Interferometry of GPS signal occultations from Low Earth Orbiters, applications on Earth and Mars

Based on the Section 335 Talk:
‘Carrier phase interferometry with GPS LEO occultations: reflected-to-direct relative delay and potential applications.’

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Section 335

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Later contribution to:
- AGU 2003 talk (same authors)
- GRL, 2004GL019775 (same authors, cover of Vol31#10 issue, AGU Highlight, NASA/EOS Highlight)
Overview

- GPS-Reflections: group vs. carrier-phase delay
- GPS LEO Occultations with leakage of reflected signal: frequency domain
- REFLECTED-TO-DIRECT RELATIVE DELAY: From interferometric fringe to interferometric delay
- GEOPHYSICAL INFORMATION CONTENT:
  - General approach
  - Corrections in the refractivity profile: lowest troposphere
  - ALTIMETRY: whole-fit
  - ALTIMETRY: point-by-point fit, topographic profile
- Validation and discussion
GPS-Reflections

• 93-03: Studies to assess about capabilities of GPS signal reflected off the Earth surface (especially Ocean) to infer surface’s properties: altimetry, scatterometry (roughness, wind)

• GPSR C/A and P(Y) group delay altimetry has limited precision and coarse spatial resolution.

• Improvement of precision by means of carrier phase (C.P.) observations appears difficult at sea, with surface roughness of the order of several carrier wavelengths. [Treuhaft et al., 2001]: 2-cm precision, C.P. altimetry at Crater Lake.

• A possible way to avoid the effect of roughness is to constrain observations to grazing elevation angles, with apparent smoother roughness: JPL GPS-R team has obtained C.P. products on ocean surfaces during 2003 (both airborne and coastal observations).

• Tropospheric effect at grazing angles? GPS-R ↔ GPS-Occ
Occultation geometry
GPS-R/Occ

- **GPS-LEO Occultations**: Some GPS reflected signals leaking into the occultation antenna, especially on ice. Grazing angles of observation help coherence of the signal at carrier level.

- **DRAWBACK** of grazing observation angles: diminished sensitivity to vertical topography component.

- Problem looks similar to coastal approach, but tropospheric contribution separable (Occultation’s goal)!!
GPS LEO Occultations with reflected signal:

- Beyerle et al. 2002 proved that horn-like signatures in the frequency domain of the Occultation event corresponded to GPS signal reflected off the Earth (ice/ocean) surface and collected in the Occultations antenna.
- They estimated radio-holographic altimetric performance of 220m/1Hz.
- **STANDARD DATA FROM OCCULTATIONS ANTENNA/RECEIVER**
Our hypothesis was: since both signals have similar Doppler frequencies, changing along the event, they should interfere with each other, as detected from coastal experiments…

… providing the total received field with interferometric fringes, both in the total amplitude (minimum/maximum at points 3/4) and total phase (or phase delay) (min/max at points 1/2). The phase fringe should be in skewed quadrature w.r.t. amplitude fringes.
Reflected-to-direct delay

- When the direct and the reflected fields reach the antenna, the total field may be modeled as sum of two fields:

\[ Ae^{i\phi} = A_d e^{i\phi_d} + A_r e^{i\phi_r} = e^{i\phi_d}[A_d + A_r e^{i(\phi_r - \phi_d)}] \]

- As first approach, the smoothed value of the amplitude within the oscillation corresponds to the direct field:

\[ A_d \sim \bar{A} \]

...same for the phase:

\[ \phi_d \sim \bar{\phi} \]
Reflected-to-direct delay

The model for complex received signal, thus, becomes:

\[ Ae^{i\phi} = e^{i\phi} [\bar{A} + \bar{A}_r e^{i\phi_I}] \]

System of 2 equations with 2 unknowns (amplitude of reflected signal and interferometric phase).

The interferometric is:

\[ \phi_I = (\phi_r - \phi_d) = k \cdot (\vec{r}_r - \vec{r}_d) \]

HENCE:

- Possibility of extracting the interferometric phase
- Each cycle in the interferometric phase is 1 GPS-L1 wavelength difference between the reflected and the direct signals (approx. 19 cm)
Reflected-to-direct delay

... writing the equation variables in their Real and Imaginary components...

\[
\tan(\phi_I) = \frac{\mathcal{R}\mathcal{I} - \mathcal{I}\mathcal{R}}{\mathcal{R}(\mathcal{R} - \mathcal{R}) + \mathcal{I}(\mathcal{I} - \mathcal{I})}
\]

The smoothed values are taken over running windows of variable size, because the interferometric frequency, and so the fringe, slow down as both signals approach each other. The size is set to two times the interferometric period, roughly estimated from the frequency domain:

\[T_w = 2 \quad T_I = 2 / f_I = 2 / (f_{\text{direct}} - f_{\text{reflected}})\]
Reflected-to-direct delay

\[ \Delta \rho = \frac{\phi I}{360^\circ} \cdot \lambda_{L1} + N \lambda_{L1} \]
Reflected-to-direct delay

T=65 → 0
T=66 → -1mm
• General remarks:
  – Observation located at ~45 deg longitude E, ~89.75 deg latitude N:

**REFLECTION ON POLAR ICE**

- Dry troposphere expected
- User Requirement for ice topography: 0.5 m precision

Current observation system: lack of coverage over extreme polar latitudes. ICESat (January 2003) having some trouble…
GPS-R/Occ: most coverage on the poles.
\[ \Delta \rho = \rho^R - \rho^D \]
\[ = (\rho^R_{geo} + \rho^R_{trop} + \rho^R_{rough} + \rho^R_{iono} + \rho^R_{instr} + n) - (\rho^D_{geo} + \rho^D_{trop} + \rho^D_{iono} + \rho^D_{instr} + n) \]
\[ = \Delta \rho_{geo}(\vec{R}, \vec{T}, \vec{S}) + \Delta \rho_{trop}(\vec{R}, \vec{T}, \vec{S}, N) + \rho^R_{rough} + n \]

- Ionosphere and instruments assumed to perturb the reflected and direct signals in the same way
- Roughness effects neglected (could introduce offsets, left as future work)
- Geometric (altimetry) and tropospheric (crossing atmosphere through different altitudes) are modeled by means of a ray tracing tool
GENERAL APPROACH:

- For given RCV and TRM locations, refractivity profile, and a-priori surface model (ellipsoid or geoid), the ray tracing tool generates the model of the interferometric delay.

- If we believe previous estimates of RCV and TRM locations, and refractivity profile provided by the inversion of the direct signal in the occultation event, the ray tracing tool only needs to correct the surface’s altitude to fit the data:

\[
\Delta \rho_{rtt}(\delta S; S, \vec{R}, \vec{T}, N) \leftrightarrow \Delta \rho_{data}
\]
EXAMPLES:

• Data (solid line), ray tracing tool model for reference surface – 8.9 m (long dashes), ray tracing tool output for reference surface + 0.9 m (short dashes)

• Data prefers –8.9 m corrected surface at the beginning, and +0.9 correction at the end

• Topography or mismodeling?

RAY-TRACING mismodeling? inaccuracies in the refractivity profile?
ALTIMETRY: point-by-point

- **Point-by-point**: find best fit at every data point (20 ms):
  - For each data point in the time series, we define the solution as the surface correction that minimizes:

\[
M_2(t, \delta S) = \frac{(\Delta \rho_{data}(t) - \Delta \rho_{RTT}(\delta S; S, \vec{R}(t), \hat{T}(t), N))^2}{\sigma_{data(t)}^2}
\]

- Uncertainty ~ 2m
- Time spacing ~20 ms
- Horizontal spacing ~83 m
ALTIMETRY: point-by-point

POINT-by-POINT APPROACH (0.2 sec averages):

<table>
<thead>
<tr>
<th>Solution:</th>
<th>Topographic profile, max grad ~ 5cm/100m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision, &lt;RMS&gt;:</td>
<td>0.70 m</td>
</tr>
<tr>
<td>Horizontal resolution</td>
<td>~1 X 1 km</td>
</tr>
</tbody>
</table>

- Polar ice: smooth topography expected
- Close to user requirements on ice
- Fine horizontal resolution
• The refractivity profile provided by the direct occultation technique is less reliable in the lowest part of the troposphere.

**QUESTION 1:** Is this kind of data sensitive to tropospheric inaccuracies?

**QUESTION 2:** Is it possible to separate the tropospheric from the geometric effects?
Interferometric delay: reflected-to-direct up to certain time, **direct-to-direct afterwards**: last seconds of interferometric observation contain direct signal only, independent of surface’s location. Are the last seconds of the observation sensitive to small perturbations in the refractivity? May we use them to refine the lowest part of the refractivity profile (< 1km)?
Tropospheric corrections

- Analysis of tropospheric perturbations of the form:

\[ N(h; \Delta N, h_0) = N(h) + \Delta N^{-h/h_0} \]

-5E-6 to +5E-6

0.1 km to 0.9 km in 0.2 km steps

- Misfit defined as:

\[ M^2(\delta N, h_0) = \frac{1}{N_s} \sum_{i=1}^{N_s} \left( \frac{\Delta \rho_{\text{data}_i} - \Delta \rho_{\text{RTT}_i}(\delta N, h_0; S, \vec{R}, \vec{T}, N)}{\sigma_{\text{data}_i}^2} \right)^2 \]
Tropospheric corrections

For $h_0 = 0.1$ km:

For the rest::

![Graph showing tropospheric corrections](image)
Tropospheric corrections

- Lowest troposphere (<1km) scanned with sensitivity better than 1% using exponential correction functions.
- Tropospheric term separable from geometric one (altimetric) when the inaccuracies are within this lowest layer.
- Inaccuracies of the refractivity profile altitudes higher than ~1 km are not separable by means of this approach.

- This particular GPS-R/Occ event: best fit for nominal refractivity profile (Occultation solution).
VALIDATION and DISCUSSION

• VALIDATION: extremely difficult, no data available for this particular event
• DISCUSSION:
  – Because of the lack of independent data, more work is needed to conclude to what extent the topographic solutions here presented are influenced by some mismodeling.
  – This work has shown that with a reliable ray tracing tool and tropospheric refractivity profile, the interferometric GPS-R/Occ data are sensitive to altimetric variations on ice at submeter level.
  – The lack of data at the poles gives more value at this GSP-R/Occ data, filling a temporal and geographical gap.
  – Data are also sensitive to slight variations in the lowest troposphere, which are separable from altimetric component.
• FUTURE WORK:
  • Need to process events at available validation locations: either external data or cross-over GPS-R/Occ events.
  • Receiver modifications to improve the precision here presented?
  • Other algorithms for retrieval of interferometric delay? (wave optics tools)
  • Confidence on horizontal location of the reflection by the ray tracing tool?
  • Reflections on ocean? Effect of wet troposphere?
  • … much more…
Later, validation on Greenland was performed:


**APPLICATIONS ON MARS:**

-Mars Express (ESA) and Mars Surveyor (NASA) carry radio instruments to analyze observations in occultation geometry. This technique could be applied to data sets from Mars to try to conduct topography of the Martian polar areas.