ICE CSIC IEEC

Efecto Casimir
Funciones Zeta
Aplicaciones en Nanotecnología

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ICE/CSIC & IEEC, Universitat Autònoma de Barcelona

II Congreso de Ingeniería Física

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Programa

- EFECTO CASIMIR EN FÍSICA Y TECNOLOGÍA

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- EFECTO CASIMIR EN FÍSICA Y TECNOLOGÍA
- La Física Cuántica (como revolución científica)

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- COLABORADORES:

  S Carloni, G Cognola, J Haro, S. Leseduarte, S Nojiri, S Odintsov, A. Romeo D Sáez-Gómez, A Saharian, P Silva, S Zerbini, ...

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- Variables ocultas, desigualdades de Bell

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Even then: Has the final value real sense ?

Bohr $\longrightarrow$ Casimir $\longrightarrow$ Pauli ...

The Casimir Effect

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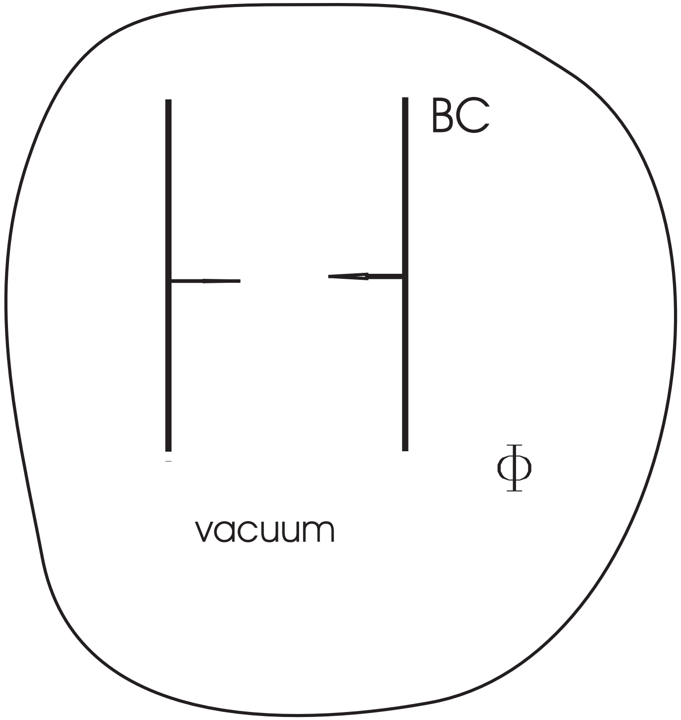

Casimir Effect

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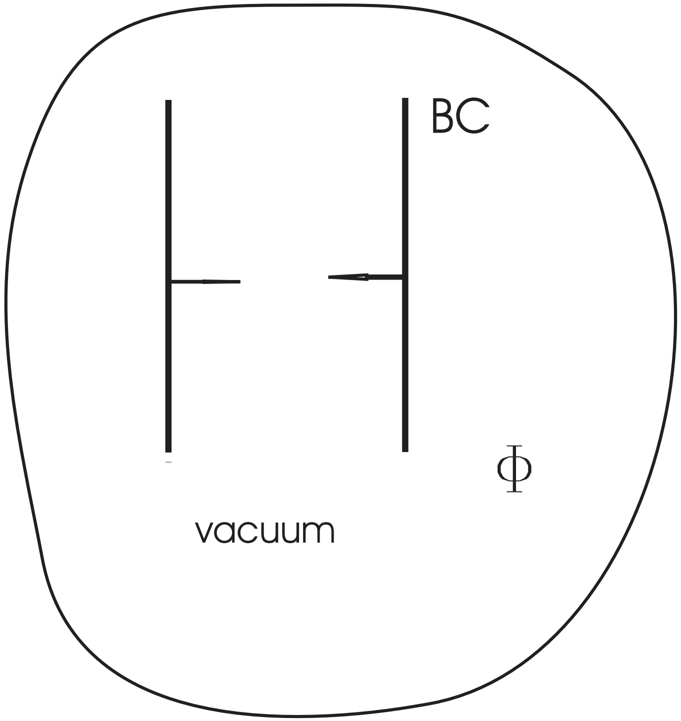

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$\Longrightarrow$ all kind of fields

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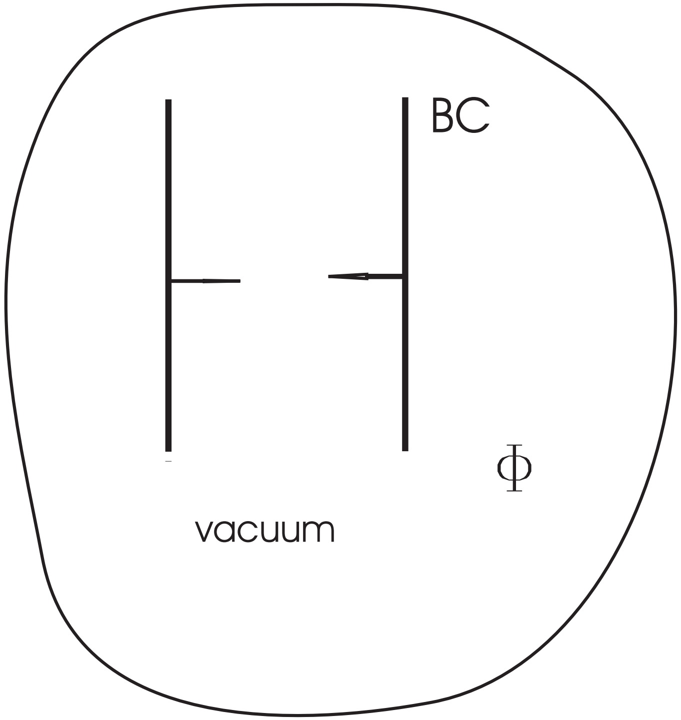

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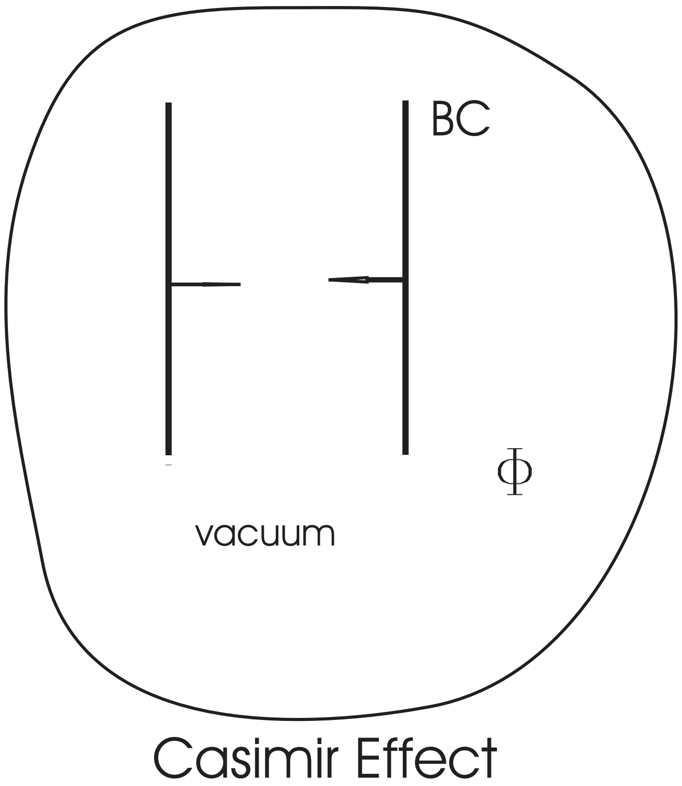

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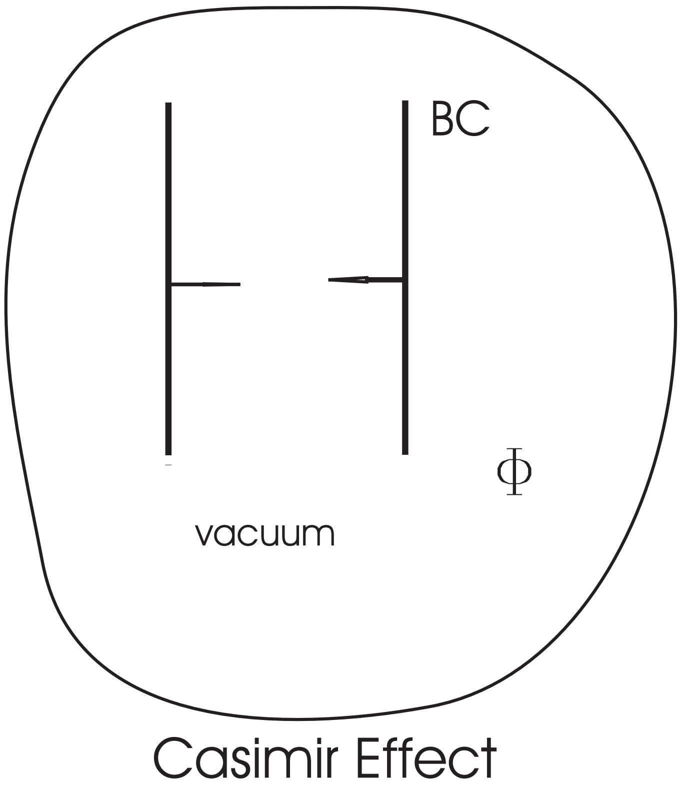
The Casimir Effect



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- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

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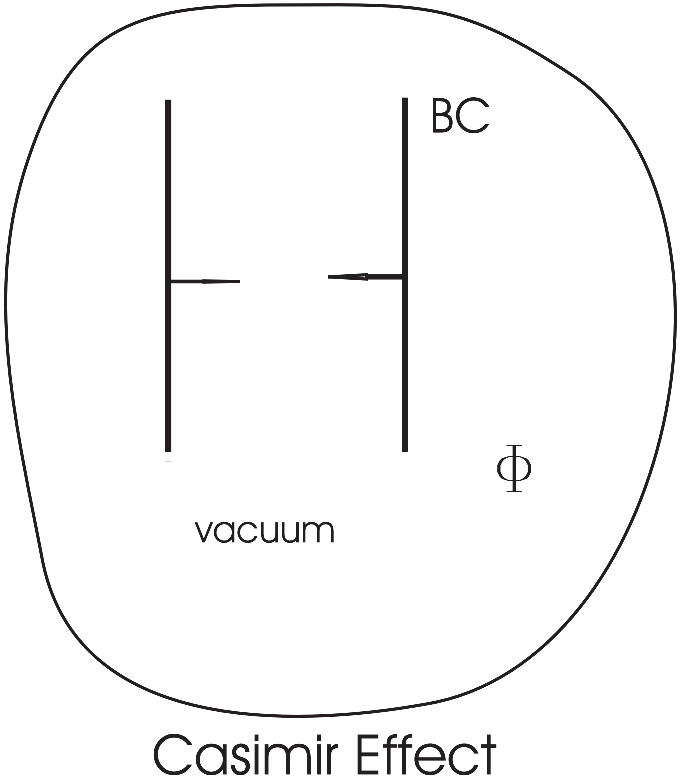

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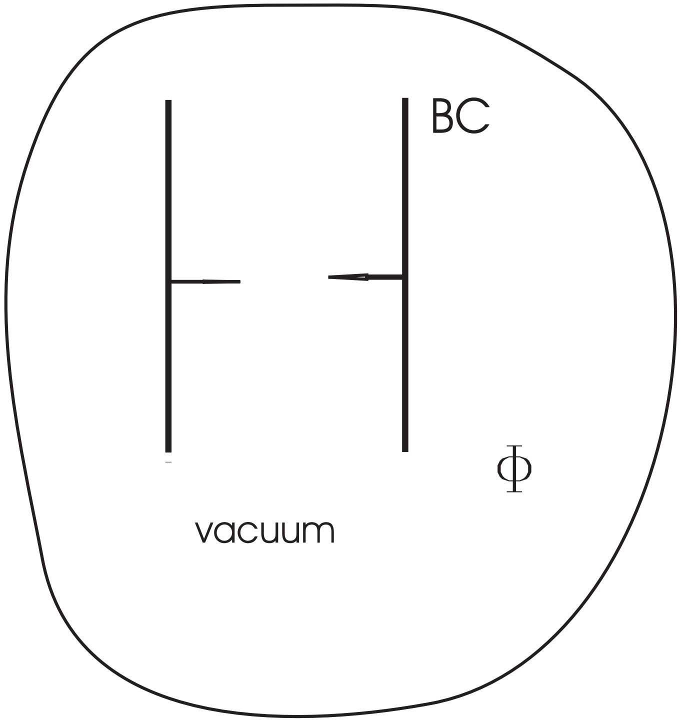

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- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)

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- Milonni has reformulated all of QED from the point of view of ZPF

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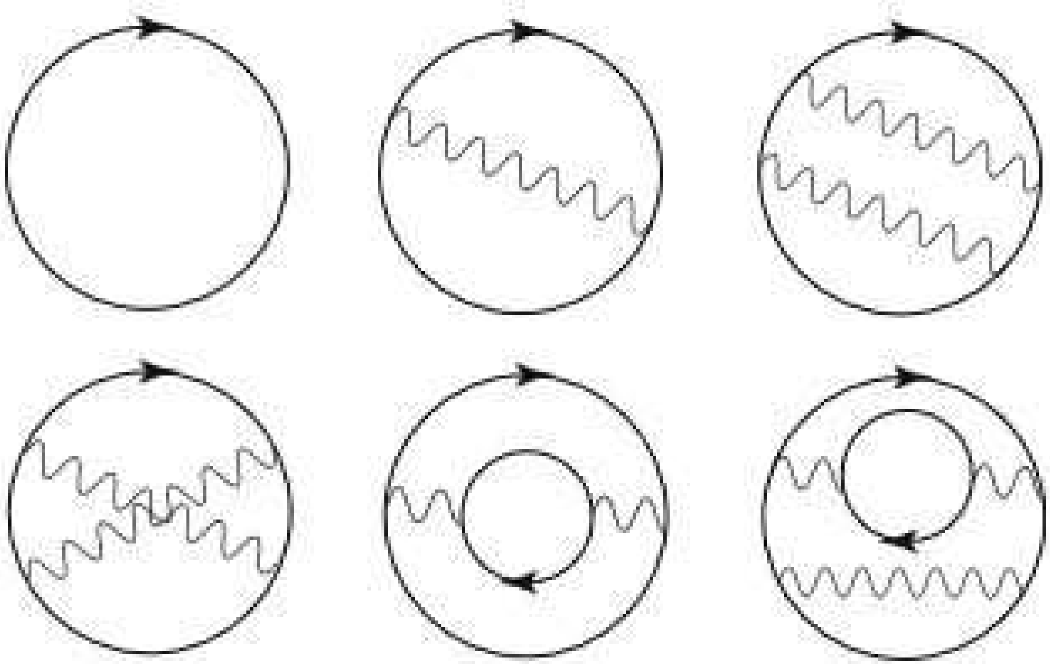

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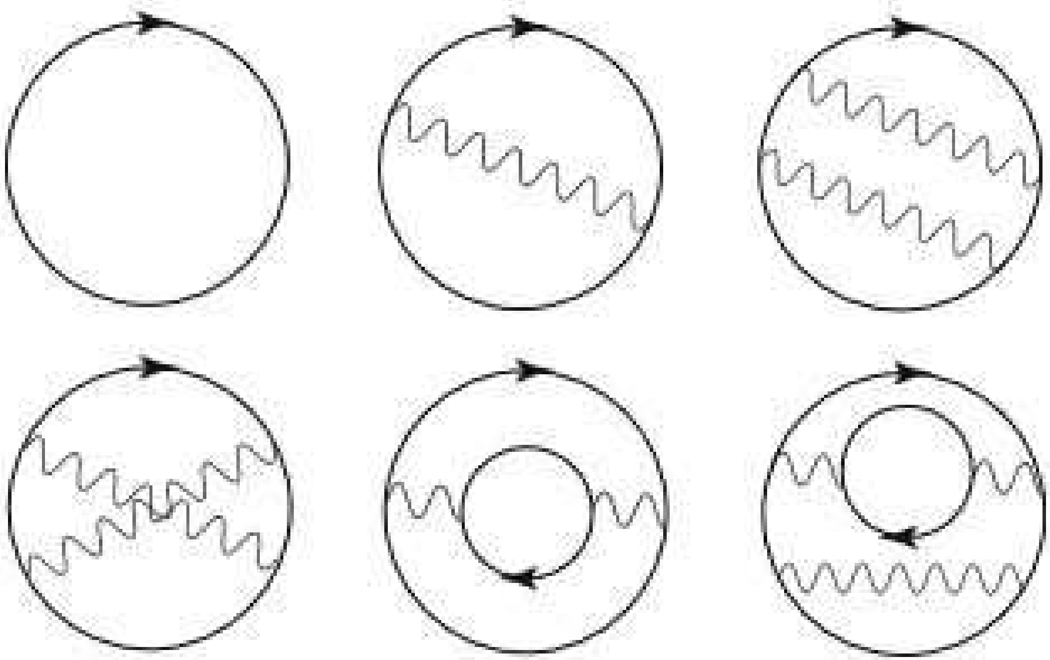

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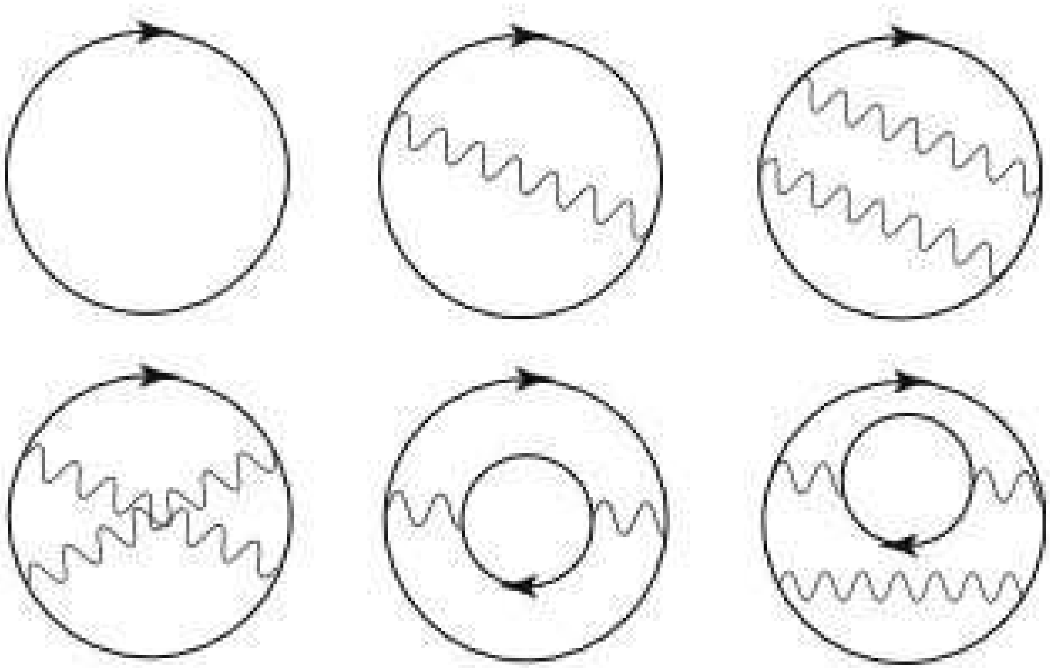

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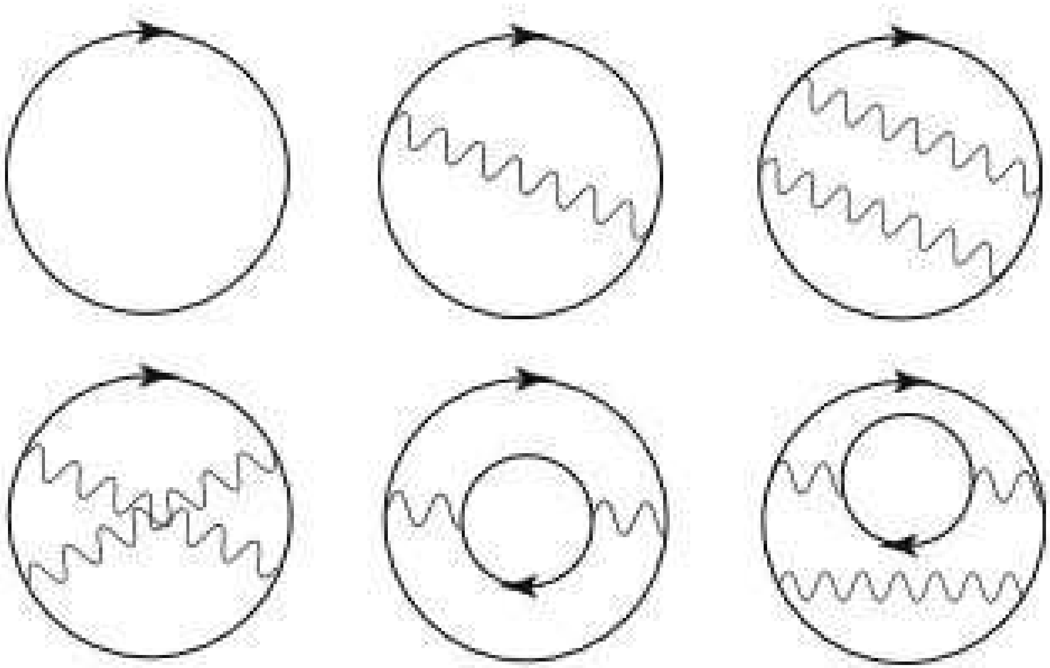

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function — Trace is over spin

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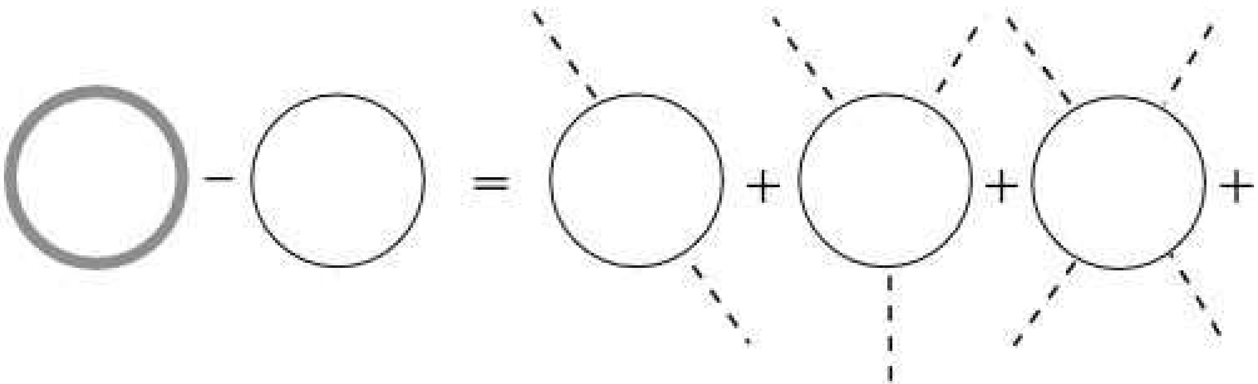

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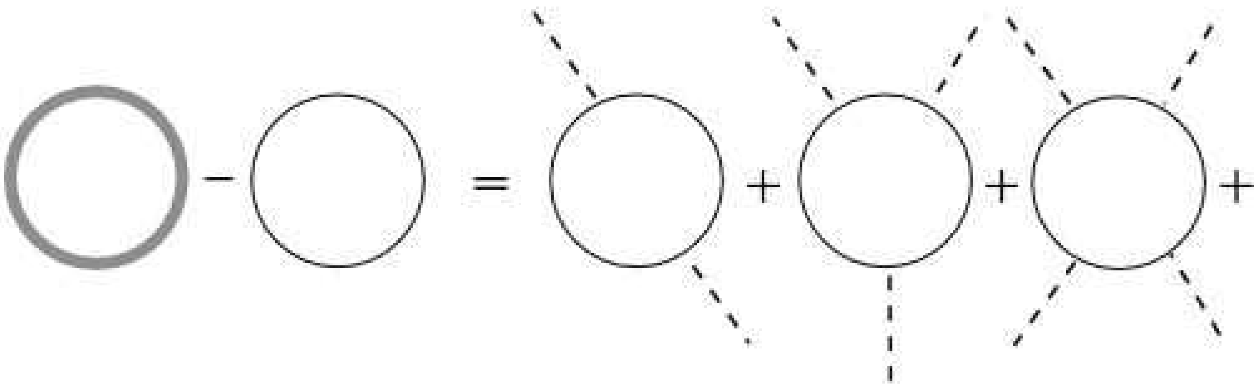

$\Longrightarrow$ Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Jaeckel, Reynaud, Lambrecht; Ford, Vilenkin; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t [cf. paragraph after Eq. (4.5)]

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) = \text{Id}$

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow \text{Id}$, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions (material's permitivity and resistivity)

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$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \text{Re} \left[e^{-i(\omega+\omega')t} \, \widehat{\dot{g}\theta_t}(\omega + \omega') \right] \times \left[|r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 \right] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

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$$\langle \hat{E}(t) \rangle = -\epsilon \int_0^t \langle \hat{F}_{Ha}(\tau) \rangle \dot{g}(\tau) d\tau$$

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- More general results: Kenneth, Klich, PRL 97, 160401 (2006)
  a mirror pair of dielectric bodies always attract each other
  CP Bachas, J Phys A40, 9089 (2007) from a general property of Euclidean QFT 'reflection positivity' (Osterwalder - Schrader 73, 75):
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  - Robin BCs in general $\Leftarrow$

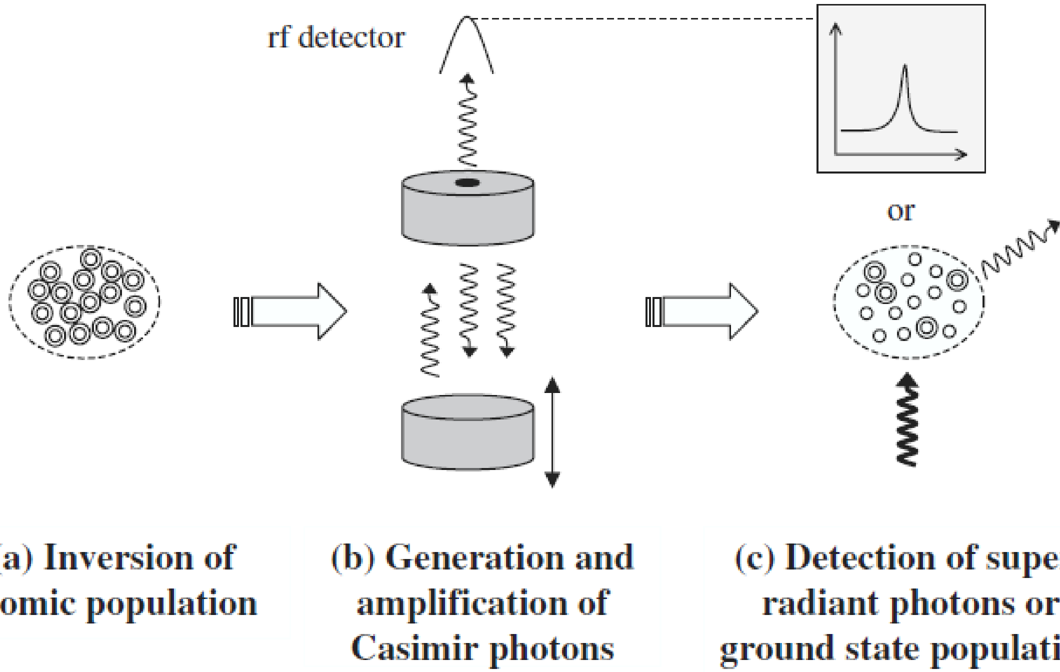

FIG. 1. Generation and detection of photons irradiated through vacuum-induced damping of motion. The two-level atoms are optically pumped to the maximum angular momentum state of the spin-orbit manifold which only allows for a single, downward, circularly polarized, magnetic dipole transition. The atoms are then sent through the cavity (a). The Casimir photons are generated through mechanical modulation of one FBAR resonator (b). An amplified superradiant pulse is triggered by a weak Casimir signal and detected by radio-frequency electronics or atomic fluorescence (c).

the relevant surfaces are separated by a distance much smaller than their typical local curvatures. This leads to an approximate expression for the Casimir force

$$F_C(d) = \frac{\pi^3 \hbar c}{360} \frac{R}{d^3}, \tag{3}$$

where R is the radius of the sphere and d is its distance from the plane, and its validity holds in the regime $R >> d$. Since this approximation can be derived in classical electrostatics by relying on the additivity of the Coulomb force, care has to be taken in the case of forces of quantum nature which have a strong geometric (and non-additive) character. Likewise, in the case of dielectric materials the comparison with theory is complicated by the necessity to know the dispersive properties of the dielectric material. This is taken into account with a formula developed by Lifshitz and collaborators [44, 71]. The loss of *universality* intrinsic in the ideal Casimir formula makes the Lifshitz formula less appealing and complicates the theory-experiment comparison requiring a detailed knowledge of the dielectric response of the materials. Nevertheless, the problem was not felt to be important, as the experimental precision was limited in comparison to the more stringent tests of quantum electroynamics at the microscopic level with Lamb shifts and $g-2$ for electron and muon; no need for refined comparison with theory was then necessary.

The outcomes of the first generation of measurements can be summarized as follows. The Sparnaay experiment, with accuracy assessed at the 100% level, was considered as inconclusive in showing the expected scaling of the force with the distance, with also evidence for repulsive forces indicating a partial control over the electrostatic background. To use Sparnaay's own words, the measurement "*did not contradict Casimir's theoretical prediction.*" The experiment by van Blokland and Overbeek was more successful from this viewpoint, obtaining agreement with the Casimir predictions at an estimated accuracy around 50%, and was thus the first uncontroversial verification of the Casimir force between metallic surfaces. Experiments with dielectric surfaces were performed using silica lenses [68, 72], crossed cylinders of muscovite mica [73, 74], thin films of liquid helium absorbed on surfaces of alkaline-earth fluoride crystals [75], flat surfaces of porosilicate glass [76]. The evidence for a crossover from the non-retarded component of the molecular force to the retarded component and an overall verification of the Lifshitz theory at the 20-40 % accuracy level, apart from the experiment by Sabinsky and Anderson reporting accuracy of order 1%, were the main results of these experiments.

After this burst of experimental activity on Casimir forces there was no further activity for many years. The spectacular success of quantum electrodynamics and its unprecedented accurate verifications at the microscopic level could not be matched by measurements of (necessarily macroscopic) forces. The attention at the macroscopic level was instead shifted on the atomic physics experiments, as the presence of a cavity with defined boundary conditions was found to affect the spontaneous emission properties of individual atoms. In this context, the microscopic counterpart of the Casimir force acting between an atom and a plane surface, also known as Casimir-Polder force [77], was measured by looking at the deflection induced on an atomic beam by two parallel plates [78] and comparing this with the theoretical predictions [79]. The new wave of Casimir force experiments was revamped after remarks by Sparnaay [80] concerning the possibility of a second generation of measurements at higher accuracy exploiting the emerging subfields of atomic force microscopy (AFM) [81] and of nanotechnology. Consequently, in partership with Carugno at the INFN in Padova, we studied a scheme to measure the Casimir force in a parallel plate configuration, starting the first tests in the early summer 1993 [82]. The apparatus capitalized on a variety of technological improvements not available at the time of the Sparnaay's measurement. Most notably, the use of microresonators and of dynamical detection techniques based on the Fourier analysis of the tunnelling current of a single axis scanning tunnelling microscope were discussed and a first prototype tested inside a scanning electron microscope. Also, consideration was given to the capability of measuring the gravitational force in the same range [16]. Unfortunately the issues of parallelization, dust in the gap, and the large $1/f$ noise present in electron tunnelling devices prevented a straightforward measurement of the Casimir force in the proposed configuration.

An attempt to measure the Casimir force using a torsional balance was initiated by Lamoreaux at the University of Washington in Seattle. The initial tests with flat plates, in 1994, also met difficulties in the alignment [83], until the experiment was reconfigured in the sphere-plane geometry by using a convex lens, and the Casimir force was then measured at distances up to 6 μm [84] with significant improvement in both range and accuracy with respect to the van Bockland and Overbeek measurement in the same configuration. These improvements were mainly due to the elimination of mechanical hysteresis in the torsion balance and the use of piezoelectric actuators for the positioning of the plates [85].

Due to the large Volta potential present between the plates even after a nominal external short-circuit, even at the closest explored distance the Casimir force was evaluated to be about 20 % of the total measured force, and required an ingenious subtraction technique to be employed. Theoretical discussions followed the appearance of the related paper, focusing on finite conductivity and temperature corrections. Given the large range investigated, this experiment with the accuracy initially quoted was in principle able to grasp both these corrections. A deeper analysis showed that the

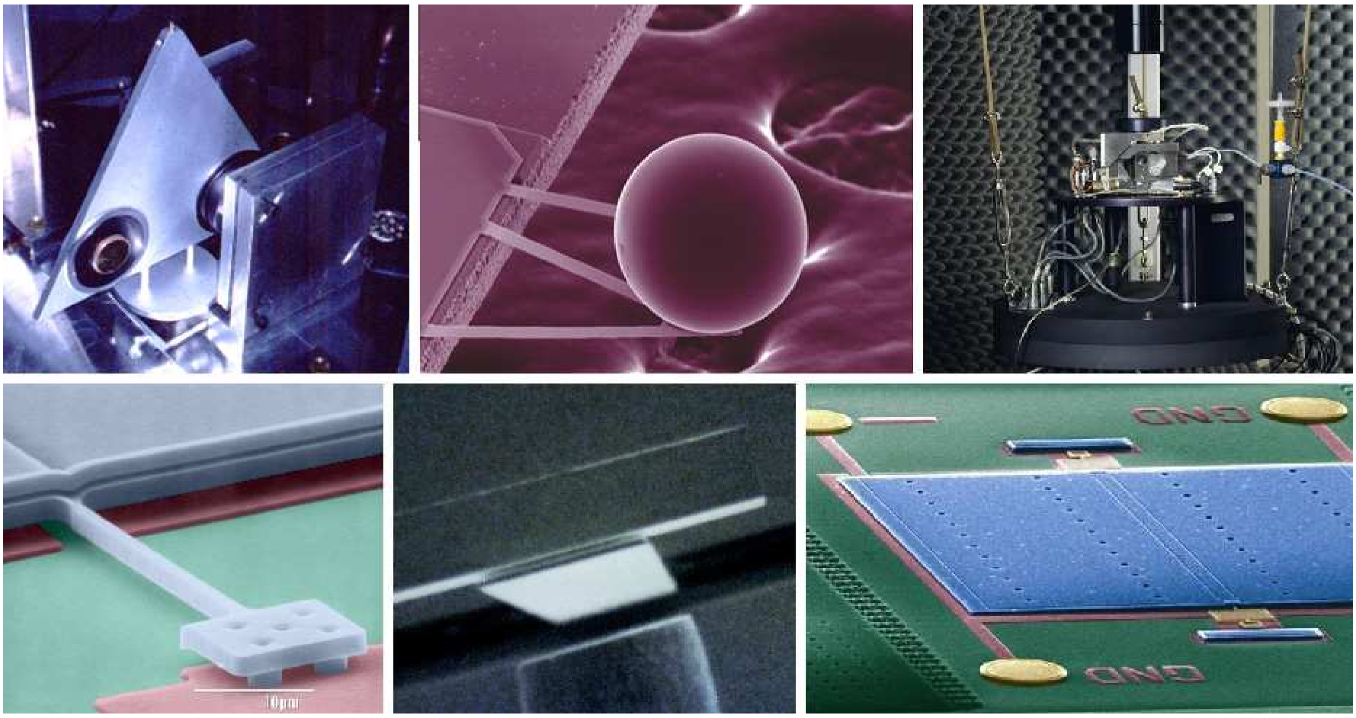

FIG. 2: Pictures from the six Casimir force experiments of the second generation. In chronological order, from top-left to bottom-right, some views of the apparatuses used in Seattle, Riverside, Stockholm, Murray Hill, Padova, and Indianapolis are depicted.

conductivity corrections were less trivial to manage due to the presence of a copper substrate deposited on the lens prior to the gold coating. Including a better assessment of the radius of curvature of the lens, found *a posteriori* to be aspheric [86], did not solve the conductivity issue. Further discussions of the experiment were carried out regarding the conductivity corrections [87, 88] and the thermal corrections [89–91]. While we suggest that the reader looks at the related interesting exchange of comments, a likely assessment of the situation can be summarized as follows: the initially quoted accuracy of 5% was probably reliable at the smallest explored distances, but it was worse at the largest distances. Lamoreaux himself pointed out the spirit of his measurements in one of the abovementioned replies [88]: "*I offer the caveat that my experiment was intended as a demonstration to show that, with modern experimental techniques, one could do a really accurate measurement of the Casimir force. As a demonstration, only minimal tests for possible systematic errors were performed: furthermore, I was satisfied with the agreement between my experimental result and my inaccurate calculation*." This remark by the pioneer of the modern generation of measurements on Casimir forces, as we will see in the following sections, is key for understanding the spirit with which the current generation of measurements on Casimir force has been carried out: they have to be considered more as demonstrations than experiments [92].

The successful use of atomic force microscopy techniques combined with the sphere-plane geometry was accomplished by Mohideen and Roy at Riverside in 1998 [93], after attempts started one year earlier. In their experiment, a metallized polysterene sphere was mounted on the tip of the AFM cantilever, and the deflection of the cantilever measured as a function of the distance between the sphere and a metallized flat surface. The metal deposited on the sphere was initially aluminum but a second version of the experiment instead used gold [94] which was predicted to provide a cleaner situation [95]. In both cases, the experiment-theory comparison took a number of corrections into account, namely the finite conductivity, the roughness, and the finite temperature, unobserved in the Lamoreaux measurement. Due to the small range of distances investigated, down to 100 nm, and the smaller Volta potential, of order 30 mV, the Casimir force dominated the electrostatic contribution over a wide range of distances, with the latter contributing to the bare force only in an amount evaluated as less than 3%.

A third successful attempt was performed by Ederth [96]. This work is remarkable for a number of reasons. The force was measured in the novel geometry of crossed cylinders (previously only used by Tabor and Winterton [73] for dielectric surfaces), and in the very short range of 20-100 nm. The expected Casimir force in such a configuration can be written as:

Lateral Casimir force beyond the Proximity Force Approximation

Robson B. Rodrigues,1 Paulo A. Maia Neto,1 Astrid Lambrecht,2 and Serge Reynaud2
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Campus Jussieu, F-75252 Paris Cedex 05, France

arXiv:quant-ph/0603120

The lateral Casimir force between corrugated surfaces has been analyzed outside the PFA domain for perfect reflectors [12] where interesting results were obtained for arbitrary values of the ratio λ_C/L of the corrugation wavelength λ_C to the interplate distance L.

In this letter, we calculate the lateral force for metallic plates modeled by the plasma model with arbitrary values of L, λ_C and λ_P. We use the perturbative approach that we developed for analyzing the effect of stochastic roughness on the normal Casimir force.

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