ICE CSIC IEEC

Vacuum Fluctuations,

The Cosmological Constant,

& Alternative Gravities

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ICE/CSIC & IEEC, UAB, Barcelona

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Outline of this presentation

- On Einstein's Cosmological Constant: a Historical Perspective

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- The Casimir Effect: Theory, Experiments, Uses

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On Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cc

- For cosmologists and general relativists: a great mistake (Einstein)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc Λ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics

    stress-energy density μ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g}\, \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g}\, \lambda$$

In other words: two contributions on the same footing (Pauli 20s, Zel'dovich '68)

$$\frac{\Lambda\, c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar\, c}{2} \sum_i \omega_i$$

Einstein Eqs, FLRW Sol, Hubble Const

Einstein Equations (1915-17): $G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu}$

Geometry = Energy-Matter

$G_{\mu\nu}$ linear combination of the metric $g_{\mu\nu}$ and 1st & 2nd derivatives

$T_{\mu\nu}$ energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

Schwarzschild solution (1916) r, θ, φ *comoving co*

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Friedmann-Lemaître-Robertson-Walker (1935-36) solut (A. Friedmann 1922)

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right)$$

gen fam: homog + isotrop, k par $\pm 1, 0$ *Keeler/Slipher/Campbell 1918, Hubble ea 1923-29*

One field eq looks like Newtonian eq for the gravit pot: $\nabla^2 \phi = 4\pi G \left(\rho + 3p/c^2\right)$

density & pressure contribute to the gravit pot $\lambda = 8\pi G \rho_{vac}$, $p_{vac} = -\rho_{vac} c^2$

From the FRW metric and Einstein Eqs, an "equation of motion" of the universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

From GR to Cosmology

With the definitions:

$$\Omega_m \equiv \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \qquad \Omega_k \equiv -\frac{k}{H^2}$$

The equation of motion becomes

$$\left(\frac{da}{dt}\right)^2 = H_0^2\left[\frac{\Omega_m^{(0)}}{a} + a^2\Omega_\lambda^{(0)} + \Omega_k^{(0)}\right]$$

(the superscript (o) represents quantities measured at the present time)

In other terms, Friedmann equation in Cosmology:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_{NR}\left(\frac{a_0}{a}\right)^3 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\lambda\right]$$

Ω_R relativistic matter ($p_R = \frac{1}{3}\rho_R;\ \ \rho_R \propto a^{-4}$) Mach's princ

Ω_{NR} nonrelativistic matter ($p_{NR} = 0;\ \ \rho_{NR} \propto a^{-3}$)

Ω_λ cosmological constant ($p_\lambda = -\rho_\lambda;\ \ \rho_\lambda = \mathrm{const}$)

$\Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda$ total energy density (cosmic triangle)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

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Even then: Has the final value real sense ?

The Casimir Effect

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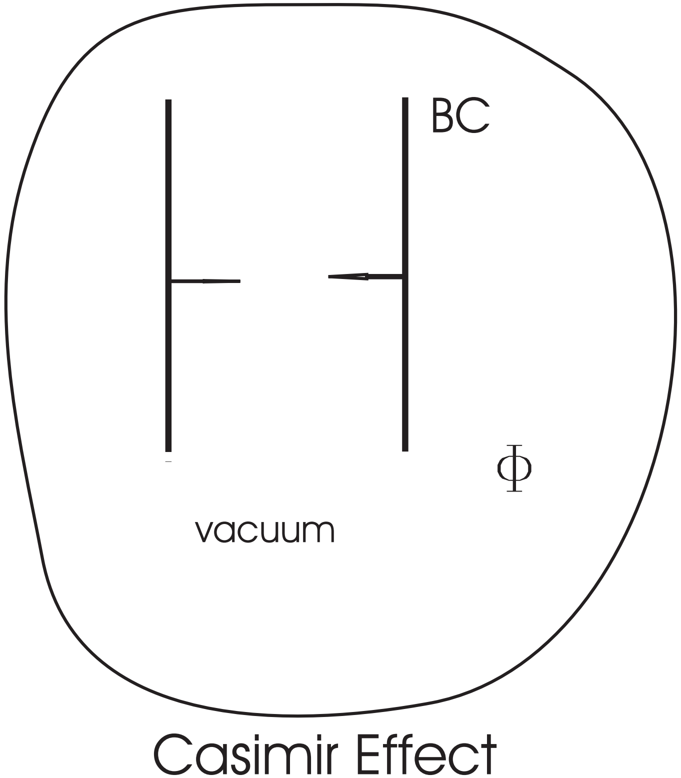
BC e.g. periodic



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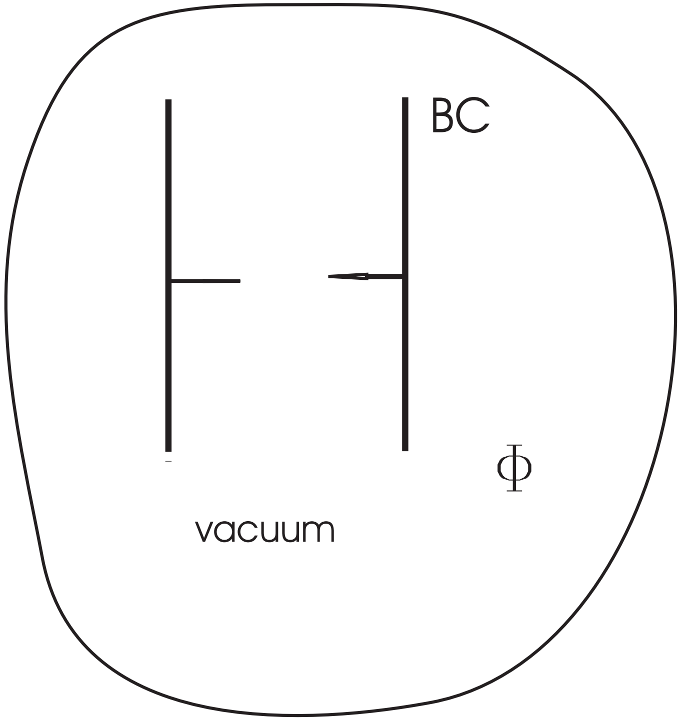

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$\Longrightarrow$ all kind of fields

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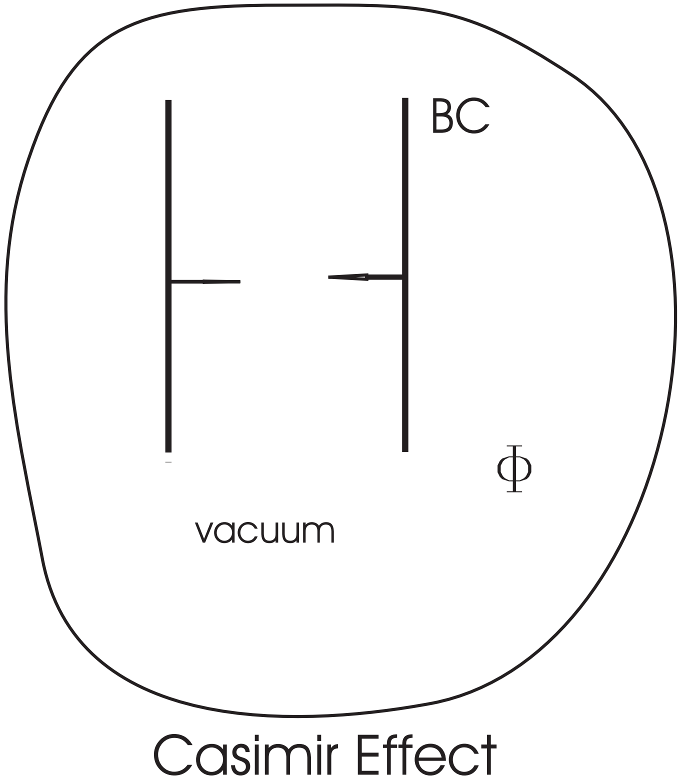

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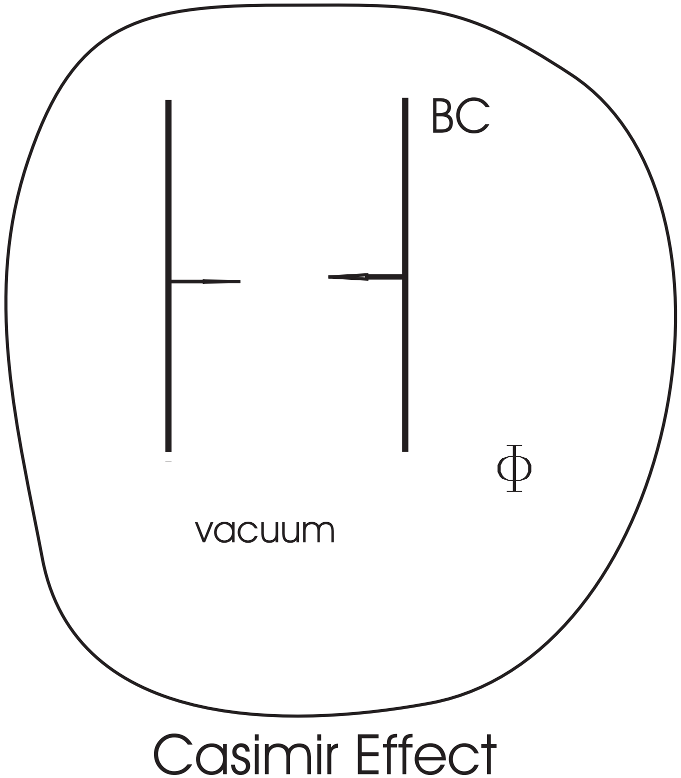

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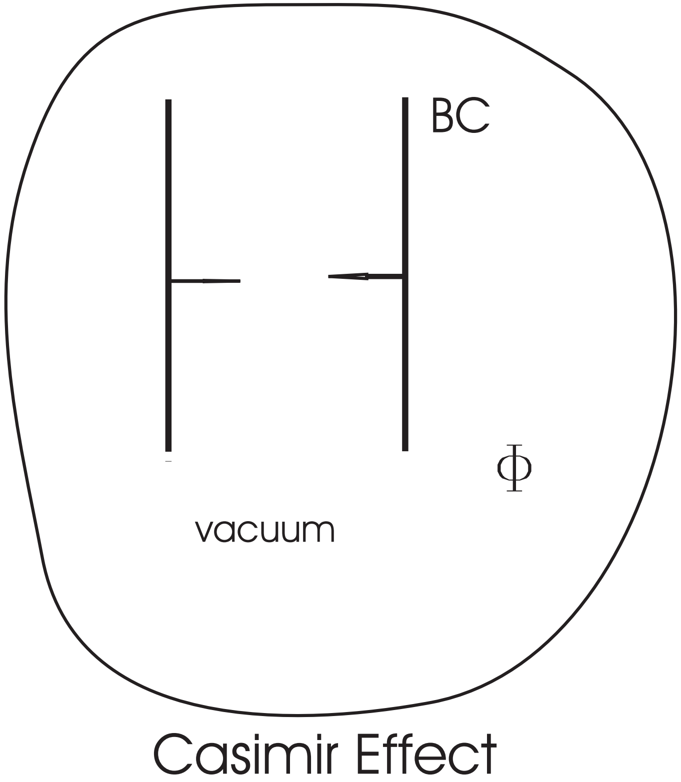
Universal process:

The Casimir Effect



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- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
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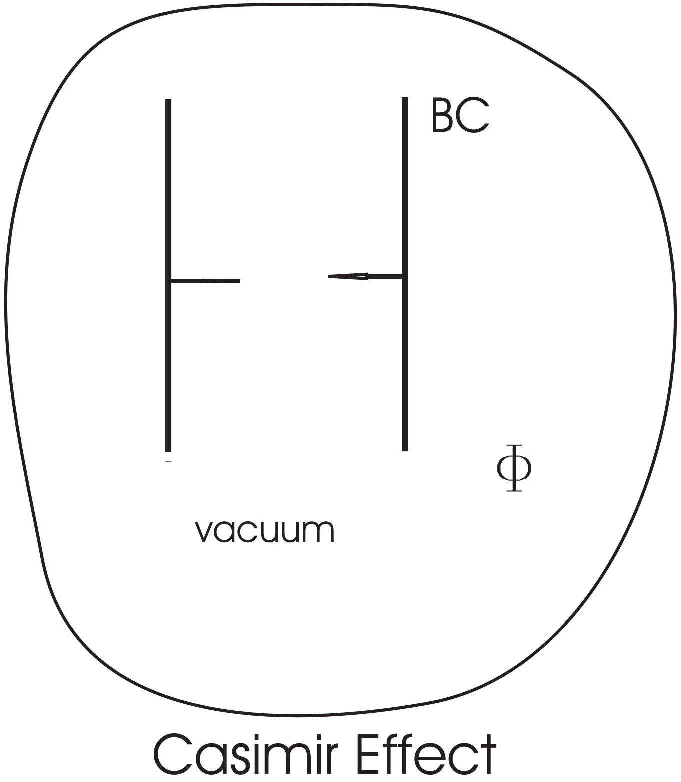

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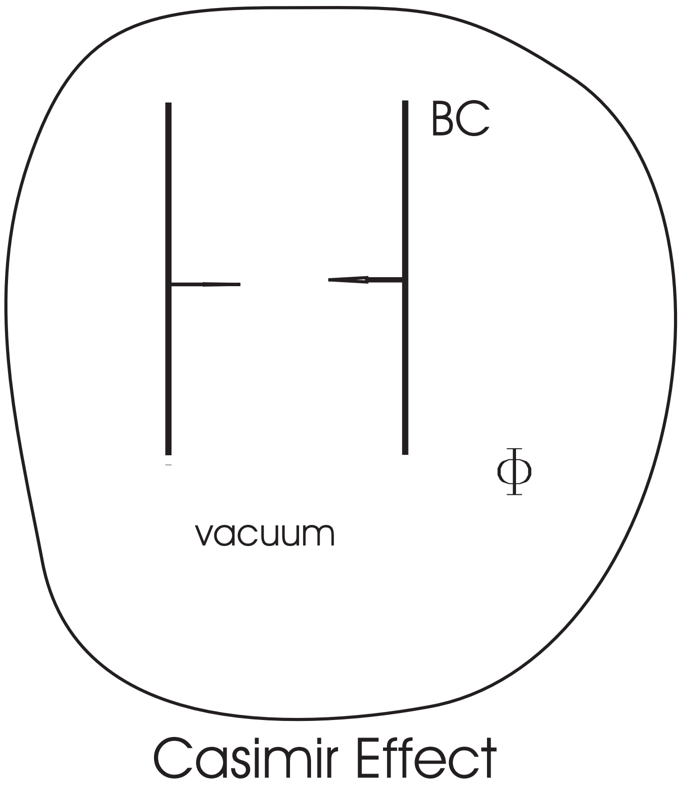

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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

The standard approach

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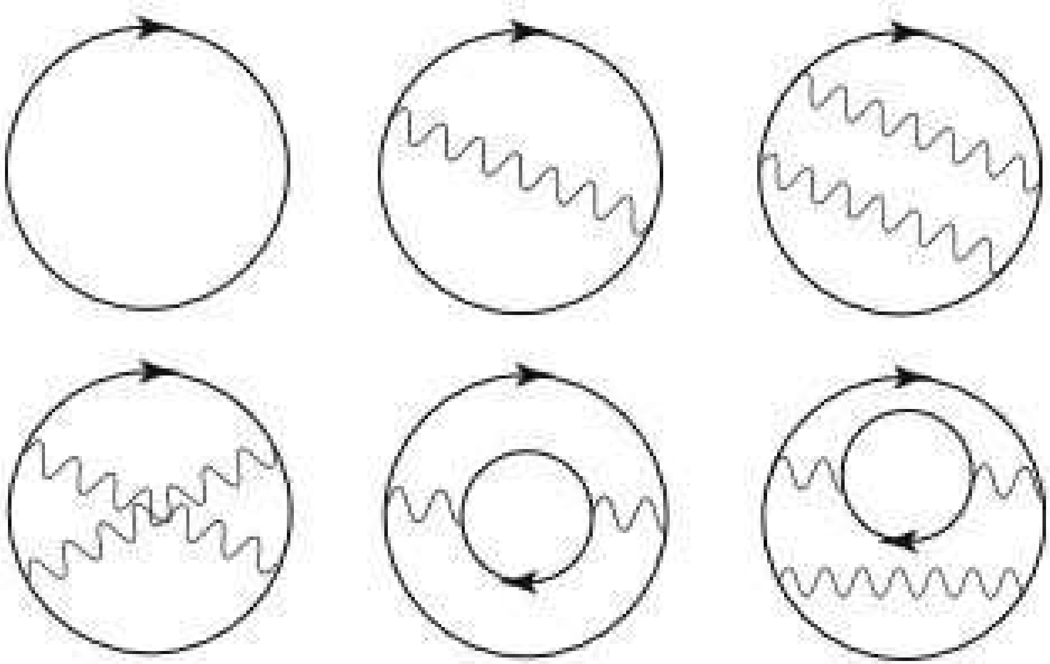

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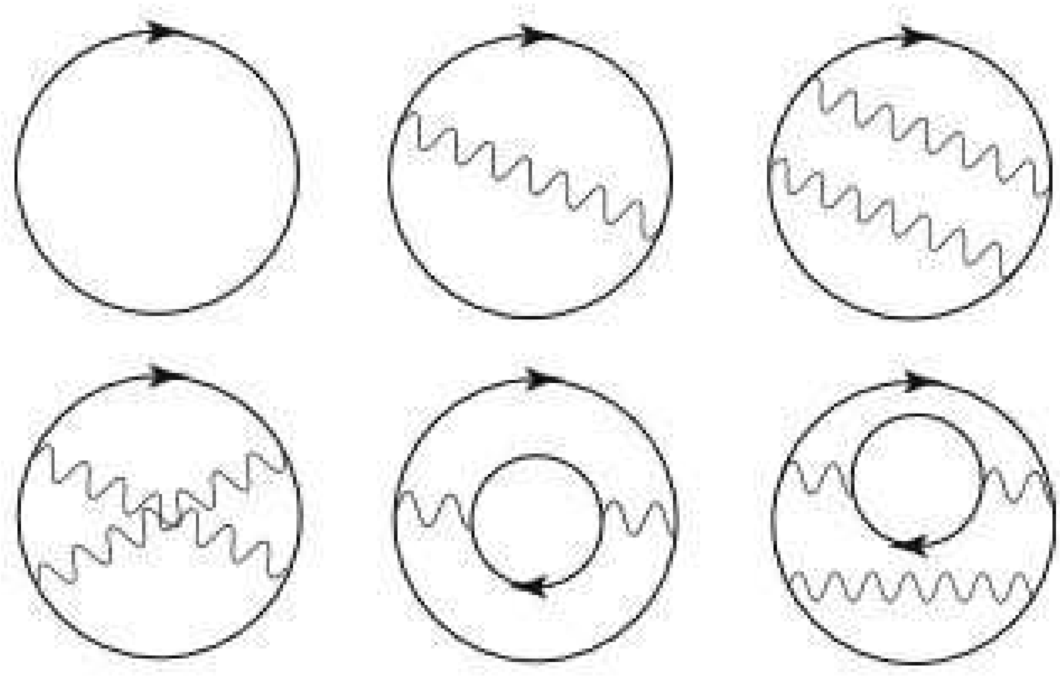

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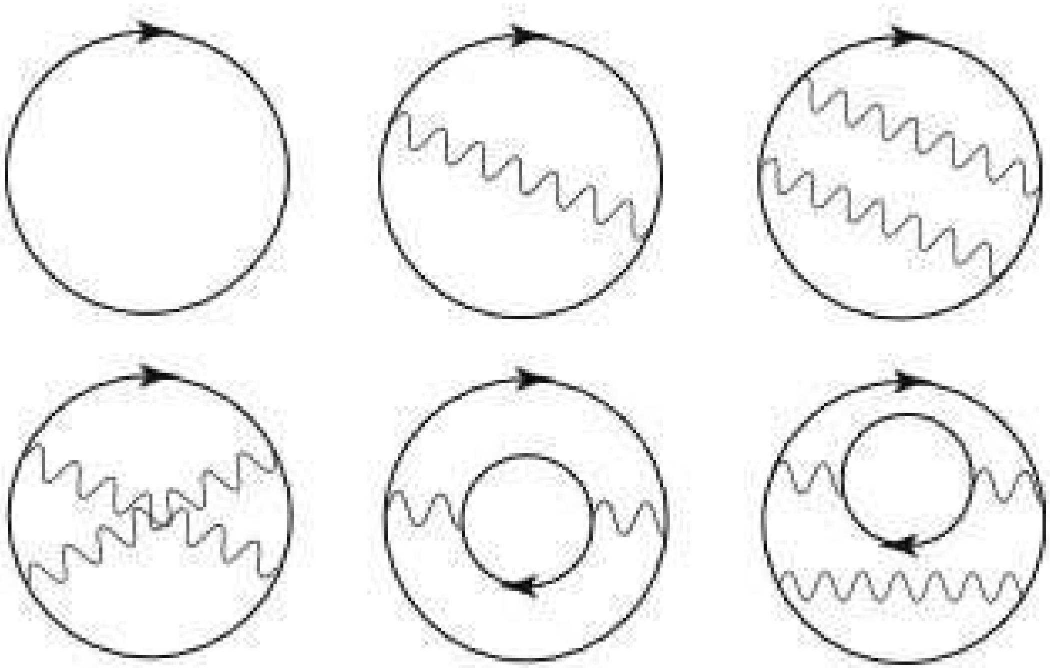

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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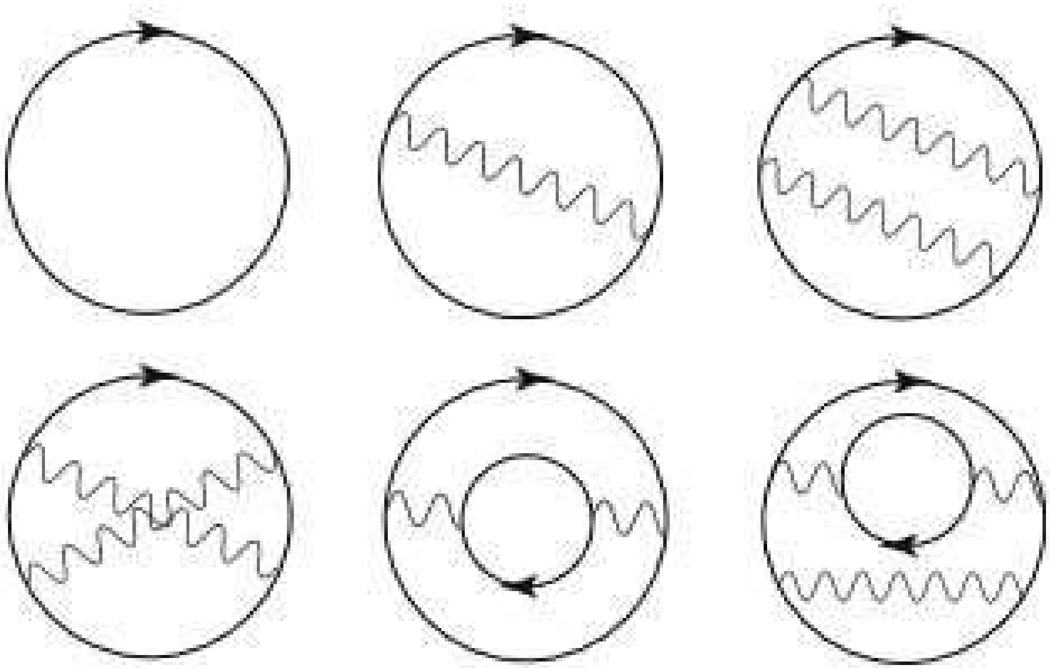

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function — Trace is over spin

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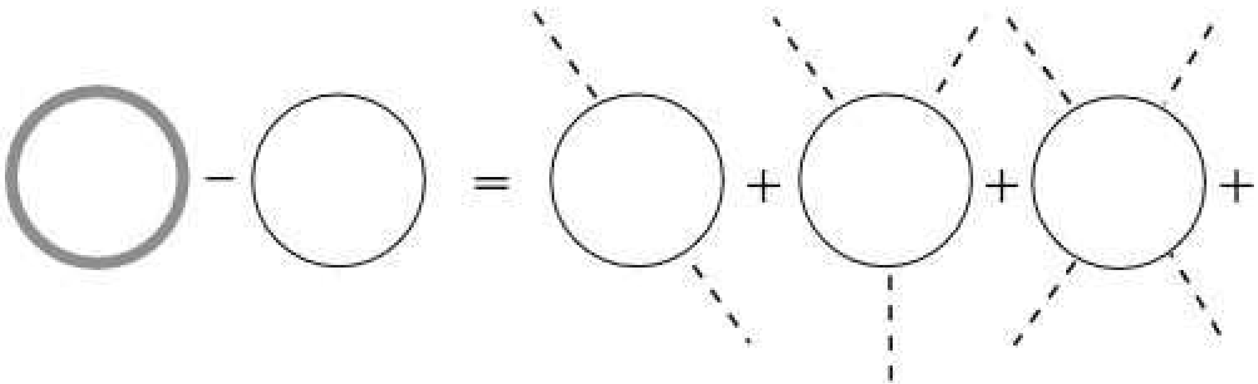

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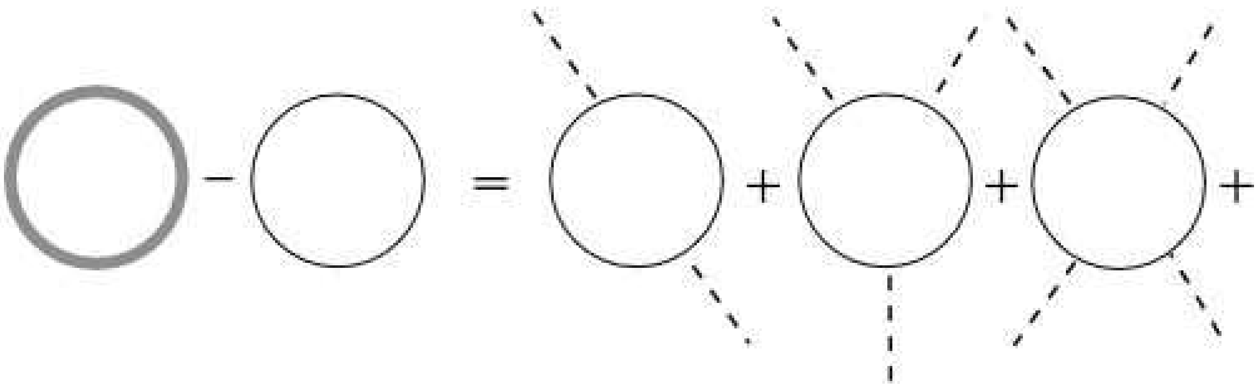

$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

The Dynamical Casimir Effect

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Ford, Vilenkin; Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

Some details of the method

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

(of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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[cf. paragraph after Eq. (4.5)]

Our interpretation: a perfectly reflecting mirror is non-physical.

Consider, instead, a partially transmitting mirror, transparent to high frequencies (math. implementation of a physical plate).

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

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- The main issue: S.A. Fulling et. al., hep-th/070209

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- Idea: zero point fluctuations can contribute to the cosmological constant Pauli 20s, Zeldovich '68

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- Boundary divergences resolved by understanding renormalization process

- Casimir stress tensor between pair of parallel perfectly conducting plates, at distance a, transverse dimensions $L >> a$

[Brown and Maclay, Phys. Rev. 184 (1969) 1272]

$$\langle T_{\mu\nu} \rangle = \frac{\mathcal{E}_{\mathcal{C}}}{a} \, \text{diag}(1, -1, -1, 3)$$

third spatial direction is normal to plates, $\mathcal{E}_{\mathcal{C}}$ Casimir energy per unit area

$$\mathcal{E}_{\mathcal{C}} = -\frac{\pi \hbar c}{720 a^3}$$

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- Gravitational interaction of the Casimir apparatus: use gravitational definition of the energy-momentum tensor as variation of matter part of action:

$$\delta W_m = \frac{1}{2} \int \sqrt{-g} \, \delta g_{\mu\nu} \, T^{\mu\nu} \qquad (*)$$

Following Schwinger (note the factor 2 in the definition), for a weak field Fulling et al define: $g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$

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  energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force
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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

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- B. Other alternatives: (i) L Faddeev 0911.0282 (Adler '82) Newton const in E-H Lag has dim of mass $\longrightarrow$ non-renormalizability Describe gravity by vector field (as Higgs mechanism) (ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in euIR sector and a mechanism to relax the cc. Also Shapiro+Sola, ...

More recent alternatives (a sample)

- (iii) E Mottola 1006.3567 Effective field theory approach
– Casimir effect in flat s-t and large quantum backreaction
are effects at the horizon scale of cosmological s-t
– imply the cosmological VE is dynamical
– its value depends on macroscopic BCs at the cosm horizon scale,
rather than on the extreme ultraviolet Planck scale [we, on both BCs]

More recent alternatives (a sample)

- (iii) E Mottola 1006.3567 Effective field theory approach
  – Casimir effect in flat s-t and large quantum backreaction
  are effects at the horizon scale of cosmological s-t
  – imply the cosmological VE is dynamical
  – its value depends on macroscopic BCs at the cosm horizon scale,
  rather than on the extreme ultraviolet Planck scale [we, on both BCs]

- (iv) T Padmanabhan Ad Sci Lett 2 74 09 cc problem and explaining
  DE independent issues: first find mechanism to make the cc vanish
  – new degrees of freedom, kind of 'gauge freedom'
  to absorb any λ while maintaining general covariance
  – could succeed in making gravity decouple from the bulk VE
  – emergent gravity approach: thermodynamic description is far more
  general than just Einstein theory
  – observed cc should be a relic of quantum gravitational physics and
  arise from degrees of freedom which scale as the surface area
  – numerics: $L_\Lambda / L_P \sim \exp \sqrt{2}\, \pi^4 \sim 10^{60}$ (hierarchy squared) $\sim 10^{61}$

- (v) Shao & Chen 1005.1920 no attempt at explaining the old cc prob
  – an extremely small quantum correction can in fact be produced quite naturally from a massive bulk field, introducing a massive bulk fermion
  – naturally as superpartner of the radion field in a SUSY theory (especially the string theory realization) of brane-world scenario
  – in particle physics Grossman and Neubert used a massive bulk fermion to understand the neutrino mass hierarchy
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- (vi) JA Dixon 1006.2334 CyberSUSY solves the cc problem
  – a new mechanism for SUSY breaking
  – its realization mixes elementary and composite states
  – SUSY anomalies present, generates spectrum for SUSY breaking consistent with known particles
  – no cc generated, because SUSY is not spontaneously broken ...

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem

2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds [A Flachi]

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, Nojiri, Odintsov, Ogushi, PRD67(2003)063515, hep-th/0209242 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, Odintsov, Saharian PRD79(2009)065023, 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account

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- More general results: Kenneth, Klich, PRL 97, 160401 (2006)
  a mirror pair of dielectric bodies always attract each other
  CP Bachas, J Phys A40, 9089 (2007) from a general property of Euclidean QFT 'reflection positivity' (Osterwalder - Schrader 73, 75):
  $\exists$ of positive Hilbert space and self-adjoint non-negative Hamiltonian

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
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  - Robin BCs in general $\Leftarrow$

Casimir eff in brworl's w large extra dim

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- Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein's entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]

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For arbitrary internal space, interaction part of the Casimir energy given by

$$\Delta E_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^\infty dx\, x(x^2 - m_\beta^2)^{D_1/2-1}$$

$$\times \ln\left[1 - \frac{(\beta_1 x + 1)(\beta_2 x + 1)}{(\beta_1 x - 1)(\beta_2 x - 1)} e^{-2ax}\right] \qquad (*)$$

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For Dirichlet and Neumann BCs on both plates this leads to

$$\Delta E_{[a_1,a_2]}^{(\mathrm{J},\mathrm{J})} = -\frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{n^{D_1+1}}$$

with $f_\nu(z) = z^\nu K_\nu(z)$ $\longrightarrow$ energy always negative

For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

$$\Delta E_{[a_1,a_2]}^{(\mathrm{D,N})} = \frac{(4\pi)^{-D_1/2} a}{\Gamma(D_1/2+1)} \sum_{\beta} \int_{m_\beta}^{\infty} dx \, \frac{\left(x^2 - m_\beta^2\right)^{D_1/2}}{e^{2ax}+1}$$

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In the case of a conformally coupled massless field on the background of a spacetime conformally related to the one described by the line element

$$ds^2 = g_{MN}dx^M dx^N = \eta_{\mu\nu}dx^\mu dx^\nu - \gamma_{il}dX^i dX^l$$

$\eta_{\mu\nu} = \mathrm{diag}(1,-1,\ldots,-1)$ metric of (D_1+1)-dim Minkowski st and X^i coordinates of Σ, with the conformal factor $\Omega^2(x^{D_1})$. Interaction part of Casimir energy is given (*), with coeffs β_j related to coeffs of the Robin BCs

$$(1+\overline{\beta}_j n^M\nabla_M)\overline{\varphi}(x) = [1+(-1)^{j-1}\Omega_j^{-1}\overline{\beta}_j\partial_{D_1}]\overline{\varphi}(x) = 0,\ \Omega_j = \Omega(x_j^{D_1})$$

& conformal factor $\beta_j = \left[\Omega_j + (-1)^j\frac{D-1}{2\Omega_j}\overline{\beta}_j\Omega'_j\right]^{-1}\overline{\beta}_j,\ \Omega'_j = \Omega'_j(x_j^{D_1})$

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\overline{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

The vacuum energy can have a minimum, for the stable equilibrium point

Can be used in braneworld models for the stabilization of the radion field

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We have considered a piston-like geometry, introducing a third plate (then this plate is sent to infinity) Casimir force

$$P = -\frac{2(4\pi)^{-D_1/2}}{V_\Sigma \Gamma(D_1/2) a^{D_1+1}} \sum_\beta \int_{am_\beta}^\infty dx \, \frac{x^2 (x^2 - a^2 m_\beta^2)^{D_1/2-1}}{\frac{(b_1 x-1)(b_2 x-1)}{(b_1 x+1)(b_2 x+1)} e^{2x} - 1}$$

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With independence of the geometry of the internal space, the force is attractive for Dirichlet or Neumann boundary conditions on both plates

$$P^{(\mathrm{J},\mathrm{J})} = -\frac{2(4\pi)^{-D_1/2}}{V_\Sigma \Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^\infty dx \, x^2 \frac{(x^2 - m_\beta^2)^{D_1/2-1}}{e^{2ax} - 1}$$
$$= \frac{2a^{-D_1-1}}{(8\pi)^{(D_1+1)/2} V_\Sigma} \sum_\beta \sum_{n=1}^\infty \frac{1}{n^{D_1+1}} \left[f_{(D_1+1)/2}(2nam_\beta) - f_{(D_1+3)/2}(2nam_\beta) \right]$$

$\mathrm{J} = \mathrm{D}, \mathrm{N}$, and repulsive for Dirichlet BC on one plate and Neumann on the other, a monotonic function of the distance

$\Longrightarrow$ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

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$\Longrightarrow$ Remarks: (i) This property could be used in the proposal of a Casimir experiment with the purpose to carry out an explicit detailed observation of 'large' extra dimensions as allowed by some models of particle physics
(ii) Possible laboratory verification (Robin BCs model skin depth of material)

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"

  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...

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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  - entropy proport to variation of the horizon area: $\delta S = \eta \, \delta \mathcal{A}$
  - local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$
  - functional dependence of S wrt energy and size of system

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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_{\Sigma} E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta \left(\eta_e A \right)$$

η_e is a function of the metric and its deriv's to a given order

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\begin{aligned}\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\ &= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad\qquad S = \frac{A}{4\,G_{eff}}\end{aligned}$$

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- Final result, for $\mathbf{f}(R)$ gravities:
the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

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- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
  T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
  C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]

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Moltes gràcies!