K} \) be a number from the field \( k(\sqrt{d}) \), then from Lemma 3 we get
\[
\frac{A(\tau_j)}{A(\tau)} = \lambda_j,
\]
where \( \lambda_j \) are algebraic numbers. Thus (2) gives
\[
(3) \quad A(\tau) = \frac{\lambda'}{\pi^6} \left\{ \prod_{m=1}^{d} \Gamma \left( \frac{m}{|d|} \right) \right\}^{\frac{d}{m}},
\]
where \( \lambda' \) is an algebraic number. Finally we have from (48)
\[
A(\tau) = \left( \frac{2K}{\pi} \right)^{12} \cdot 2^{-s} (kk')^4 = \lambda'' \left( \frac{K}{\pi} \right)^{12},
\]
where \( \lambda'' \) is an algebraic number. This gives, when inserted in (3)
\[
(4) \quad K = \lambda''' \sqrt{\pi} \left\{ \prod_{m=1}^{d} \Gamma \left( \frac{m}{|d|} \right) \right\}^{\frac{d}{m}} \frac{\pi^6}{2^s},
\]
which is the desired expression for \( K \) in finite terms.

References (in the order of appearance in the text)

The Chowla-Selberg Formula (CS)


A. Selberg and S. Chowla, On Epstein’s Zeta function (I), Proc. Nat. Acad. Sci. 35 (1949) 371-74


K. Ramachandra, Some applications of Kronecker’s limit formulas, Ann. Math. 80 (1964) 104-148

A. Weil, Elliptic functions according to Eisenstein and Kronecker (Springer, Berlin, 1976)


B.H. Gross, On the periods of abelian integrals and a formula of Chowla and Selberg, Inv. Math. 45 (1978) 193-211

P. Deligne, Valeurs de fonctions L et periodes d’integrales, PSPM 33 (1979) 313-346
History

Lerch (1897):

\[
\sum_{\lambda=1}^{[D]} \left( \frac{D}{\lambda} \right) \log \Gamma \left( \frac{\lambda}{D} \right) = h \log |D| - \frac{h}{3} \log(2\pi) - \sum_{(a,b,c)} \log a
\]

+ \frac{2}{3} \sum_{(a,b,c)} \log [\theta'_1(0|\alpha)\theta'_1(0|\beta)]

\[D\] discriminant, \(\theta'_1 \sim \eta^3\)

\(h\) class number of binary quadratic forms \((a, b, c)\)
History

Lerch (1897):

\[
\sum_{\lambda=1}^{\lfloor D \rfloor} \left( \frac{D}{\lambda} \right) \log \Gamma \left( \frac{\lambda}{D} \right) = h \log |D| - \frac{h}{3} \log(2\pi) - \sum_{(a,b,c)} \log a
\]

\[
+ \frac{2}{3} \sum_{(a,b,c)} \log \left[ \theta'_1(0|\alpha)\theta'_1(0|\beta) \right]
\]

\(D\) discriminant, \(\theta'_1 \sim \eta^3\)

\(h\) class number of binary quadratic forms \((a, b, c)\)

**Eta evaluations** Dedekind eta function for \(\text{Im} (\tau) > 0\)

\[
\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad q := e^{2\pi i \tau}
\]

It is a 24-th root of the discriminant func \(\Delta(\tau)\) of an elliptic curve \(\mathbb{C}/L\) from a lattice \(L = \{a\tau + b \mid a, b \in \mathbb{Z}\}\)

\[
\Delta(\tau) = (2\pi)^{12} q \prod_{n=1}^{\infty} (1 - q^n)^{24}
\]
Properties & Recent Results

⇒ The C-S formula gives the value of a product of eta functions

⇒ If there is only one form in the class, it yields the value of a single eta function in terms of gamma functions

⇒ Long series of improvements: Kaneko (90), Nakajima and Taguchi (91), Williams et al. (95)

⇒ In the last years the C-S formula has been ‘broken’ to isolate the eta functions:
  Williams, van Poorten, Chapman, Hart

Extended CS Series Formulas (ECS)

Consider the zeta function \( \Re s > p/2, A > 0, \Re q > 0 \)

\[
\zeta_{A, \vec{c}, q}(s) = \sum_{\vec{n} \in \mathbb{Z}^p} \left[ \frac{1}{2} (\vec{n} + \vec{c})^T A (\vec{n} + \vec{c}) + q \right]^{-s} = \sum_{\vec{n} \in \mathbb{Z}^p} \left[ Q (\vec{n} + \vec{c}) + q \right]^{-s}
\]

prime: point \( \vec{n} = \vec{0} \) to be excluded from the sum

(inescapable condition when \( c_1 = \cdots = c_p = q = 0 \))

\[
Q (\vec{n} + \vec{c}) + q = Q(\vec{n}) + L(\vec{n}) + \bar{q}
\]

Case \( q \neq 0 (\Re q > 0) \)

\[
\zeta_{A, \vec{c}, q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s - p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2} \pi^s q^{-s/2+p/4}}{\sqrt{\det A} \Gamma(s)} \times \sum_{\vec{m} \in \mathbb{Z}_{1/2}^p} \cos(2\pi \vec{m} \cdot \vec{c}) (\vec{m}^T A^{-1} \vec{m})^{s/2-p/4} K_{p/2-s} \left( 2\pi \sqrt{2q \vec{m}^T A^{-1} \vec{m}} \right)
\]

Pole: \( s = p/2 \)  
Residue: 

\[
\text{Res}_{s=p/2} \zeta_{A, \vec{c}, q}(s) = \frac{(2\pi)^{p/2}}{\Gamma(p/2)} (\det A)^{-1/2}
\]

EE JPA34 (2001) 3025  [ECS1]
Gives (analytic cont of) multidimensional zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane

Exhibits singularities (simple poles) of the meromorphic continuation—with the corresponding residua—explicitly

Only condition on matrix $A$: corresponds to (non negative) quadratic form, $Q$. Vector $\vec{c}$ arbitrary, while $q$ is (to start) a non-neg constant

$K_\nu$ modified Bessel function of the second kind and the subindex $1/2$ in $\mathbb{Z}^{p}_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbb{Z}^p$ participate in the sum. E.g., if we take an $\vec{m} \in \mathbb{Z}^p$ we must then exclude $-\vec{m}$

[simple criterion: one may select those vectors in $\mathbb{Z}^p \setminus \{0\}$ whose first non-zero component is positive]

Case $c_1 = \cdots = c_p = q = 0$ [true extens of CS, diag subcase]

$$\zeta_{A_p}(s) = \frac{2^{1+s}}{\Gamma(s)} \sum_{j=0}^{p-1} (\det A_j)^{-1/2} \left[ \pi^{j/2} a_{p-j}^{j/2-s} \Gamma \left( s - \frac{j}{2} \right) \zeta_R(2s-j) + 4\pi^{s} a_{p-j}^{\frac{j-s}{2}} \sum_{n=1}^{\infty} \sum_{\vec{m}_j \in \mathbb{Z}^j} n^{j/2-s} \left( \vec{m}_j^t A_j^{-1} \vec{m}_j \right)^{s/2-j/4} K_{j/2-s} \left( 2\pi n \sqrt{a_{p-j} \vec{m}_j^t A_j^{-1} \vec{m}_j} \right) \right]$$

[ECS3d]
Definition of Determinant

\[ H \quad \Psi \text{DO operator} \quad \{ \varphi_i, \lambda_i \} \quad \text{spectral decomposition} \]

\[ \prod_{i \in I} \lambda_i \quad \ln \prod_{i \in I} \lambda_i = \sum_{i \in I} \ln \lambda_i \]

Riemann zeta func: \( \zeta(s) = \sum_{n=1}^{\infty} n^{-s}, \quad \text{Re} \ s > 1 \) (& analytic cont)

Definition: zeta function of \( H \)

\[ \zeta_H(s) = \sum_{i \in I} \lambda_i^{-s} = \text{tr} \ H^{-s} \]

As Mellin transform:

\[ \zeta_H(s) = \frac{1}{\Gamma(s)} \int_{0}^{\infty} dt \ t^{s-1} \text{tr} \ e^{-tH}, \quad \text{Re}s > s_0 \]

Derivative:

\[ \zeta'_H(0) = -\sum_{i \in I} \ln \lambda_i \]

Determinant: Ray & Singer, ’67

\[ \text{det}_{\zeta} H = \exp \left[ -\zeta'_H(0) \right] \]

Weierstrass def: subtract leading behavior of \( \lambda_i \) in \( i \), as \( i \to \infty \), until series \( \sum_{i \in I} \ln \lambda_i \) converges \( \implies \) non-local counterterms !!

C. Soulé et al, Lectures on Arakelov Geometry, CUP 1992; A. Voros,...
Multipl or N-Comm Anomaly, or Defect

Given $A, B,$ and $AB \psi$ DOs, even if $\zeta_A, \zeta_B,$ and $\zeta_{AB}$ exist, it turns out that, in general,

$$\det_\zeta(AB) \neq \det_\zeta A \det_\zeta B$$

The multiplicative (or noncommutative) anomaly (defect) is defined as

$$\delta(A, B) = \ln \left[ \frac{\det_\zeta(AB)}{\det_\zeta A \det_\zeta B} \right] = -\zeta_{AB}'(0) + \zeta_A'(0) + \zeta_B'(0)$$

Wodzicki formula

$$\delta(A, B) = \text{res} \left\{ \left[ \ln \sigma(A, B) \right]^2 \right\} \frac{2 \ord A \ord B (\ord A + \ord B)}{2 \ord A \ord B (\ord A + \ord B)}$$

where $\sigma(A, B) = A^{\ord B} B^{\ord A}$
Consequences of the Multipl Anomaly

In the path integral formulation

$$\int [d\Phi] \exp \left\{ - \int d^D x \left[ \Phi^\dagger(x)( \ldots ) \Phi(x) + \cdots \right] \right\}$$

Gaussian integration:

$$\rightarrow \quad \det(\quad )^\pm$$

$$\begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix} \rightarrow \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\det(AB) \quad \text{or} \quad \det A \cdot \det B$$

In a situation where a superselection rule exists, $AB$ has no sense (much less its determinant):

$$\Rightarrow \quad \det A \cdot \det B$$

But if diagonal form obtained after change of basis (diag. process), the preserved quantity is:

$$\Rightarrow \quad \det(AB)$$
Ten Physical Applications of Spectral Zeta Functions

Second Edition
Zero point energy

**QFT** vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left( n + \frac{1}{2} \right) \lambda_n \ a_n \ a_n^\dagger$$

$$\langle 0 | H | 0 \rangle = \frac{\hbar c}{2} \sum_n \lambda_n = \frac{1}{2} \text{tr} \ H$$

gives $\infty$ physical meaning?

Regularization + Renormalization (cut-off, dim, $\zeta$)

Even then: Has the final value real sense?
Vacuum Fluct & the Equival Principle

The main issue: energy ALWAYS gravitates therefore the energy density of the vacuum appears on the rhs of Einstein’s equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

Equivalent to a cosmological const
\[ \Lambda = 8\pi G \mathcal{E}, \quad \rho_c = \frac{3H^2}{8\pi G} \]

Observations: M. Tegmark et al. [SDSS Collab.] PRD 2004
\[ \Lambda = (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \sim 4.32 \times 10^{-9} \text{ erg/cm}^3 \]

Question: how finite Casimir energy of pair of plates couples to gravity?

Two ways to proceed. Gauge-invariant procedure:
energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force
\[ \rightarrow \text{ Leads to complicated model-dependent calculations} \]

Alternative: find a physically natural coordinate system, more realistic than another: Fermi coord system [Marzlin ’94]

Calculations done also in Rindler coord (uniform accel obs)
The accelerating Universe

Dark Energy: effects on the expansion rate of Universe, 3 approaches:

— **Standard candles**: measure luminosity distance as function of redshift
— **Standard rulers**: angular diam distance & expansion rate as f of redsh
— **Growth of fluctuations**: generated at origin of U & amplified by inflation

Both angular diameter and luminosity distances are integrals of (inverse) expansion rate: encode effects of DE

To get competitive constraints on DE need see changes in \( H(z) \) at 1% would give statistical errors in DE EoS of \( O(10\%) \)

— calibrate the ruler accurately over most of the age of the universe
— measure the ruler over much of the volume of the universe
— make ultra-precise measurements of the ruler

On large scales or early times the perturbative treatment is valid: calculations are perfectly under control

Length scales from physics of the early universe are imprinted on the distribution of mass and radiation: they form time-independent rulers (M White, Berkeley)
Evidence for the acceleration of the Universe expansion:

- distant supernovae

- the cosmic microwave background

- baryon acoustic oscillations BAO
  Martin White mwhite/bao/

- the galaxy distribution
  AG Sanchez et al. ’12, Gaztanaga et al.

- correlations of galaxy distributions
  R. Scranton et al. ’03,

Multiple sets of evidence: no systematics affect the conclusion that $\dot{a} > 0$, a scale factor of the Universe
On 17 Mar 2014, John Kovac announced that, by looking at the CMB signal, BICEP2 had found the imprint of gravitational waves from the Big Bang:

- polarization of the CMB
- curly patterns known as B modes
- generated by gravitational waves during inflation
BICEP2 I: DETECTION OF B-mode POLARIZATION AT DEGREE ANGULAR SCALES


to be submitted to a journal TBD

ABSTRACT

We report results from the BICEP2 experiment, a Cosmic Microwave Background (CMB) polarimeter specifically designed to search for the signal of inflationary gravitational waves in the B-mode power spectrum around $\ell \sim 80$. The telescope comprised a 26 cm aperture all-cold refracting optical system equipped with a focal plane of 512 antenna coupled transition edge sensor (TES) 150 GHz bolometers each with temperature sensitivity of $\approx 300 \mu K_{\text{cmb}} \sqrt{s}$. BICEP2 observed from the South Pole for three seasons from 2010 to 2012. A low-foreground region of sky with an effective area of 380 square degrees was observed to a depth of 87 nK-degrees in Stokes $Q$ and $U$. In this paper we describe the observations, data reduction, maps, simulations and results. We find an excess of $B$-mode power over the base lensed-$\Lambda$CDM expectation in the range $30 < \ell < 150$, inconsistent with the null hypothesis at a significance of $>5\sigma$. Through jackknife tests and simulations based on detailed calibration measurements we show that systematic contamination is much smaller than the observed excess. We also estimate potential foreground signals and find that available models predict these to be considerably smaller than the observed signal. These foreground models possess no significant cross-correlation with our maps. Additionally, cross-correlating BICEP2 against 100 GHz maps from the BICEP1 experiment, the excess signal is confirmed with $3\sigma$ significance and its spectral index is found to be consistent with that of the CMB, disfavoring synchrotron or dust at $2.3\sigma$ and $2.2\sigma$, respectively. The observed $B$-mode power spectrum is well-fit by a lensed-$\Lambda$CDM + tensor theoretical model with tensor/scalar ratio $r = 0.20^{+0.07}_{-0.05}$, with $r = 0$ disfavored at 7.0$\sigma$. Subtracting the best available estimate for foreground dust modifies the likelihood slightly so that $r = 0$ is disfavored at 5.9$\sigma$.

Subject headings: cosmic background radiation — cosmology: observations — gravitational waves — inflation — polarization

1. INTRODUCTION

The discovery of the Cosmic Microwave Background (CMB) by Penzias & Wilson (1965) confirmed the hot big bang paradigm and established the CMB as a central tool for the study of cosmology. In recent years, observations of its temperature anisotropies have helped establish and refine the “standard” cosmological model now known as $\Lambda$CDM, under which our universe is understood to be spatially flat, dominated by cold dark matter, and with a cosmological constant ($\Lambda$) driving accelerated expansion at late times. CMB temperature measurements have now reached remarkable precision over angular scales ranging from the whole sky to arcminute resolution, producing results in striking concordance with predictions of $\Lambda$CDM and constraining its key parameters to sub-percent precision (e.g. Bennett et al. 2013; Hin-
Important about the finding

Prediction about B modes from inflation relies not just on the phenomenon of gravitational waves but on the quantization of gravity

Indeed, inflation assumes that everything started out as quantum fluctuations
* of a scalar field?
* of spacetime itself?

that then got amplified by inflation

B-mode polarization signal is twice as high as that from PLANCK!
* Planck data comes from temperature maps of the CMB, not from a direct polarization measure
* Team thought over the past three years about every possible systematic explanation that could have falsified the signal
* Done the most extensive systematic analysis that the PIs have ever been involved in by far
* Compared the BICEP2 signal with BICEP1, very different detectors
Could Quantum Gravity ever be detected?

Freeman Dyson at Singapore conference Aug 2013, celebrating his 90th Birthday:

* physically impossible

* to detect individual graviton, mirrors need be so heavy they would collapse to form a Black Hole


• gravitational waves stretch spacetime along one direction while contracting it along the other

• would affect how electromagnetic radiation travels through space, causing it to be polarized

• “…measurement of polarization of CMB due to gravitational waves from Inflation would firmly establish the quantization of gravity”

• gravitational waves can be traced back to individual gravitons, “…what we finally hope to detect is the signal from a single graviton amplified by the Universe expansion”

And Alan Guth “…the expected gravitational waves arise from the quantum properties of the gravitational field itself, and are not merely a by-product of the gravitational field interacting with the quantum fluctuations of other fields”
Using cosmology to establish the quantization of gravity

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While many aspects of general relativity have been tested, and general principles of quantum dynamics demand its quantization, there is no direct evidence for that. It has been argued that development of detectors sensitive to individual gravitons is unlikely, and perhaps impossible. We argue here, however, that measurement of polarization of the cosmic microwave background due to a long wavelength stochastic background of gravitational waves from inflation in the early Universe would firmly establish the quantization of gravity.

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Direct detection of gravitational waves is an exciting frontier of experimental physics, with positive results anticipated soon (e.g., Ref. [1]). The anticipated signals are classical disturbances, comprised of coherent superpositions of many individual quanta. The possibility of detecting individual gravitons is far more daunting. Indeed, recently Freeman, Dyson, and colleagues [2] have cogently estimated that it may in fact be infinitely more daunting, namely, that it is likely to be impossible, to physically realize a detector sensitive to individual gravitons without having the detector collapse into a black hole in the process.

If that is the case, one might wonder whether we can ever hope to detect a gravitational wave. The answer is yes, if we manage to do so at a frequency and magnitude, even within the inflationary scenario, depends on the rate of expansion during inflation. If the background is not observed, it could simply indicate a relatively small rate of expansion. But detection is a plausible possibility, as we describe, and major efforts are underway to achieve it. We should also emphasize that no essentially new predictions or calculations are presented here; we are merely bringing to the foreground an implication of existing results that seems particularly noteworthy.

The fact that quantization associated with gravity appears to be an essential feature of a gravitational wave background generated by inflation is suggested by existing calculations, including the following. A period
“Good Morning, Inflation! Hello, Multiverse!” (Max Tegmark)

Hawking tells Turok: “You owe me!” But Turok's sticking with his cyclic universe

“I believe that if both Planck and the new results agree, then together they would give substantial evidence against inflation!” (Neil Turok)

“There's a significant possibility of some of the polarization signal in E and B modes not being cosmological. If genuine, then the spectrum is a bit strange and may indicate something added to the normal inflationary recipe” (Peter Coles)

“What is strange is that all the blue dots lie so close to zero. Statistically speaking this is extremely unlikely and it may suggest that the noise levels have been over-estimated” (Hans Kristian Eriksen)

Reserve judgement until this is confirmed by other experiments: STP, POLARBEAR, ABS, ACTPOL, CLASS, EBEX, SPIDER, PIPER, and PLANCK