Simulations of Extreme-Mass-Ratio Inspirals Using Pseudospectral Methods

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Extreme-mass-ratio inspirals (EMRIs), stellar-mass compact objects inspiraling into a massive black hole, are one of the main sources of gravitational waves expected for the Laser Interferometer Space Antenna (LISA) observatory. To extract the EMRI signals from the LISA data stream we need very accurate theoretical templates of the gravitational waves that they produce. In order to do so, we need to consider the gravitational back-reaction, that is, how the gravitational field of the stellar-mass compact object affects its own trajectory. In general relativity, the back-reaction is described in terms of a gravitational ‘self-force’, and the foundations to compute it have been laid several years ago. However, parts of the calculation of the self-force have to be performed numerically. In this poster we describe an effort towards the computation of the self-force based on time-domain pseudospectral methods, in which we try to take advantage of the high precision that this method provides.

Main Motivation
To have a theoretical understanding of astrophysical sources of Gravitational Waves that the future NASA/ESA mission LISA [H] will be able to observe. Simulations of sources like Extreme-Mass Ratio Binaries (EMRBs) can provide key information for the development of techniques of Gravitational Wave Astronomy.

Calculations for a Charged Particle orbiting a non-rotating Black Hole. I
Our goal is, using spectral methods, to solve the equations describing the scalar field generated by a charged particle orbiting a non-rotating black hole.

\[ \psi = f(r) \delta (r - a) + \phi (r) \]

where the particle energy density is given by \( E = \int \delta (r - a) \Phi (r) d^3r \) and \( \Phi (r) \) is the scalar dynamical gravitational field.

The scalar particle has charge \( q \) and moves along the world line \( \gamma \) parameterized by the proper time \( \tau \).

As a first step we have to compute the field generated by the particle moving in a geodesic of the Schwarzschild spacetime. Expanding the scalar field in spherical harmonics

\[ \Phi (r) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \Phi _{lm} Y _{lm}(\theta , \phi ) \]

we obtain, for each harmonic component, 1+1 wave equations of the form

\[ \left( -\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} \right) \Phi _{lm} = S _{lm} \]

where the singular source is

\[ S _{lm} = A \Phi _{lm} \]

and the black hole potential is given by the Regge-Wheeler potential for scalar fields.

\[ V _1 = f \left( \frac{a}{r} \right) + \frac{2 l}{r} \]

Results from the Simulations
In this simulations we have used two computational domains:

- Convergence test for the evolution of a gaussian wave packet traveling in flat space. The truncation error decays as \( e^{-N} \).
- Scalar gravitational waves generated by the evolution of a charged particle orbiting a non rotating black hole.

Future Work
- Extension of these techniques to general orbits and to the gravitational case.
- To perform self-force computations.
- To transfer these techniques to the case of a spinning (Kerr) Black Hole.

References

Acknowledgments
We acknowledge financial support from the European Commission under a Marie Curie International Reintegration Grant (MIRG-CT-2007-205005) and the Spanish Ministry of Science and Education (MEC,ESP 2007 - AEI).