Vacuum Fluctuations: Regularization & Cosmological Issues

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Outline

- On Einstein’s Cosmological Constant: a Historical Perspective
- Quantum Vacuum Fluctuations: the Casimir Effect
- Vacuum Fluctuations and the Equivalence Principle
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs
- CE and Accelerated Expansion (Dark Energy): a Cosmo-Topological Casimir Effect?
- Gravity Equations as Equations of State
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Led to Big Bang theory (Fred Hoyle, BBC radio’s Third Programme, 18:30 GMT, 28 March 1949)
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On a different level: Richard Dawkins, Hoyle’s fallacy evolutionary biology $\longleftrightarrow$ intelligent design
Trying to solve these puzzles!

The $\lambda$ is indeed a peculiar quantity.
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The cosmological constant $\Lambda$ is indeed a peculiar quantity that has to do with cosmology, Einstein's equations, and the FRW universe.
Trying to solve these puzzles!

- The $\lambda$ is indeed a peculiar quantity
  - has to do with cosmology Einstein’s eqs., FRW universe
  - has to do with the local structure of elementary particle physics

stress-energy density $\mu$ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g} \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g} \lambda$$
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In other words: two contributions on the same footing

[Pauli 20’s, Zel’dovich ’68]

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\frac{\Lambda c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar c}{2} \sum_i \omega_i
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For elementary particle physicists: a great embarrassment

no way to get rid off

Coleman, Hawking, Weinberg, Polchinski, ... ’88–’89
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THE COSMOLOGICAL CONSTANT PROBLEM
Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$
Zero point energy

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Spectrum, normal ordering (harm oscill):

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H = \left( n + \frac{1}{2} \right) \lambda_n \ a_n \ a_n^\dagger
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Regularization \((\text{cut-off, dim, } \zeta)\) + Renormalization

Even then: Has the final value real sense?
The Casimir Effect
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BC e.g. periodic
⇒ all kind of fields

Casimir Effect

vacuum

Φ
The Casimir Effect

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Universal process:
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Universal process:
- Sonoluminiscence (Schwinger)
- Cond. matter (wetting $^3$He alc.)
- Optical cavities
- Direct experim. confirmation
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Van der Waals, Lifschitz theory

- Dynamical CE  ⇐
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant  ⇐
The standard approach
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Casimir force: calculated by computing change in zero point energy of the em field
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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi} \text{Im} \int d\omega \text{ Tr } \int d^3x [G(x, x, \omega + i\epsilon) - G_0(x, x, \omega + i\epsilon)]$$
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$\mathcal{G}$ full Greens function for the fluctuating field
$\mathcal{G}_0$ free Greens function

Trace is over spin
\( E_C = \langle \text{plates} \rangle - \langle \text{no plates} \rangle \)
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change in the density of states due to the background
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⇒ A restatement of the Casimir sum over shifts in zero-point energies

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\[ \text{“Experimental confirmation of the Casimir effect doesn’t establish the reality of zero point fluct’s better than say the Lamb shift does” [R Jaffe e a] } \]
The main issue: S.A. Fulling et. al., hep-th/070209

energy **ALWAYS gravitates** therefore the energy density of the vacuum appears on the rhs of Einstein’s equations:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu}) \]
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Question: how finite Casimir energy of pair of plates couples to gravity?

Two ways to proceed. *Gauge-invariant* procedure:

energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force

\[ \rightarrow \text{ Leads to complicated model-dependent calculations} \]
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- **Observations:** M. Tegmark et al. [SDSS Collab.] PRD 2004  
  \[ \Lambda = (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \sim 4.32 \times 10^{-9} \text{ erg/cm}^3 \]

- **Question:** how finite Casimir energy of pair of plates **couples** to gravity?

- **Two ways** to proceed. **Gauge-invariant** procedure:  
  energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force  
  \( \rightarrow \) Leads to **complicated** model-dependent calculations

- Alternative: find a **physically natural** coordinate system, more realistic than another: **Fermi** coord system [Marzlin '94]
Vacuum Fluct & the Equival Principle

- The main issue: energy ALWAYS gravitates therefore the energy density of the vacuum appears on the rhs of Einstein’s equations:

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- Calculations done also in Rindler coord (uniform accel obs)
Relativistic field: collection of harmonic oscill’s (scalar field)

\[ E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \quad \omega = k^2 + \frac{m^2}{\hbar^2}, \quad k = \frac{2\pi}{\lambda} \]
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Evaluating in a box and putting a cut-off at maximum \( k_{\text{max}} \) corresponding to QFT physics (e.g., Planck energy)

\[ \frac{M_P}{M_{\text{ew}}} \sim 10^{16}, \quad \frac{M_P}{M_{\text{cc}}} \sim 10^{31}, \quad \rho \sim \frac{\hbar k_{\text{Planck}}^4}{16\pi^2} \sim 10^{123} \rho_{\text{obs}} \]

a thick aether! Caldwell, Carroll but Gómez, Dvali: species ↓ 10^{30}
**CC Problem**

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What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

\[ \implies \text{kind of cosmological Casimir effect} \]
A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998] → left with this incremental value coming from the topology or BCs
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B. Other alternatives: (i) L Faddeev 0911.0282 (Adler ’82) Newton const in E-H Lag has dim of mass → non-renormalizability
Describe gravity by vector field (as Higgs mechanism)
(ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in euIR sector and a mechanism to relax the cc. Also Shapiro+Sola, ...
More recent alternatives (a sample)

(iii) E Mottola 1006.3567  Effective field theory approach
- Casimir effect in flat s-t and large quantum backreaction are effects at the horizon scale of cosmological s-t
- imply the cosmological VE is dynamical
- its value depends on macroscopic BCs at the cosm horizon scale, rather than on the extreme ultraviolet Planck scale  [we, on both BCs]
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(iv) T Padmanabhan Ad Sci Lett 2 74 09  cc problem and explaining
DE independent issues: first find mechanism to make the cc vanish
– new degrees of freedom, kind of ‘gauge freedom’
  to absorb any $\lambda$ while maintaining general covariance
– could succeed in making gravity decouple from the bulk VE
– emergent gravity approach: thermodynamic description is far more
  general than just Einstein theory
– observed cc should be a relic of quantum gravitational physics and
  arise from degrees of freedom which scale as the surface area
– numerics: $L_A/L_P \sim \exp \sqrt{2} \pi^4 \sim 10^{60}$ (hierarchy squared) $\sim 10^{61}$
no attempt at explaining the old cc prob
– an extremely small quantum correction can in fact be produced quite naturally from a massive bulk field, introducing a massive bulk fermion
– naturally as superpartner of the radion field in a SUSY theory (especially the string theory realization) of brane-world scenario
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(v) Shao & Chen 1005.1920  no attempt at explaining the old cc prob
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(vi) JA Dixon 1006.2334  CyberSUSY solves the cc problem
– a new mechanism for SUSY breaking
– its realization mixes elementary and composite states
– SUSY anomalies present, generates spectrum for SUSY breaking
consistent with known particles
– no cc generated, because SUSY is not spontaneously broken...
The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem

2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds
   [A Flachi]
   - Bulk Casimir effect (effective potential) for a conformal or massive scalar field
   - Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
   - Consistent with observational data even for relatively large extra dimension

Previous work:
- flat space brane
- bulk conformal scalar field
- conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, Odintsov, Saharian PRD79(2009)065023, 0902.0717 Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons
The Sign of the Casimir Force

Many papers dealing on this issue: here just short account
The Sign of the Casimir Force

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a mirror pair of dielectric bodies always attract each other
∃ of positive Hilbert space and self-adjoint non-negative Hamiltonian
E.g. \( \exists \) correlation inequality: \( \langle f \Theta(f) \rangle > 0 \)

\( \Theta \) reflection with respect to a 3-dim hyperplane in \( \mathbb{R}^4 \)

the action of \( \Theta \) on \( f \) is anti-unitary \( \Theta(cf) = c^* \Theta(f) \)
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- Robin BCs in general
Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, $\Sigma$ compact internal space.
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Most general case: constants in the BCs different for the two plates. It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces.
Casimir eff in brworl’s w large extra dim

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Genuinely appear in: vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83], spinor and gauge field theories, quantum gravity and supergravity [Luckock ea 91].

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Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein’s entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]
Gravity Eqs as Eqs of State: $f(R)$ Case

The cosmological constant as an “integration constant”
T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
Unimodular Gravity Also I Shapiro, J Solà, ... cc RG flow
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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn. Also Erik Verlinde (private discussions)
Jacobson’s argument: basic thermodynamic relation

\[ \delta Q = T \delta S \]

- entropy proportional to variation of the horizon area: \[ \delta S = \eta \delta A \]
- local temperature \( T \) defined as Unruh temp: \( T = \frac{\hbar k}{2\pi} \)
- functional dependence of \( S \) wrt energy and size of system
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Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

\[ S = -2\pi \int_{\Sigma} E_{pq}^{pqrs} \epsilon_{pq} \epsilon_{rs}, \quad \delta S = \delta (\eta e A) \]

\( \eta e \) is a function of the metric and its deriv’s to a given order

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Case of \( f(R) \) gravities: \( L = f(R, \nabla^n R) \)
Also the concept of an **effective Newton constant** for graviton exchange *(effective propagator)*

\[
\frac{1}{8\pi G_{\text{eff}}} = E_R^{pqrs} \epsilon_{pq} \epsilon_{rs} = \frac{\partial f}{\partial R} (g^{pr} g^{qs} - g^{qr} g^{ps}) \epsilon_{pq} \epsilon_{rs} \\
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For these theories, the different polarizations of the gravitons only enter in the definition of the effective Newton constant through the metric itself.
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Final result, for \( f(R) \) gravities:

*the local field equations can be thought of as an equation of state of equilibrium thermodynamics* (as in the GR case)
Jacobson’s argument non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics

EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2
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By means of a more general definition of local entropy, using Wald’s definition of dynamic BH entropy

RM Wald PRD1993; V Iyer, RM Wald PRD1994
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