Repulsive Casimir Forces from Additional Dimensions, and GR Alternative Cosmologies

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Outline

On Einstein’s Cosmological Constant (in the Year of Galileo and Astronomy)
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- CE and Accelerated Expansion (Dark Energy)
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The Sign of the Casimir Force
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- Repulsion from Higher Dimensions and BCs
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- Phase Space of Hořava-Lifshitz Cosmologies
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With THANKS to:
S. Carloni, G. Cognola, J. Haro, S.D. Odintsov, A. Saharian, P.J. Silva, S. Zerbini, ...
On Einstein’s Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cosmological constant

- For cosmologists and general relativists: a great mistake (Einstein)

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -(8\pi G/c^4)T_{\mu\nu} + \lambda g_{\mu\nu} \]

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc $\Lambda$ is indeed a peculiar quantity

  - has to do with cosmology Einstein’s eqs., FRW universe
  - has to do with the local structure of elementary particle physics

stress-energy density $\mu$ of the vacuum

\[ L_{cc} = \int d^4x \sqrt{-g} \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g} \lambda \]

In other words: two contributions on the same footing (Zel’dovich, 68)

\[ \frac{\Lambda c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar c}{2} \sum_i \omega_i \]
Einstein Equations (1915-17): \[ G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu} \]

Geometry = Energy-Matter

\( G_{\mu\nu} \) linear combination of the metric \( g_{\mu\nu} \) and 1st & 2nd derivatives

\( T_{\mu\nu} \) energy-momentum tensor

Schwarzschild solution (1916)
\[
ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2
\]

Friedmann-Lemaître-Robertson-Walker (1935-36) sol (A. Friedmann 1922)
\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right)
\]

gen fam: homog + isotrop, \( k \) par \( \pm 1, 0 \)

Hubble ea 1923-29, Keeler Slipher Campbell 1918

One field eq looks like Newtonian eq for the gravit pot:
\[
\nabla^2 \phi = 4\pi G (\rho + 3p/c^2)
\]

density & pressure contribute to the gravit pot
\[
\lambda = 8\pi G \rho_{vac}, \quad p_{vac} = -\rho_{vac} c^2
\]

From the FRW metric and Einstein Eqs, an "equation of motion" of the universe
\[
H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\lambda}{3} - \frac{k}{a^2}
\]
From GR to Cosmology

With the definitions:

\[ \Omega_m \equiv \frac{8\pi G}{3H^2} \rho_m, \quad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \quad \Omega_k \equiv -\frac{k}{H^2} \]

The equation of motion becomes

\[ \left( \frac{da}{dt} \right)^2 = H_0^2 \left[ \frac{\Omega_m(0)}{a} + a^2 \Omega_\lambda(0) + \Omega_k \right] \]

*(the superscript (o) represents quantities measured at the present time)*

In other terms, Friedmann equation in Cosmology:

\[ \frac{\dot{a}^2}{a^2} = H_0^2 \left[ \Omega_R \left( \frac{a_0}{a} \right)^4 + \Omega_{NR} \left( \frac{a_0}{a} \right)^3 + \Omega_k \left( \frac{a_0}{a} \right)^2 + \Omega_\lambda \right] \]

- \( \Omega_R \) relativistic matter \( (p_R = \frac{1}{3}\rho_R; \quad \rho_R \propto a^{-4}) \)
- \( \Omega_{NR} \) nonrelativistic matter \( (p_{NR} = 0; \quad \rho_{NR} \propto a^{-3}) \)
- \( \Omega_\lambda \) cosmological constant \( (p_\lambda = -\rho_\lambda; \quad \rho_\lambda = \text{const}) \)

\[ \Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda \quad \text{total energy density} \quad \text{(cosmic triangle)} \]
Zero point energy

QFT vacuum to vacuum transition: \( \langle 0 | H | 0 \rangle \)
Zero point energy

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Spectrum, normal ordering (harm oscill): 

$$H = \left( n + \frac{1}{2} \right) \lambda_n \ a_n \ a_n^\dagger$$
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\langle 0|H|0 \rangle = \frac{\hbar c}{2} \sum_n \lambda_n = \frac{1}{2} \text{tr} \ H = \frac{1}{2} \zeta_H^{-1}(-1)
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Even then: Has the final value real sense?
The Casimir Effect
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BC e.g. periodic
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BC e.g. periodic
⇒ all kind of fields

vacuum

Casimir Effect
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⇒ all kind of fields
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Universal process:
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Universal process:
- Sonoluminiscence  (Schwinger)
- Cond. matter  (wetting \(^3\)He alc.)
- Optical cavities
- Direct experim. confirmation
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Van der Waals, Lifschitz theory

- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant ⇐
The main issue: energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor

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\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu}) \]

It affects cosmology: \( \tilde{T}_{\mu\nu} \) excitations above the vacuum
Quantum Vacuum Fluct’s & the CC

The main issue: S.A. Fulling et. al., hep-th/070209v2

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Recent observations: M. Tegmark et al. [SDSS Collab.] PRD 2004

\[ \Lambda = (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \sim 4.32 \times 10^{-9} \text{ erg/cm}^3 \]
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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich ’68
Relativistic field: collection of harmonic oscillation's (scalar field)

\[ E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \quad \omega = k^2 + \frac{m^2}{\hbar^2}, \quad k = \frac{2\pi}{\lambda} \]
CC Problem

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- Evaluating in a box and putting a cut-off at maximum \( k_{max} \) corresponding to QFT physics (e.g., Planck energy)

\[ \rho \sim \frac{\hbar k_{Planck}^4}{16\pi^2} \sim 10^{123} \rho_{obs} \]

kind of (thick!) aether R Caldwell, S Carroll but C Gómez, G Dvali
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What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

\[ \implies \text{kind of cosmological Casimir effect} \]
A. Assuming one is able to prove that the ground value of the cc is zero (Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998) we will be left with this incremental value coming from the topology or BCs.
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Cosmo-Topol Casimir Eff’t & Alternat’s

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B. Recent alternatives:
   (i) L. Faddeev 0911.0282 (Adler ’82)
   Newton const in E-H Lag has dim of mass \(\rightarrow\) non-renormalizability
   Describe gravity by vector field (as Higgs mechanism)
   (ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in EuIR sector and a mechanism to relax the cc

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The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem

2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds

   Bulk Casimir effect (effective potential) for a conformal or massive scalar field

   Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)

   Consistent with observational data even for relatively large extra dimension

   Previous work:
   → flat space brane
   → bulk conformal scalar field
   → conclusion: no CE

   We used zeta regularization at full power, with positive results!

EE, S Nojiri, SD Odintsov, S Ogushi, Phys Rev D67 (2003) 063515 Casimir effect in de Sitter and Anti-de Sitter braneworlds EE, SD Odintsov, AA Saharian 0902.0717 Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons
The Sign of the Casimir Force

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Casimir calculation: attractive force

Boyer got repulsion [TH, Phys Rev, 174 (1968)] for a spherical shell. It is a special case requiring stringent material properties of the sphere and a perfect geometry and BC.
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a mirror pair of dielectric bodies always attract each other
CP Bachas, J Phys A40, 9089 (2007) from a general property of
Euclidean QFT ‘reflection positivity’ (Osterwalder - Schrader 73, 75):
∃ of positive Hilbert space and self-adjoint non-negative Hamiltonian
E.g. \( \exists \) correlation inequality: \( \langle f \Theta(f) \rangle > 0 \)

\( \Theta \) reflection with respect to a 3-dim hyperplane in \( \mathbb{R}^4 \)

the action of \( \Theta \) on \( f \) is anti-unitary \( \Theta(cf) = c^{*} \Theta(f) \)
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\Theta \textbf{reflection} with respect to a 3-dim hyperplane in $\mathbb{R}^4$

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The existence of the reflection operator $\Theta$ is a consequence of \textbf{unitarity} only, and makes no assumptions about the discrete $C, P, T$ symmetries
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Boyer’s result does not contradict the theorem, since cutting an elastic

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  - mirror probes in a Fermi sea (chemical-potential term), eg when electron-gas fluctuations become important
  - periodic BCs for fermions
  - Robin BCs in general \( \Leftarrow \)
Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, $\Sigma$ compact internal space.
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Most general case: constants in the BCs different for the two plates. It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces.
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Genuinely appear in:
\[ \Rightarrow \] vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83],
\[ \Rightarrow \] spinor and gauge field theories,
\[ \Rightarrow \] quantum gravity and supergravity [Luckock ea 91]
Can be made conformally invariant, purely-Neumann conditions cannot
\[ \Rightarrow \] needed for conformally invariant theories with BC, to preserve cf invar
Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1 - 1, 1)} \times \Sigma$, $\Sigma$ compact internal space.

Most general case: constants in the BCs different for the two plates. It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces.

Robin type BCs are an extension of Dirichlet and Neumann’s → most suitable to describe physically realistic situations.

Genuinely appear in: → vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83], → spinor and gauge field theories, → quantum gravity and supergravity [Luckock ea 91]

Can be made conformally invariant, purely-Neumann conditions cannot → needed for conformally invariant theories with BC, to preserve cf invar

Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein’s entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]
Robin BCs can model the finite penetration of the field through the boundary: the ‘skin-depth’ param related to Robin coefficient [Mostep ea 85, Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]
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For arbitrary internal space, interaction part of the Casimir energy given by

\[ \Delta E_{[a_1, a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_{\beta} \int_{m_\beta}^{\infty} dx \left( x^2 - m_\beta^2 \right)^{D_1/2-1} \]

\[ \times \ln \left[ 1 - \frac{(\beta_1 x + 1)(\beta_2 x + 1)}{(\beta_1 x - 1)(\beta_2 x - 1)} e^{-2ax} \right] \quad (*) \]
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\]

For Dirichlet and Neumann BCs on both plates this leads to

\[
\Delta E_{[a_1,a_2]}^{(J,J)} = -\frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^{\infty} \frac{f((D_1+1)/2(2n a m_\beta)}{n^{D_1+1}}
\]

with \( f_\nu(z) = z^\nu K_\nu(z) \) \( \rightarrow \) energy always negative
For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

\[
\Delta E_{[a_1, a_2]}^{(D,N)} = \frac{(4\pi)^{-D_1/2} a}{\Gamma(D_1/2 + 1)} \sum_{\beta} \int_{m_\beta}^{\infty} dx \frac{(x^2 - m^2)^{D_1/2}}{e^{2ax} + 1}
\]

\[
= \frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_{\beta} \sum_{n=1}^{\infty} \frac{f_{(D_1+1)/2}(2n a m_\beta)}{(-1)^{n+1} n^{D_1+1}}
\]

positive for all values of the inter-plate distance.
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In the case of a conformally coupled massless field on the background of a spacetime conformally related to the one described by the line element

\[
ds^2 = g_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu - \gamma_{il} dX^i dX^l
\]

\(\eta_{\mu\nu} = \text{diag}(1, -1, \ldots, -1)\) metric of \((D_1 + 1)\)-dim Minkowski st and \(X^i\) coordinates of \(\Sigma\), with the conformal factor \(\Omega^2(x^{D_1})\). Interaction part of Casimir energy is given (*), with coeffs \(\beta_j\) related to coeffs of the Robin BCs

\[
(1 + \bar{\beta}_j n^M \nabla_M) \varphi(x) = [1 + (-1)^{j-1} \Omega_j^{-1} \bar{\beta}_j \partial_{D_1}] \varphi(x) = 0, \quad \Omega_j = \Omega(x_j^{D_1})
\]

\& conformal factor \(\beta_j = \left[ \Omega_j + (-1)^j \frac{D_1-1}{2\Omega_j} \bar{\beta}_j \Omega_j' \right]^{-1} \bar{\beta}_j, \quad \Omega_j' = \Omega_j'(x_j^{D_1})\)
In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are \( \beta^{-1}_j = (-1)^j \frac{c_j}{2} - 2D \zeta / r_D \), \( c_1, c_2 \) mass parameters in the surface action of the scalar field for the left and right branes, respectively. The vacuum energy can have a minimum, for the stable equilibrium point. Can be used in braneworld models for the stabilization of the radion field.
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We have considered a **piston-like geometry**, introducing a third plate (then this plate is sent to infinity): Casimir force

\[
P = - \frac{2(4\pi)^{-D_1/2}}{V \Sigma \Gamma(D_1/2) a^{D_1+1}} \sum \int_{am_\beta}^{\infty} dx \frac{x^2(x^2 - a^2 m^2) a^{D_1/2-1}}{(b_1 x-1)(b_2 x-1)(b_1 x+1)(b_2 x+1) e^{2x} - 1}
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\]

With independence of the geometry of the internal space, the force is attractive for Dirichlet or Neumann boundary conditions on both plates.

\[
P^{(J,J)} = - \frac{2(4\pi)^{-D_1/2}}{V_N \Gamma(D_1/2)} \sum_{j} \int_{0}^{\infty} dx x^2 \frac{(x^2 - m_j^2)^{D_1/2-1}}{e^{2ax} - 1}
= 2a^{-D_1-1} \frac{1}{(8\pi)^{(D_1+1)/2} V_N} \sum_{j} \sum_{n=1}^{\infty} \frac{1}{n^{D_1+1}} \left[ f(D_1+1)/2(2n m_j) - f(D_1+3)/2(2n m_j) \right]
\]

\( J = D, N \), and repulsive for Dirichlet BC on one plate and Neumann on the other, a monotonic function of the distance.
For general Robin BCs the Casimir force can be either attractive (negative $P$) or repulsive (positive $P$), depending on the Robin coefficients and distance between plates. [also L P Teo arXiv:0907.2989 arXiv:0907.5258]
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Remarks: (i) This property could be used in the proposal of a Casimir experiment with the purpose to carry out an explicit detailed observation of ‘large’ extra dimensions as allowed by some models of particle physics.
(ii) Possible laboratory verification (Robin BCs model skin depth of material)
Gravity Eqs as Eqs of State: $f(R)$ Case

The cosmological constant as an “integration constant”

T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...

Unimodular Gravity 

Also I Shapiro, J Solà,... cc RG flow
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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)
Jacobson’s argument: basic thermodynamic relation

\[ \delta Q = T \delta S \]

- entropy proportional to variation of the horizon area: \( \delta S = \eta \delta A \)
- local temperature \( T \) defined as Unruh temp: \( T = \hbar k / 2\pi \)
- functional dependence of \( S \) wrt energy and size of system
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Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

\[ S = -2\pi \int_{\Sigma} E_{pqrs}^{pr} \epsilon_{pq} \epsilon_{rs}, \quad \delta S = \delta (\eta_e A) \]

\( \eta_e \) is a function of the metric and its deriv’s to a given order

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Case of \( f(R) \) gravities: \( L = f(R, \nabla^n R) \)
Also the concept of an effective Newton constant for graviton exchange (effective propagator)

\[
\frac{1}{8\pi G_{\text{eff}}} = E_R^{pqrs} \varepsilon_{pq} \varepsilon_{rs} = \frac{\partial f}{\partial R} (g^{pr} g^{qs} - g^{qr} g^{ps}) \varepsilon_{pq} \varepsilon_{rs}
\]

\[
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Final result, for \( f(R) \) gravities:

the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)
Jacobson’s argument non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics

EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2
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S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities

T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];

Hořava made a proposal for an ultraviolet completion of GR: Hořava-Lifshitz gravity [arXiv:0901.3775]
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Consistency status of the theory not yet completely clear, nor its low energy limit, and how GR is recovered at the different regimes.
Dynamical system approach → properties of cosmological models based on the Hořava-Lifshitz (HL) gravity
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The cosmological phase space of the HL model is characterized
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We show that the cosmological models generated from HL gravity without the detailed balance assumption have the potential to describe the transition between the Friedmann and the dark energy eras.
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Grazie Mille!
"Cosmology, the Quantum Vacuum, and Zeta Functions"

A workshop with a celebration of Emilio Elizalde's sixtieth birthday
ICE/CSIC, Universitat Autònoma de Barcelona, 8-10th March, 2010

Speakers:
I. Aref'eva (Steklov Math Inst, Moscow)
M. Asorey (Zaragoza Univ)
I. Brevik (Trondheim Univ)
I.L. Buchbinder (TSPU, Tomsk)
S. Capozziella (Naples Univ)
S. Carloni (IEEC, Barcelona)
M. Chaichian, A. Tureanu (Helsinki Univ)
G. Cognola, S. Zerbini (Trento Univ)
V. Faraoni (Bishops Univ, Canada)
A. Feinstein (Univ Basque Country)
E. Gaztañaga (ICE/CSIC-IEEC)
K. Ghoroku (Fukuoka Univ)
J. Gomis (UB, Barcelona)
A. Gonzalez-Arroyo (UAM, Madrid)
P. Gonzalez-Diaz (IAEFT, CSIC, Madrid)
T. Inagaki (Hiroshima Univ)
K.I. Maeda (Waseda Univ, Tokyo)
A. Maroto (Complutense Univ, Madrid)
G. Montani (ICRA, Roma)
S. Nojiri (Nagoya Univ)
V.V. Obukhov, K.E. Osetrin (TSPU, Tomsk)
S.D. Odintsov (ICREA, ICE/CSIC-IEEC)
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