TOTAL-VARIATION-BASED METHODS FOR GRAVITATIONAL WAVE DENOSING

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Introduction

- Methods based in **Total Variation (TV)** are considered to be one of the **best denoising models**.
- TV uses the $L_1$ norm → hard to compute.
- Developed and tested for image processing. (Osher, Goldstein, Candès, Marquina, Donoho, Tibshirani,...)
- We have adapted these techniques to Gravitational Waves signals.
- Not need any **a priori information** about the source.
- Can be applied both time and frequency domain.
- Can be extended easily to higher dimensionality.
- Complementary to existing data analysis methods.
**Fundaments**

**Linear degradation model**

\[ f = u + n \]

- \( f \): measured signal.
- \( n \): noise.
- \( u \): signal to be recovered.

Solution: Find a function \( u \) whose L2-norm distance to \( f \) is noise standard deviation.

\[ \| f - u \|_{L_2}^2 = \sigma^2 \]

Gibbs phenomena.

Non-unique solution.

Use an auxiliary energy 'prior' to make \( u \) regular:

Unique solution if \( R(u) \) is convex.

\[ \min_u R(u) \]

subject to \( \| f - u \|_{L_2}^2 = \sigma^2 \)
Unconstrained variational problem

\[ u = \arg\min_u \left\{ R(u) + \frac{\mu}{2} \| f - u \|_{L^2}^2 \right\}; \quad \mu > 0 \]

- **Unique solution** for a given value of \( \mu \).
- \( \mu \) becomes the scale parameter \( \rightarrow \) Larger values allow to recover finer scales.

\[ R(u) := \int |\nabla u|_{L^2}^2 \quad \Delta u + \mu (f - u) = 0 \]

- Easy to solve due to differentiability and strict convexity.
- Noise amplifies high frequencies and solution shows spurious oscillations.
Rudin Osher Fatemi Model

**ROF Model**

\[
    u = \arg\min_u \left\{ \text{TV}(u) + \frac{\mu}{2} \| f - u \|^2_{L_2} \right\}; \quad \text{TV}(u) := \int |\nabla u|
\]

- Convex problem.
- Preserves steep gradients or edges and avoids spurious oscillations.
- Fine scales are destroyed by the effect of TV norm.

**Bregman Iteration**

- The **residual error** is added back to the constraint to solve a new ROF variational problem in each iteration.
- Allow to recover scales in a progressive way.
Regularized ROF (rROF)

- TV functional is slightly perturbed:

$$TV_\beta(u) := \int \sqrt{|\nabla u|^2 + \beta}$$

- Associated Euler-Lagrange equation is elliptical and non-degenerate.

Split Bregman Algorithm

- Efficient implementation to solve ROF model.
- Bregman Iteration + **decoupling** of the TV problem into $L_1$ and $L_2$ terms.

$$\arg\min_{u,d} \left\{ |d| + \frac{\mu}{2} \|f - u\|_{L_2}^2 + \frac{\lambda}{2} \|b - \nabla_x u - d\|_{L_2}^2 \right\}$$
Gravitational Waves Catalogs

Rotating Core Collapse

- 128 Waveforms.
- Short duration $\sim 10$ ms.

Binary Black Holes merger

- 174 waveforms of BBH.
- High accuracy and very low eccentricity.


Credit: A.H. Mroué et al.
Simulated Gaussian Noise

- Advanced LIGO proposed broadband configuration.
- rROF applied in the time domain.
- SB applied in the frequency domain.
- Simulated non-white Gaussian noise.
- Sampling frequency 16384 Hz.
- Signal-to-noise ratio.

\[
\sqrt{4\Delta t^2 \Delta f \sum_{k=1}^{N_f} \frac{|\tilde{h}(f_k)|^2}{S(f_k)}}
\]

\[
\sum_{k=1}^{N_f} \frac{|\tilde{h}(f_k)|^2}{S(f_k)}
\]
Detector noise is **non-white** → regularization parameter must be weighted with coefficients related with detector noise curve.

Detector noise is **not stationary** and its standard deviation is unknown.

The optimal value of $\mu$ cannot be set up a priori.

Perform simulations with different values of $\mu$ and compare with original signal.

\[
\text{PSNR(dB)} = 10 \log_{10} \left( \frac{N}{\text{MSE}} \right); \quad \text{MSE} = \frac{||x-u||^2_{L^2}}{N}
\]
Noise generation dependence

Random noise generations

Variation with SNR
Signal Denoising: Example of Simulated Gaussian Noise

SNR = 20

![Graph showing SNR = 20 with plots for Original, SB, and rROF]
Real Noise Setup

- Real Noise from LIGO (LIGO Open Science Center)
- GPS Time = \(840114176\) \(\Leftrightarrow\) October, 20, 2006
- Data quality flags for Burst and Binaries.
- 4 s of data, \(F_s = 4096\) Hz
- rROF + Bregman Iteration with very low \(\mu\).
BBH example

Torres, Alejandro (UV)
Burst example
We have adapted total variation based denoising algorithms to GW signals. The algorithms can remove noise independently of the type of signal. The rROF method has led to satisfactory results without any assumption about the noise distribution. The SB method results are determined by the weight curve we have chosen → other possible distributions are allowed. SB method must be optimized to work with real noise. A whitening process before the TV-methods could improve the results with real noise. The methods have to be tested jointly with other techniques of data analysis.
Thank you for your attention!

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