ICE CSIC IEEC

The Vacuum Energy: Casimir Effect and the Cosmological Constant

EMILIO ELIZALDE

ICE/CSIC & IEEC, UAB, Barcelona

web: google → emilio elizalde

FFP8, Madrid, October 17th to 19th, 2006

Outline of the talk

- On the Zero Point Energy & the Casimir Effect

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- Vacuum Fluctuations & the Cosmological Constant

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- Conclusions

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Even then: Has the final value real sense ?

The Casimir Effect

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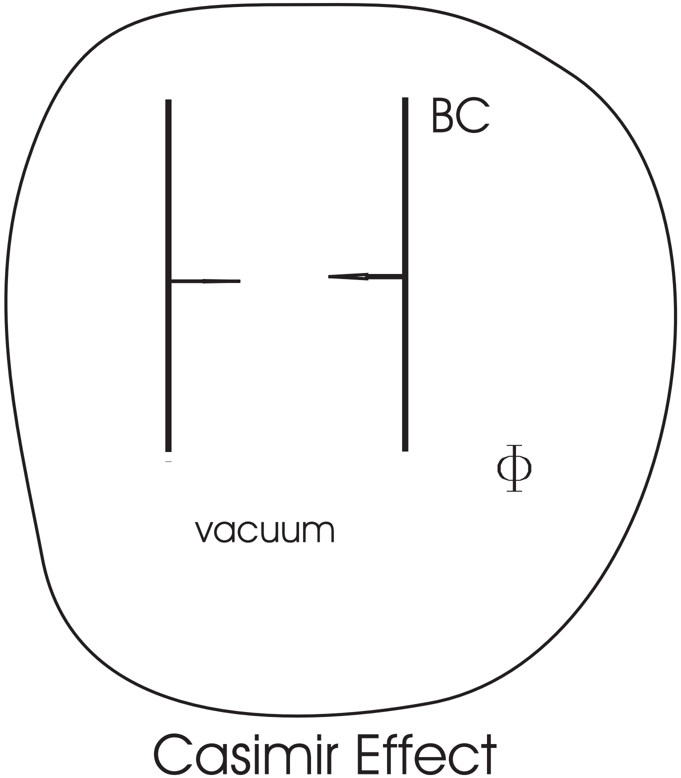

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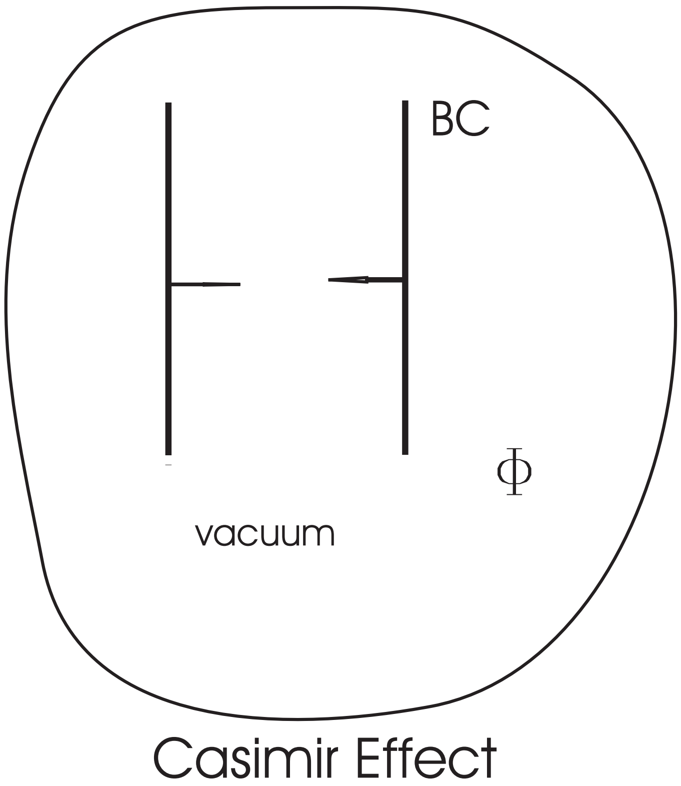

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$\Longrightarrow$ all kind of fields

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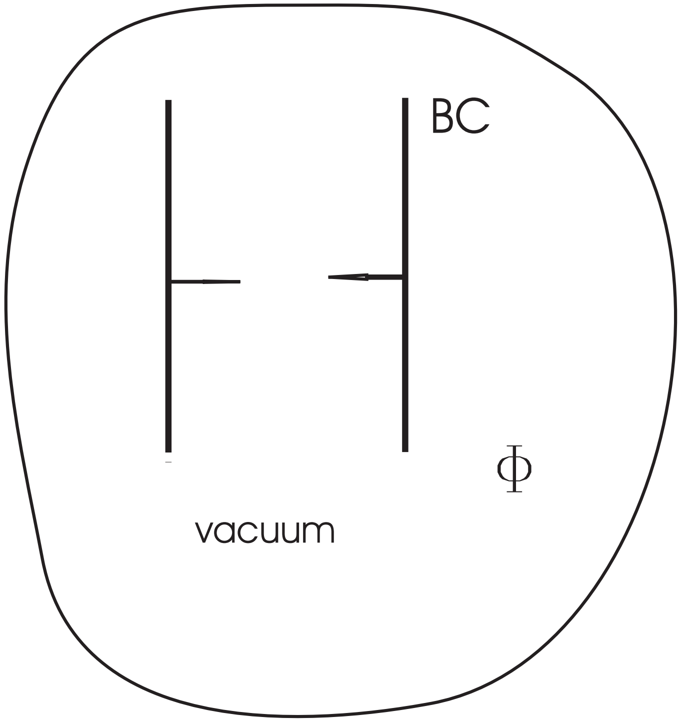

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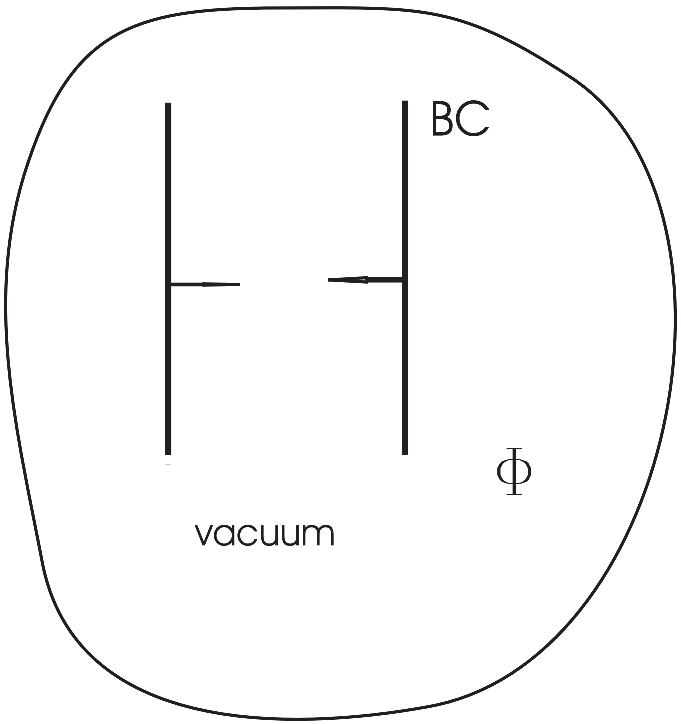

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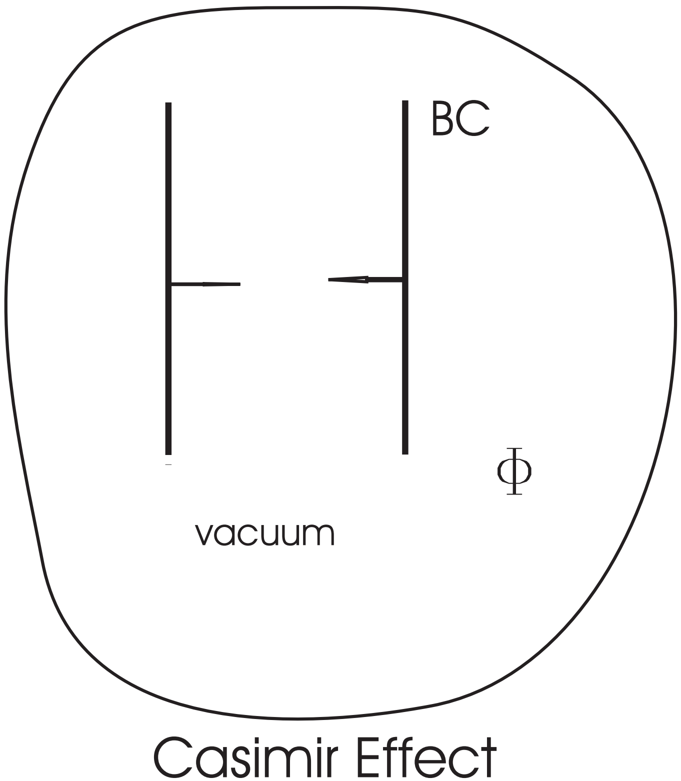

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- Optical cavities
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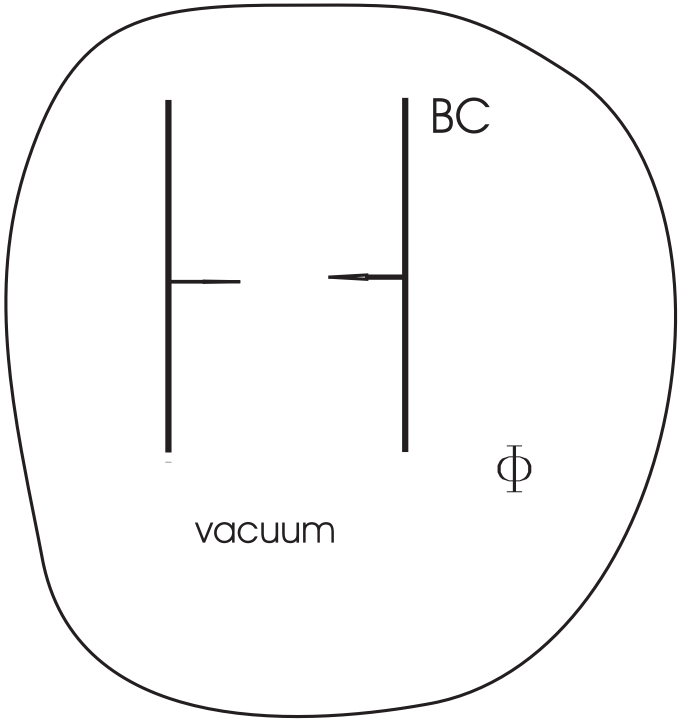

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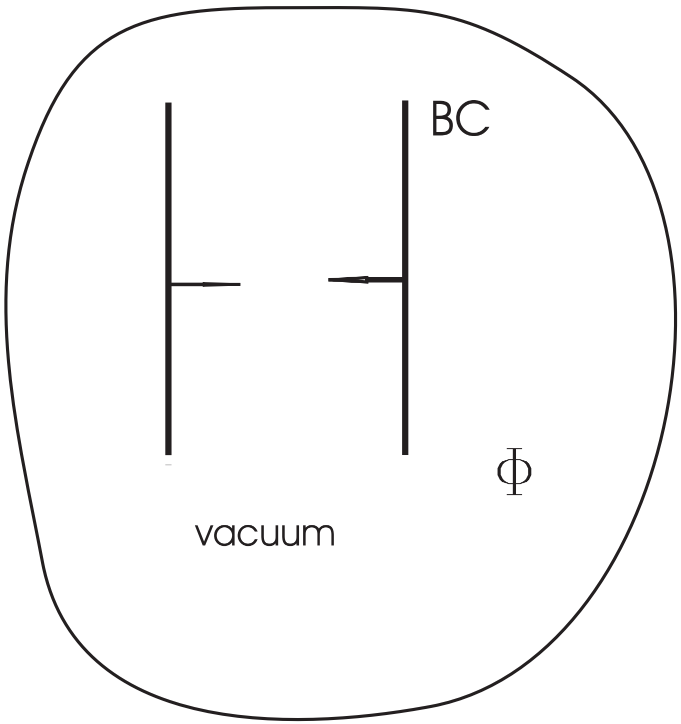

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- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Jaeckel, Reynaud, Lambrecht; Ford, Vilenkin; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani,...

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

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$$\lambda = (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \quad \sim 4.32 \times 10^{-9} \text{ erg/cm}^3$$

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- Idea: zero point fluctuations can contribute to the cosmological constant — Ya.B. Zeldovich '68

- Relativistic field: collection of harmonic oscill's (scalar field)

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$$\rho \;\sim\; \frac{\hbar\, k^4_{Planck}}{16\pi^2} \;\sim\; 10^{123}\rho_{obs}$$

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}\mu$m dims, bulk vs brane Susy breaking scales
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  - (b) dS & AdS worldbranes
  - (c) supergraviton theories (discret dims, deconstr)

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$

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- Scalar field, ϕ, pervading the universe ($\hbar = c = 1$)

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- M effective mass term, m arbitrarily small
  (a tiny mass for the field cannot be excluded, and fits well)

* L. Parker & A. Raval, PRL86 749 (2001); PRD62, 083503 (2000)
* V.G. Gurzadyan & S.-S. Xue, Mod Phys Lett A18, 561 (2003)

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^{p} a_j \prod_{h=1}^{q} b_h}\int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{\mathbf{m}_q=-\infty}^{\infty}\left[\sum_{j=1}^{p}\left(\frac{2\pi n_j}{a_j}\right)^2+\sum_{h=1}^{q}\left(\frac{2\pi m_h}{b_h}\right)^2+\mathbf{k}_d^2+M^2\right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty}\left(\frac{1}{a^2}\sum_{j=1}^{p} n_j^2+\frac{1}{b^2}\sum_{h=1}^{q} m_h^2+M^2\right)^{\frac{d+1}{2}+1}$$

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$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty}\left(\frac{1}{a^2}\sum_{j=1}^p n_j^2+\frac{1}{b^2}\sum_{h=1}^q m_h^2+M^2\right)^{\frac{d+1}{2}+1}$$

- For d–open, (p–toroidal, q–spherical) universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j\, b^q}\int_0^\infty dk\, k^{d-1}\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty}P_{q-1}(l)$$

$$\left[\sum_{j=1}^p\left(\frac{2\pi n_j}{a_j}\right)^2+\frac{Q_2(l)}{b^2}+\mathbf{k}_d^2+M^2\right]^{1/2}$$

[$P_{q-1}(l)$ poly in l deg $q-1$]

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty}P_{q-1}(l)\left(\frac{4\pi^2}{a^2}\sum_{j=1}^p n_j^2+\frac{l(l+q)}{b^2}+\frac{M^2}{4\pi^2}\right)^{\frac{d+1}{2}+1}$$

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

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- For the zeta function (Re $s > p/2$):

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[\frac{1}{2}\left(\vec{n}+\vec{c}\right)^T A\left(\vec{n}+\vec{c}\right)+q\right]^{-s}$$

$$\equiv \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[Q\left(\vec{n}+\vec{c}\right)+q\right]^{-s}$$

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$$\sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c})\left(\vec{m}^T A^{-1}\vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

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- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbb{Z}^p_{1/2}$ means that only half of the vectors $\vec{m}\in\mathbb{Z}^p$ are summed over. That is, if we take an $\vec{m}\in\mathbb{Z}^p$ we must then exclude $-\vec{m}$ (as simple criterion one can, for instance, select those vectors in $\mathbb{Z}^p\backslash\{\vec{0}\}$ whose first non-zero component is positive).

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2} \Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

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Yields the vacuum energy density:

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sqrt{\frac{\sum_{k=1}^{q} m_k^2 + M^2}{\sum_{j=1}^{h} n_j^2}}$$

$$\times K_1 \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

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$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p}\binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

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$\Longrightarrow$ Sign may change with BC (e.g., Dirichlet): a problem

Matching the obs. results for the CC

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ρ_ϕ	$p=0$	$p=1$	$p=2$	$p=3$
$b = l_P$	10^{-13}	10^{-6}	1	10^{5}
$b = 10 l_P$	10^{-14}	$[10^{-8}]$	10^{-3}	10
$b = 10^2 l_P$	10^{-15}	(10^{-10})	10^{-6}	10^{-3}
$b = 10^3 l_P$	10^{-16}	10^{-12}	$[10^{-9}]$	(10^{-7})
$b = 10^4 l_P$	10^{-17}	10^{-14}	10^{-12}	10^{-11}
$b = 10^5 l_P$	10^{-18}	10^{-16}	10^{-15}	10^{-15}

Table 2: Vacuum energy density in units of erg/cm^3, for p large compactified dimensions a, and $q = p+1$ small compactified dimensions b, $p = 0, \ldots, 3$, for different values of b, proportional to the Planck length l_P

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$\Longrightarrow$ To examine $\rightarrow$ couplings in GR

$\rightarrow$ alternative theories

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

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$\Longrightarrow$ Casimir energy density and effective potential for a de Sitter (dS) brane in a five-dimensional anti-de Sitter (AdS) background

$\Longrightarrow$ Action for conformally inv massless scalar with scalar-gravit coupling

$$\mathcal{S} = \frac{1}{2} \int d^5x \sqrt{g} \left[-g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi_5 R^{(5)} \phi^2 \right]$$

$\xi_5 = -3/16$ $\qquad$ $R^{(5)}$ 5-dim scalar curvature

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2 \right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2 \end{aligned}$$

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$$\mathcal{E}_{\text{Cas}} = \frac{\hbar c}{2L\mathcal{R}^4}\zeta\left(-\frac{1}{2}|L_5\right) = -\frac{\hbar c\pi^3}{36L^6}\left[\frac{\pi^2}{315} - \frac{1}{240}\left(\frac{L}{\mathcal{R}}\right)^2 + \mathcal{O}\left(\frac{L}{\mathcal{R}}\right)^4\right]$$

C. Supergraviton Theories

⟹ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

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- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)
- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$
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C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70
$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435
$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127
$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91
$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757
$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96
$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)
- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$ $\Longrightarrow$ shown the induced cosmological constant could be positive due to topological contributions
- Previously considered in $\mathbb{R}^4$

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$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

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$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h^\lambda_{n\mu}\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$
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$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k}$, $\Delta^\dagger \phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n-k}$, $\sum_{k=0}^{N-1} a_k = 0$,

a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

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(Δ becomes usual differentiation operator in properly defined continuum limit)

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- The value of the cc is regulated by the corresponding size of the torus (one can most naturally use the minimum number, N = 3, of copies of bosons and fermions), and is not far from the observational values

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Thank You !