ICE CSIC IEEC

First consistent formulation of the Dynamical Casimir Effect, and its possible impact on Cosmology

EMILIO ELIZALDE

ICE/CSIC & IEEC, UAB, Barcelona

web: google → emilio elizalde

Cosmology Workshop Montpellier06, Nov. 23-24, 2006

Outline of the talk

- On the Zero Point Energy & the Casimir Effect

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant
- New CC Theories & Cosmo-topological Casimir Effect

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant
- New CC Theories & Cosmo-topological Casimir Effect
- A Simple Model with large + small dimensions

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant
- New CC Theories & Cosmo-topological Casimir Effect
- A Simple Model with large + small dimensions
- Mathematical Tools: Zeta's, Det's, Residues, Anomalies

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant
- New CC Theories & Cosmo-topological Casimir Effect
- A Simple Model with large + small dimensions
- Mathematical Tools: Zeta's, Det's, Residues, Anomalies
- Vacuum Energy in More Realistic Models:
  - dS & AdS Braneworlds
  - Supergraviton Theories

Outline of the talk

- On the Zero Point Energy & the Casimir Effect
- Dynamical Casimir Effect: First Consistent Formulation
- Vacuum Fluctuations & the Cosmological Constant
- New CC Theories & Cosmo-topological Casimir Effect
- A Simple Model with large + small dimensions
- Mathematical Tools: Zeta's, Det's, Residues, Anomalies
- Vacuum Energy in More Realistic Models:
  - dS & AdS Braneworlds
  - Supergraviton Theories
- Conclusions

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \; a_n \; a_n^\dagger$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0 | H | 0 \rangle = \frac{\hbar \, c}{2} \sum_n \lambda_n = \frac{1}{2} \, \mathrm{tr} \, H$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle \;=\; \frac{\hbar\, c}{2} \sum_n \lambda_n \;=\; \frac{1}{2}\, \mathrm{tr}\, H$$

gives ∞ physical meaning?

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0 | H | 0 \rangle \; = \; \frac{\hbar\, c}{2} \sum_n \lambda_n \; = \; \frac{1}{2} \, \mathrm{tr} \, H$$

gives ∞ physical meaning?

Regularization $+$ Renormalization (cut-off, dim, ζ)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{\hbar\, c}{2} \sum_n \lambda_n = \frac{1}{2} \,\text{tr}\, H$$

gives ∞ physical meaning?

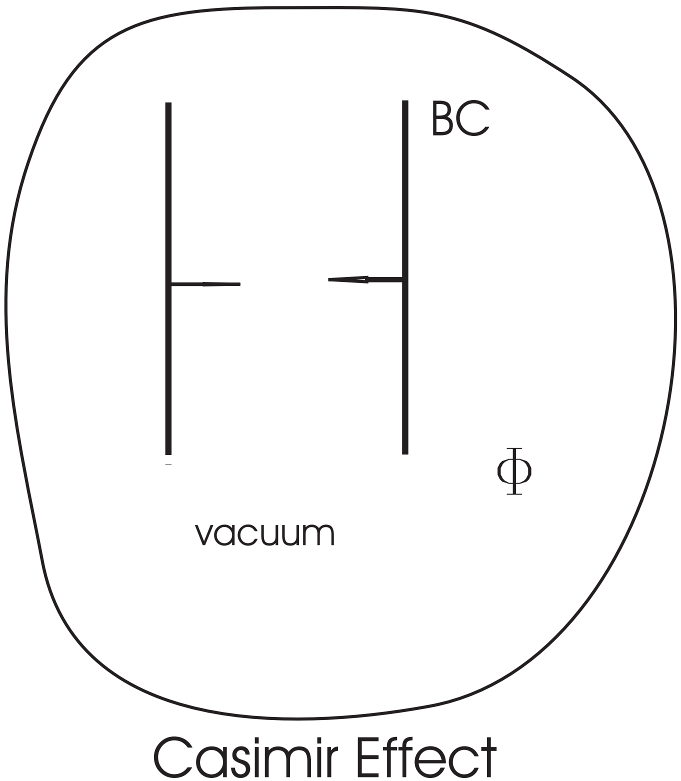
Regularization $+$ Renormalization (cut-off, dim, ζ)

Even then: Has the final value real sense ?

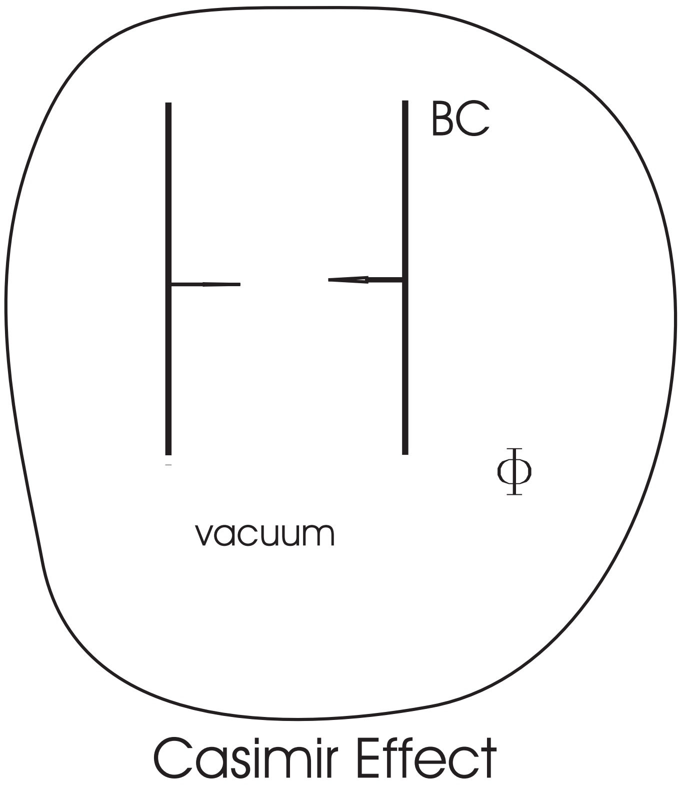
The Casimir Effect

The Casimir Effect

BC e.g. periodic



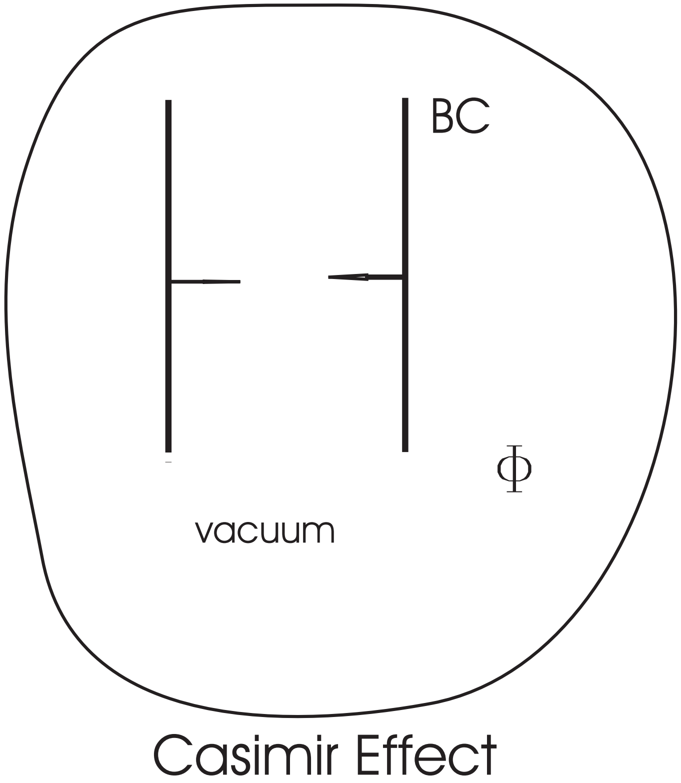
The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

The Casimir Effect

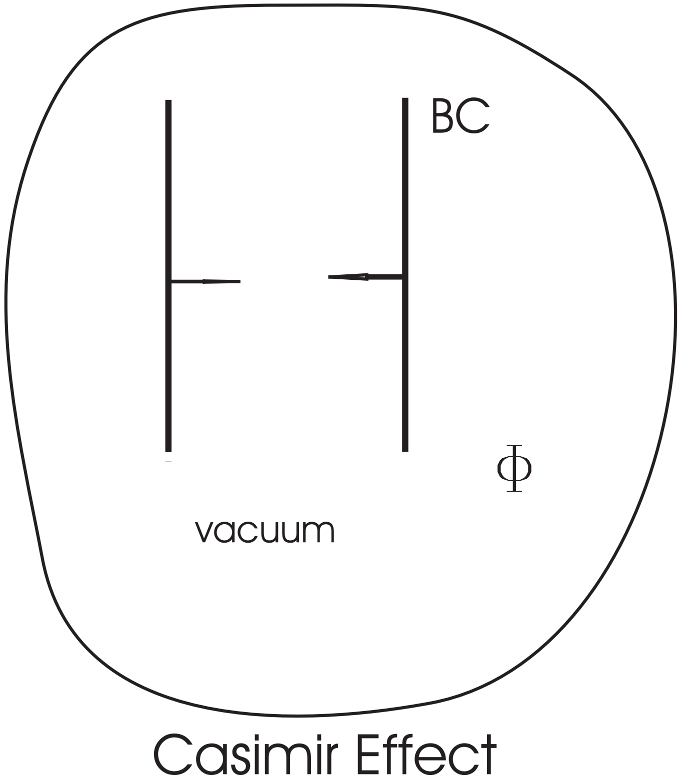


BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

The Casimir Effect



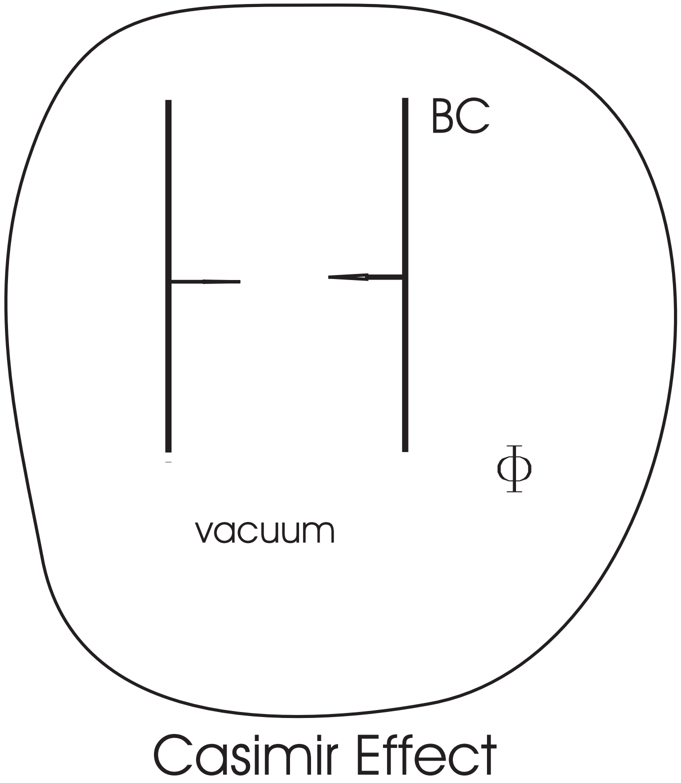
BC e.g. periodic

⟹ all kind of fields

⟹ curvature or topology

Universal process:

The Casimir Effect



BC e.g. periodic

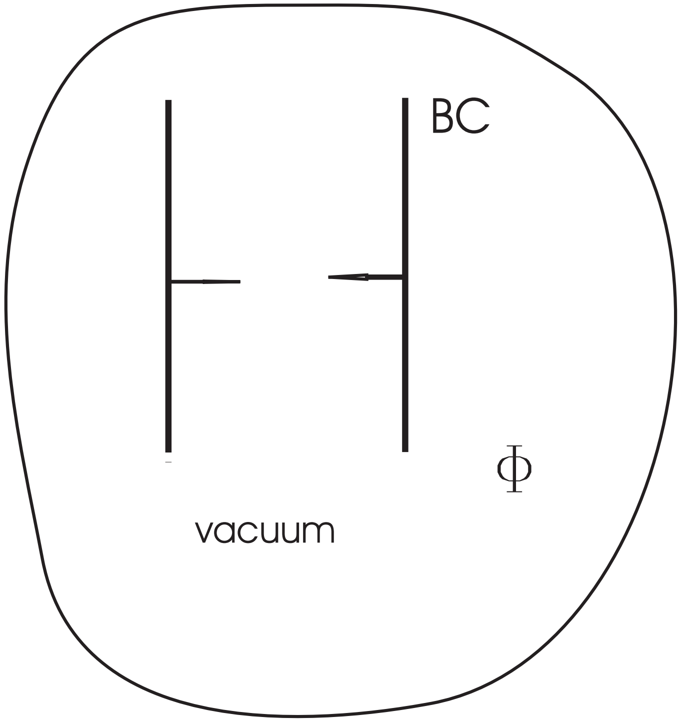
$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

The Casimir Effect



Casimir Effect

BC e.g. periodic

$\Longrightarrow$ all kind of fields

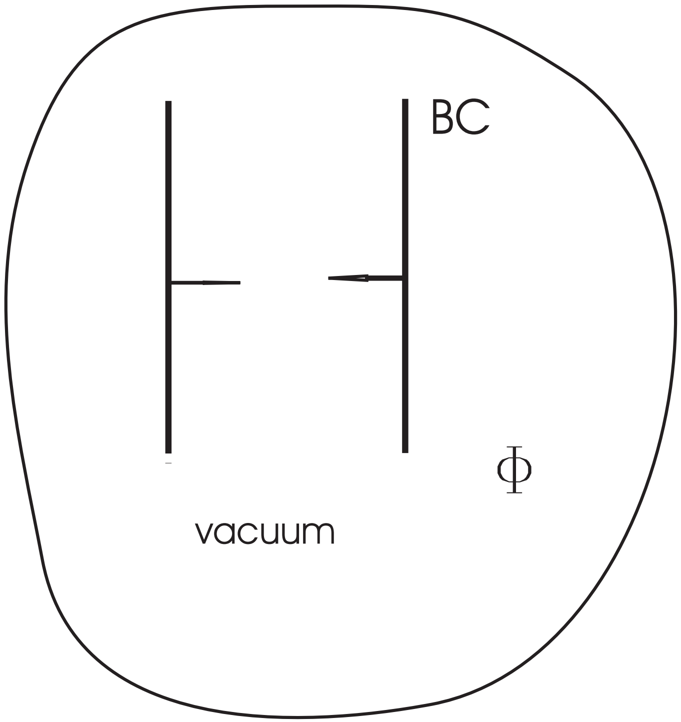
$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lifschitz theory

The Casimir Effect



Casimir Effect

BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lifschitz theory

- Dynamical CE $\Longleftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Longleftarrow$

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

- Moving mirrors modify structure of quantum vacuum
- Creation and annihilation of photons; once mirrors return to rest, some produced photons may still remain: flux of radiated particles

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

- Moving mirrors modify structure of quantum vacuum
- Creation and annihilation of photons; once mirrors return to rest, some produced photons may still remain: flux of radiated particles
- For a single, perfectly reflecting mirror:
  # photons & energy diverge while mirror moves

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

- Moving mirrors modify structure of quantum vacuum
- Creation and annihilation of photons; once mirrors return to rest, some produced photons may still remain: flux of radiated particles
- For a single, perfectly reflecting mirror:
  # photons & energy diverge while mirror moves
- Several renormalization prescriptions have been used in order to obtain a well-defined energy

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

- Moving mirrors modify structure of quantum vacuum
- Creation and annihilation of photons; once mirrors return to rest, some produced photons may still remain: flux of radiated particles
- For a single, perfectly reflecting mirror:
  # photons & energy diverge while mirror moves
- Several renormalization prescriptions have been used in order to obtain a well-defined energy
- Problem: for some trajectories this finite energy is not a positive quantity and cannot be identified with the energy of the photons

The Dynamical Casimir Effect

S.A. Fulling & P.C.W. Davies, Proc Roy Soc A348 (1976)

- Moving mirrors modify structure of quantum vacuum
- Creation and annihilation of photons; once mirrors return to rest, some produced photons may still remain: flux of radiated particles
- For a single, perfectly reflecting mirror:
  # photons & energy diverge while mirror moves
- Several renormalization prescriptions have been used in order to obtain a well-defined energy
- Problem: for some trajectories this finite energy is not a positive quantity and cannot be identified with the energy of the photons

Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Jaeckel, Reynaud, Lambrecht; Ford, Vilenkin; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

1ST CONSISTENT APPROACH: J. Haro & E.E., PRL 97 (2006)

1st Consistent Approach: J. Haro & E.E., PRL 97 (2006)

- Partially transmitting mirrors, which become transparent to very high frequencies (analytic matrix)
- Proper use of a Hamiltonian method & corresponding renormalization

1ST CONSISTENT APPROACH: J. Haro & E.E., PRL 97 (2006)

- Partially transmitting mirrors, which become transparent to very high frequencies (analytic matrix)
- Proper use of a Hamiltonian method & corresponding renormalization
- Proved both: # of created particles is finite & their energy is always positive, for the whole trajectory during the mirrors' displacement

1st Consistent Approach: J. Haro & E.E., PRL 97 (2006)

- Partially transmitting mirrors, which become transparent to very high frequencies (analytic matrix)
- Proper use of a Hamiltonian method & corresponding renormalization
- Proved both: # of created particles is finite & their energy is always positive, for the whole trajectory during the mirrors' displacement
- The radiation-reaction force acting on the mirrors owing to emission-absorption of particles is related with the field's energy through the energy conservation law: energy of the field at any t equals (with opposite sign) the work performed by the reaction force up to time t

1ST CONSISTENT APPROACH: J. Haro & E.E., PRL 97 (2006)

- Partially transmitting mirrors, which become transparent to very high frequencies (analytic matrix)
- Proper use of a Hamiltonian method & corresponding renormalization
- Proved both: # of created particles is finite & their energy is always positive, for the whole trajectory during the mirrors' displacement
- The radiation-reaction force acting on the mirrors owing to emission-absorption of particles is related with the field's energy through the energy conservation law: energy of the field at any t equals (with opposite sign) the work performed by the reaction force up to time t
- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest

1st Consistent Approach: J. Haro & E.E., PRL 97 (2006)

- Partially transmitting mirrors, which become transparent to very high frequencies (analytic matrix)
- Proper use of a Hamiltonian method & corresponding renormalization
- Proved both: # of created particles is finite & their energy is always positive, for the whole trajectory during the mirrors' displacement
- The radiation-reaction force acting on the mirrors owing to emission-absorption of particles is related with the field's energy through the energy conservation law: energy of the field at any t equals (with opposite sign) the work performed by the reaction force up to time t
- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

SOME DETAILS OF THE METHOD

Some details of the method

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$ (of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)
- Assume boundary at rest for time $t \leq 0$ and returns to its initial position at time T

Some details of the method

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$ (of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)
- Assume boundary at rest for time $t \leq 0$ and returns to its initial position at time T
- Hamiltonian density conveniently obtained using the method in Johnston, Sarkar, JPA 29 (1996) 1741

  Coordinate change used to convert the moving boundary Ω_t into a fixed one $\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

  ($\bar{t}$ the new time)

Some details of the method

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$ (of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)

- Assume boundary at rest for time $t \leq 0$ and returns to its initial position at time T

- Hamiltonian density conveniently obtained using the method in Johnston, Sarkar, JPA 29 (1996) 1741

  Coordinate change used to convert the moving boundary Ω_t into a fixed one $\widetilde{\Omega}$: $(t(\bar{t},\mathbf{y}),\mathbf{x}(\bar{t},\mathbf{y})) = \mathcal{R}(\bar{t},\mathbf{y}) = (\bar{t},\mathbf{R}(\bar{t},\mathbf{y}))$

  ($\bar{t}$ the new time)

- Hamiltonian density

$$\widetilde{\mathcal{H}}(\bar{t},\mathbf{y}) = \frac{1}{2}\left(\widetilde{\xi}^2 + J|\nabla_{\mathbf{x}}\phi|^2\right) + \widetilde{\xi}\left(\partial_{\bar{t}}\widetilde{\phi} - \sqrt{J}\partial_t\phi\right)$$

$\widetilde{\phi}$ field, $\widetilde{\xi}$ conjugate momentum, J Jacobian: $d^3\mathbf{x} \equiv J d^3\mathbf{y}$

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t (cf. paragraph after Eq. (4.5))

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t (cf. paragraph after Eq. (4.5))

Our interpretation: a perfectly reflecting mirror is non-physical. Decided to consider instead a partially transmitting mirror, transparent to high frequencies.

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t (cf. paragraph after Eq. (4.5))

Our interpretation: a perfectly reflecting mirror is non-physical. Decided to consider instead a partially transmitting mirror, transparent to high frequencies.

Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)e^{-2i\omega L} \\ r(\omega)e^{2i\omega L} & s(\omega) \end{pmatrix}$$

$\Longrightarrow$ S matrix is taken to be: $x = L$ position of the mirror

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

Seminal Davis-Fulling model [PRSL A348 (1976) 393] renormalized energy is negative: while the mirror moves, the renormalized energy cannot be considered as the energy of the produced particles at time t (cf. paragraph after Eq. (4.5))

Our interpretation: a perfectly reflecting mirror is non-physical. Decided to consider instead a partially transmitting mirror, transparent to high frequencies.

Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)e^{-2i\omega L} \\ r(\omega)e^{2i\omega L} & s(\omega) \end{pmatrix}$$

$\Longrightarrow$ S matrix is taken to be: $x = L$ position of the mirror

$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions (material's permitivity and resistivity)

RESULTS ARE REWARDING:

RESULTS ARE REWARDING:

In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \text{Re} \left[e^{-i(\omega+\omega')t} \, \widehat{\dot{g}\theta_t}(\omega + \omega') \right]$$
$$\times \left[|r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 \right] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

RESULTS ARE REWARDING:

In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t)\rangle = -\frac{\epsilon}{2\pi^2}\int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega+\omega'} \text{Re}\left[e^{-i(\omega+\omega')t}\, \widehat{\dot{g}\theta_t}(\omega+\omega')\right]$$
$$\times[|r(\omega)+r^*(\omega')|^2 + |s(\omega)-s^*(\omega')|^2] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

Energy conservation is fulfilled: the dynamical energy at any time t equals, with the opposite sign, the work performed by the reaction force up to that time t

$$\langle \hat{E}(t)\rangle = -\epsilon \int_0^t \langle \hat{F}_{Ha}(\tau)\rangle \dot{g}(\tau) d\tau$$

COMPARISON WITH OTHER RESULTS

Heisenberg picture approach:

- Got the “in” modes when the mirror describes the trajectory
- Obtained the average number of produced particles after the mirror returns to rest, by calculating the Bogoliubov coefficients

Comparison with other results

Heisenberg picture approach:

- Got the “in” modes when the mirror describes the trajectory
- Obtained the average number of produced particles after the mirror returns to rest, by calculating the Bogoliubov coefficients
- The radiation-reaction force in the Heisenberg picture, $\langle \hat{F}_H(t) \rangle$, is the difference between the energy density of the “in” vacuum state on the left and right sides of the mirror

COMPARISON WITH OTHER RESULTS

Heisenberg picture approach:

- Got the "in" modes when the mirror describes the trajectory
- Obtained the average number of produced particles after the mirror returns to rest, by calculating the Bogoliubov coefficients
- The radiation-reaction force in the Heisenberg picture, $\langle \hat{F}_H(t) \rangle$, is the difference between the energy density of the "in" vacuum state on the left and right sides of the mirror
- Regularization is needed to obtain a well-defined (cut-off independent) quantity

Comparison with other results

Heisenberg picture approach:

- Got the “in” modes when the mirror describes the trajectory
- Obtained the average number of produced particles after the mirror returns to rest, by calculating the Bogoliubov coefficients
- The radiation-reaction force in the Heisenberg picture, $\langle \hat{F}_H(t) \rangle$, is the difference between the energy density of the “in” vacuum state on the left and right sides of the mirror
- Regularization is needed to obtain a well-defined (cut-off independent) quantity
- Final formula disagrees with the radiation-reaction force obtained using the Hamiltonian approach

Comparison with other results

Heisenberg picture approach:

- Got the “in” modes when the mirror describes the trajectory
- Obtained the average number of produced particles after the mirror returns to rest, by calculating the Bogoliubov coefficients
- The radiation-reaction force in the Heisenberg picture, $\langle \hat{F}_H(t) \rangle$, is the difference between the energy density of the “in” vacuum state on the left and right sides of the mirror
- Regularization is needed to obtain a well-defined (cut-off independent) quantity
- Final formula disagrees with the radiation-reaction force obtained using the Hamiltonian approach
- Been able to prove that the force coincides with the radiation-reaction force calculated by Jaekel and Reynaud after renormalization:

$$\langle \hat{F}_{J,R,ren}(t) \rangle \equiv \langle \hat{F}_{H,ren}(t) \rangle$$

We conclude that the method of Jaekel and Reynaud is equivalent to the quantum theory in the Heisenberg picture

We conclude that the method of Jaekel and Reynaud is equivalent to the quantum theory in the Heisenberg picture

- The dissipative parts of $\langle \hat{F}_{Ha}(t) \rangle$ and $\langle \hat{F}_{J,R,ren}(t) \rangle$ always agree
- The reactive parts do not match, there is the relation

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\alpha\epsilon}{2\pi}\ddot{g}(t) + \langle \hat{F}_{J,R,ren}(t) \rangle$$

We conclude that the method of Jaekel and Reynaud is equivalent to the quantum theory in the Heisenberg picture

- The dissipative parts of $\langle \hat{F}_{Ha}(t) \rangle$ and $\langle \hat{F}_{J,R,ren}(t) \rangle$ always agree
- The reactive parts do not match, there is the relation

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\alpha \epsilon}{2\pi} \ddot{g}(t) + \langle \hat{F}_{J,R,ren}(t) \rangle$$

- Crucial point: during the movement of the mirror, the work done by the motion force $\langle \hat{F}_{J,R,ren}(t) \rangle$ is *not* a negative quantity: the dynamical energy is not positive and a meaningless result is obtained because the dynamical energy is the energy of the emitted photons

We conclude that the method of Jaekel and Reynaud is equivalent to the quantum theory in the Heisenberg picture

- The dissipative parts of $\langle \hat{F}_{Ha}(t) \rangle$ and $\langle \hat{F}_{J,R,ren}(t) \rangle$ always agree
- The reactive parts do not match, there is the relation

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\alpha\epsilon}{2\pi}\ddot{g}(t) + \langle \hat{F}_{J,R,ren}(t) \rangle$$

- Crucial point: during the movement of the mirror, the work done by the motion force $\langle \hat{F}_{J,R,ren}(t) \rangle$ is *not* a negative quantity: the dynamical energy is not positive and a meaningless result is obtained because the dynamical energy is the energy of the emitted photons
- Barton and Calogeracos [95,00]: two important differences with our results. First, to obtain the Schrödinger eq they make a unitary transformation not easily generalizable to the case of two moving mirrors. Second, a mass renormalization is performed to eliminate the reactive part, where the energy of the field is not a positive quantity at all time t. Again, the concept of particle is ill-defined during the mirror's displacement

TWO PARTIALLY TRANSMITTING MIRRORS

TWO PARTIALLY TRANSMITTING MIRRORS

- Impossible, in practice, to work in the Heisenberg picture
- Approach of Jaekel and Reynaud, starts from the effective Hamiltonian, does not seem useful to obtain the reactive part of the motion force and the dynamical energy while the mirrors move

TWO PARTIALLY TRANSMITTING MIRRORS

- Impossible, in practice, to work in the Heisenberg picture
- Approach of Jaekel and Reynaud, starts from the effective Hamiltonian, does not seem useful to obtain the reactive part of the motion force and the dynamical energy while the mirrors move
- As before, in order to get those quantities we are led to use our Hamiltonian approach. This demands now considerable effort

TWO PARTIALLY TRANSMITTING MIRRORS

- Impossible, in practice, to work in the Heisenberg picture
- Approach of Jaekel and Reynaud, starts from the effective Hamiltonian, does not seem useful to obtain the reactive part of the motion force and the dynamical energy while the mirrors move
- As before, in order to get those quantities we are led to use our Hamiltonian approach. This demands now considerable effort
- We see no basic obstruction to extend our procedure to higher dimensions and fields of any kind

Two partially transmitting mirrors

- Impossible, in practice, to work in the Heisenberg picture
- Approach of Jaekel and Reynaud, starts from the effective Hamiltonian, does not seem useful to obtain the reactive part of the motion force and the dynamical energy while the mirrors move
- As before, in order to get those quantities we are led to use our Hamiltonian approach. This demands now considerable effort
- We see no basic obstruction to extend our procedure to higher dimensions and fields of any kind
- There are proposals to detect the radiated photons: Kim, Brownell, Onofrio, PRL 96 (2006) 200402

TWO PARTIALLY TRANSMITTING MIRRORS

- Impossible, in practice, to work in the Heisenberg picture
- Approach of Jaekel and Reynaud, starts from the effective Hamiltonian, does not seem useful to obtain the reactive part of the motion force and the dynamical energy while the mirrors move
- As before, in order to get those quantities we are led to use our Hamiltonian approach. This demands now considerable effort
- We see no basic obstruction to extend our procedure to higher dimensions and fields of any kind
- There are proposals to detect the radiated photons: Kim, Brownell, Onofrio, PRL 96 (2006) 200402

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)

  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]

- The Casimir force can be calculated without reference to vacuum fluctuations

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]
- The Casimir force can be calculated without reference to vacuum fluctuations
- Are zero point fluctuations of the vacuum real?

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]
- The Casimir force can be calculated without reference to vacuum fluctuations
- Are zero point fluctuations of the vacuum real?
- Schwinger attempted to formulate QED without reference to ZPF

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]
- The Casimir force can be calculated without reference to vacuum fluctuations
- Are zero point fluctuations of the vacuum real?
- Schwinger attempted to formulate QED without reference to ZPF
- No one could show that source theory or another S-matrix based approach can provide a complete description of QED to all orders

On the ‘reality’ of zero point fluctuations

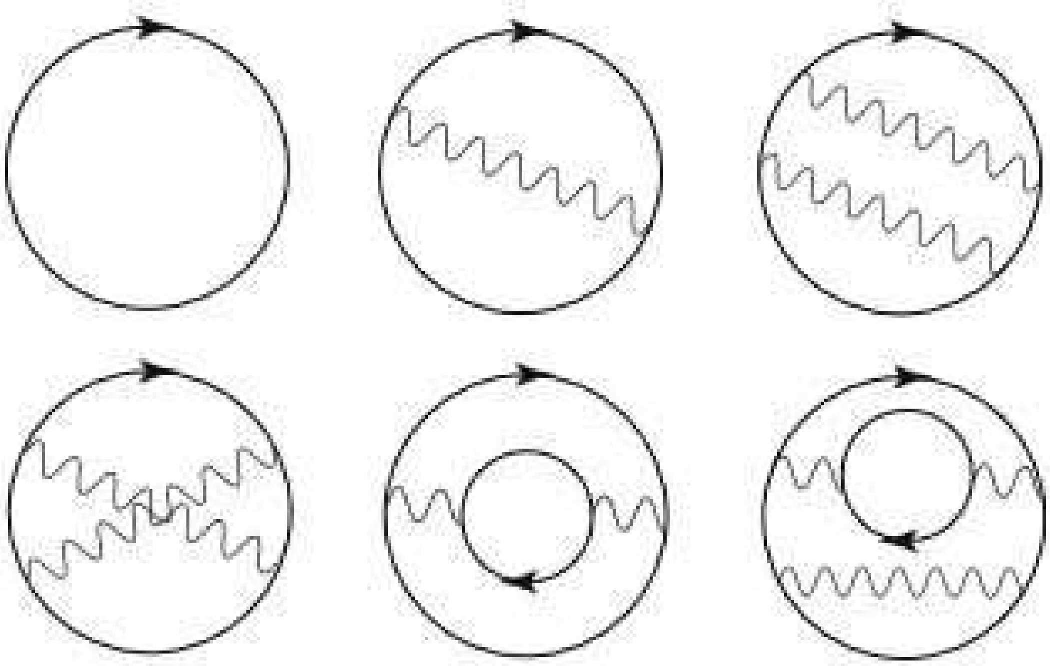
- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]
- The Casimir force can be calculated without reference to vacuum fluctuations
- Are zero point fluctuations of the vacuum real?
- Schwinger attempted to formulate QED without reference to ZPF
- No one could show that source theory or another S-matrix based approach can provide a complete description of QED to all orders
- In QCD confinement seems to present an insuperable challenge, since quarks and gluons do not appear in the physical S-matrix

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)
  [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]
- The Casimir force can be calculated without reference to vacuum fluctuations
- Are zero point fluctuations of the vacuum real?
- Schwinger attempted to formulate QED without reference to ZPF
- No one could show that source theory or another S-matrix based approach can provide a complete description of QED to all orders
- In QCD confinement seems to present an insuperable challenge, since quarks and gluons do not appear in the physical S-matrix
- Milonni has reformulated all of QED from the point of view of ZPF

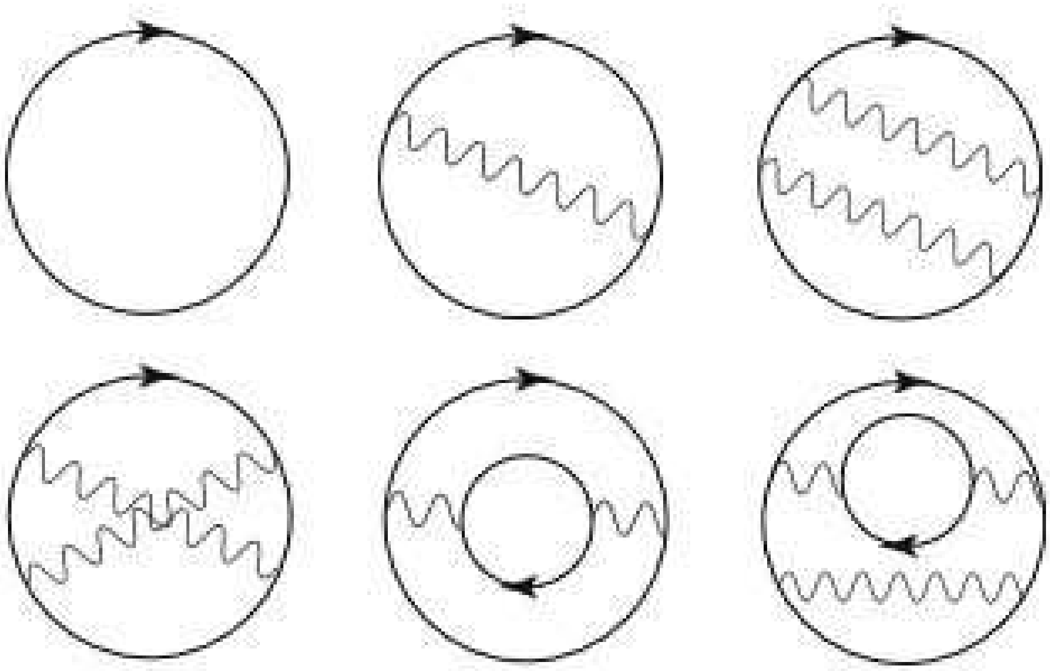
The standard approach

The standard approach



$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

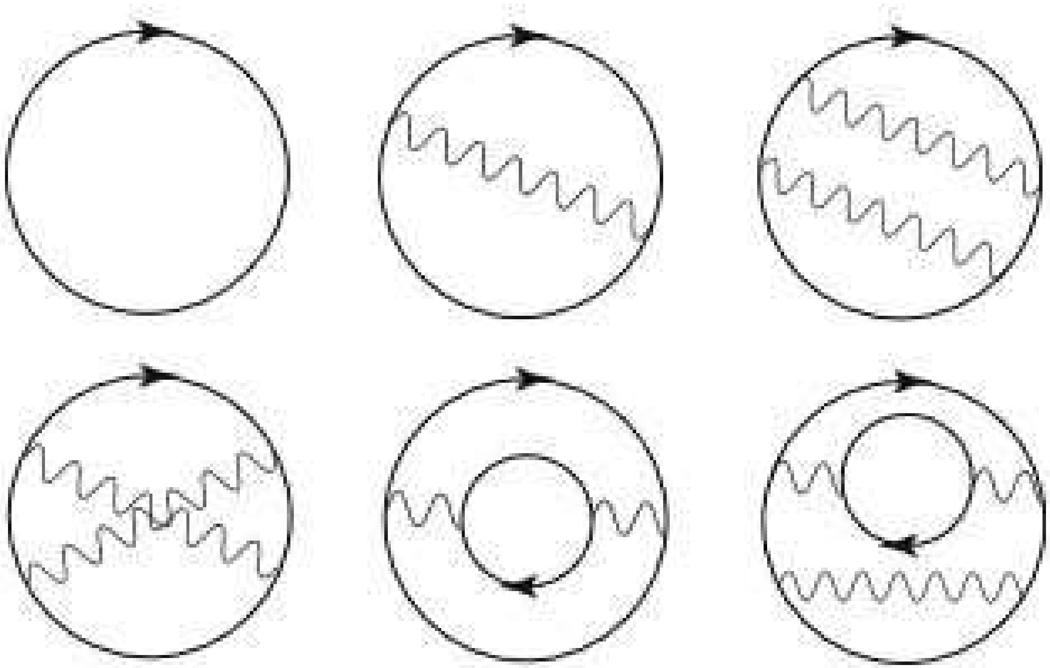
The standard approach



$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

$\Longrightarrow$ But Casimir effects can be calculated as S-matrix elements: Feynman diagrs with ext. lines

The standard approach



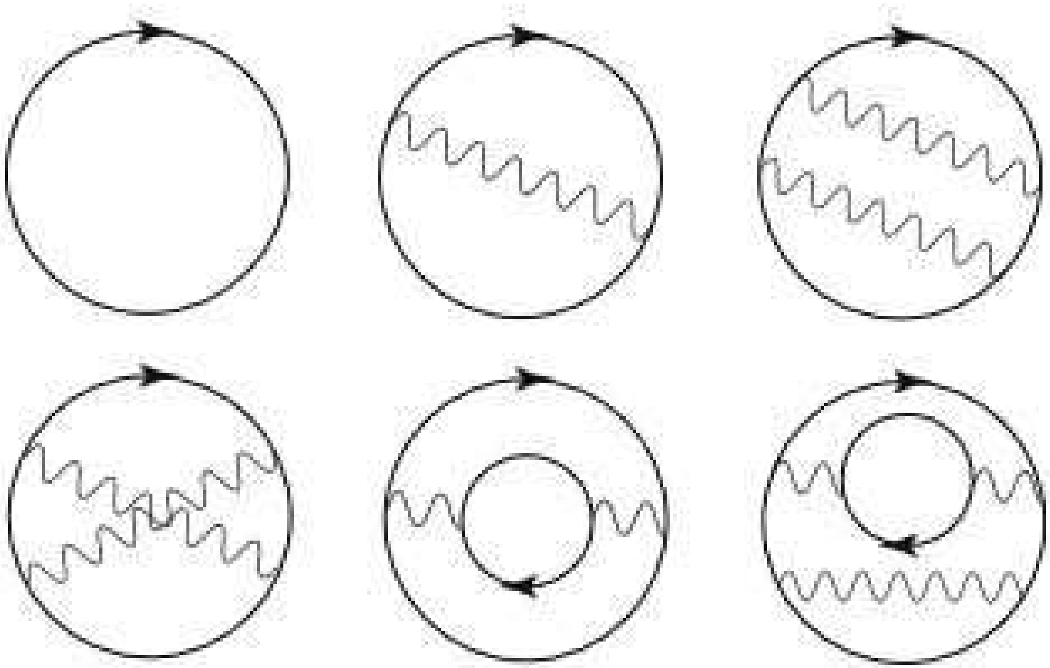
$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

$\Longrightarrow$ But Casimir effects can be calculated as S-matrix elements: Feynman diagrs with ext. lines

In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega \; \mathrm{Tr}\int d^3x \left[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)\right]$$

The standard approach



$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

$\Longrightarrow$ But Casimir effects can be calculated as S-matrix elements: Feynman diagrs with ext. lines

In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon)-\mathcal{G}_0(x,x,\omega+i\epsilon)]$$

$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function

Trace is over spin

$$E_C = \langle \qquad \rangle_{\textbf{plates}} - \langle \qquad \rangle_{\textbf{no plates}}$$

$$E_C = \langle \qquad \rangle_{\textbf{plates}} - \langle \qquad \rangle_{\textbf{no plates}}$$

$$\frac{1}{\pi}\mathrm{Im}\int [\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

change in the density of states due to the background

$$E_C = \langle \qquad \rangle_{\textbf{plates}} - \langle \qquad \rangle_{\textbf{no plates}}$$

$$\frac{1}{\pi}\mathrm{Im}\int [\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

change in the density of states due to the background

$\Longrightarrow$ A restatement of the Casimir sum over shifts in zero-point energies

$$\frac{\hbar}{2}\sum(\omega - \omega_0)$$

$$E_C = \langle \qquad \rangle_{\textbf{plates}} - \langle \qquad \rangle_{\textbf{no plates}}$$

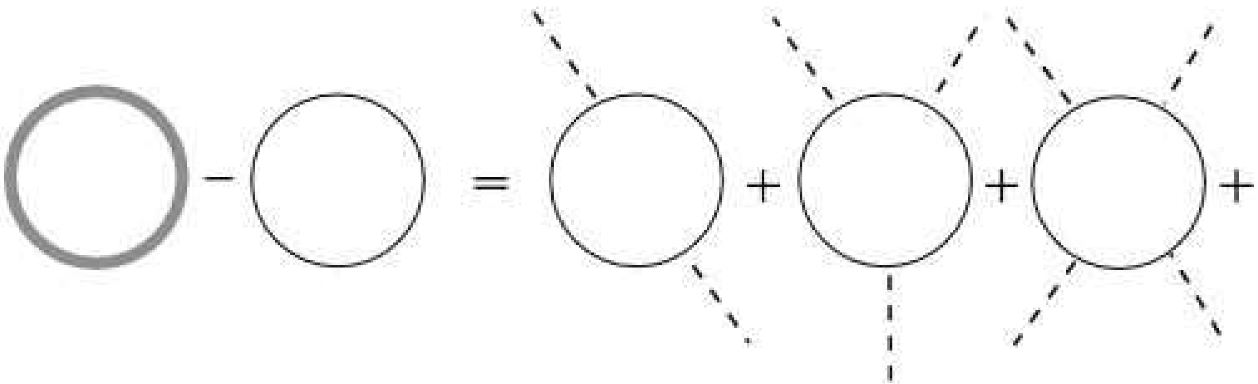
$$\frac{1}{\pi}\mathrm{Im}\int [\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

change in the density of states due to the background

$\Longrightarrow$ A restatement of the Casimir sum over shifts in zero-point energies

$$\frac{\hbar}{2}\sum(\omega - \omega_0)$$

$\Longrightarrow$ Lippman-Schwinger eq. allows full Greens f, $\mathcal{G}$, be expanded as a series in free Green's f, $\mathcal{G}_0$, and the coupling to the external field



$$E_C = \langle \qquad \rangle_{\textbf{plates}} - \langle \qquad \rangle_{\textbf{no plates}}$$

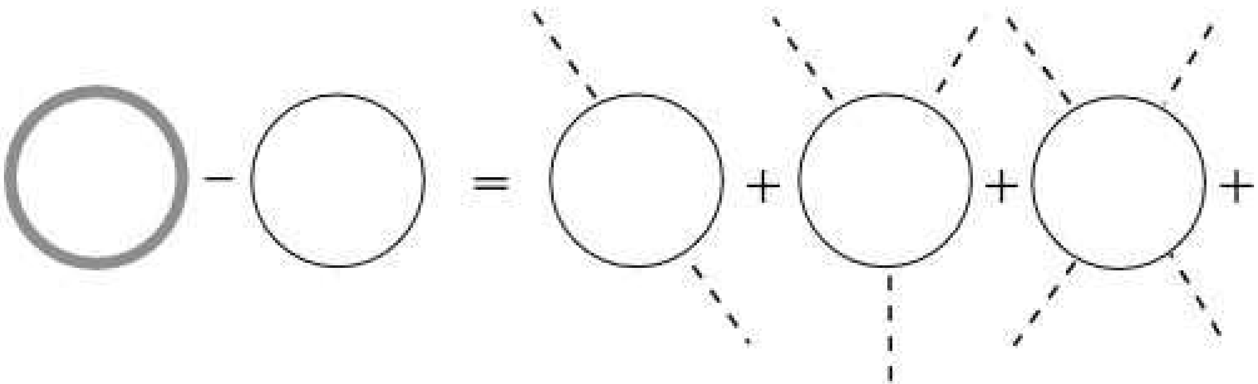
$$\frac{1}{\pi}\text{Im}\int [\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

change in the density of states due to the background

$\Longrightarrow$ A restatement of the Casimir sum over shifts in zero-point energies

$$\frac{\hbar}{2}\sum(\omega-\omega_0)$$

$\Longrightarrow$ Lippman-Schwinger eq. allows full Greens f, $\mathcal{G}$, be expanded as a series in free Green's f, $\mathcal{G}_0$, and the coupling to the external field



$\Longrightarrow$ Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations

Similar situation in electrostatics

- The energy of a smooth charge distribution, $\rho(x)$, can be calculated directly

$$\frac{1}{2}\int dx\,dy\,\frac{\rho(x)\rho(y)}{|x-y|}$$

Similar situation in electrostatics

- The energy of a smooth charge distribution, $\rho(x)$, can be calculated directly

$$\frac{1}{2}\int dx\, dy\ \frac{\rho(x)\rho(y)}{|x-y|}$$

- Or alternatively, from the energy stored in the electric field

$$\frac{1}{8\pi}\int dx\, |\vec{E}(x)|^2$$

Similar situation in electrostatics

- The energy of a smooth charge distribution, $\rho(x)$, can be calculated directly

$$\frac{1}{2}\int dx\, dy\, \frac{\rho(x)\rho(y)}{|x-y|}$$

- Or alternatively, from the energy stored in the electric field

$$\frac{1}{8\pi}\int dx\, |\vec{E}(x)|^2$$

- The existence of the latter formula is not an evidence for the "reality" of the electric field

Similar situation in electrostatics

- The energy of a smooth charge distribution, $\rho(x)$, can be calculated directly

$$\frac{1}{2}\int dx\,dy\;\frac{\rho(x)\rho(y)}{|x-y|}$$

- Or alternatively, from the energy stored in the electric field

$$\frac{1}{8\pi}\int dx\,|\vec{E}(x)|^2$$

- The existence of the latter formula is not an evidence for the "reality" of the electric field
- But propagating electromagnetic waves were discovered

Similar situation in electrostatics

- The energy of a smooth charge distribution, $\rho(x)$, can be calculated directly

$$\frac{1}{2}\int dx\, dy\, \frac{\rho(x)\rho(y)}{|x-y|}$$

- Or alternatively, from the energy stored in the electric field

$$\frac{1}{8\pi}\int dx\, |\vec{E}(x)|^2$$

- The existence of the latter formula is not an evidence for the “reality” of the electric field
- But propagating electromagnetic waves were discovered
- Look for direct evidence of vacuum fluctuations!

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G\mathcal{E}$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G \mathcal{E}$

- Recent observations: M. Tegmark *et al.* [SDSS Collab.] PRD 2004

$$\lambda \; = \; (2.14 \pm 0.13 \times 10^{-3}\ \text{eV})^4 \quad \sim \; 4.32 \times 10^{-9}\ \text{erg/cm}^3$$

Vacuum energy density and the CC

- The main issue:

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological constant $\lambda = 8\pi G\mathcal{E}$

- Recent observations: M. Tegmark *et al.* [SDSS Collab.] PRD 2004

$$\lambda = (2.14 \pm 0.13 \times 10^{-3}\ \text{eV})^4 \sim 4.32 \times 10^{-9}\ \text{erg/cm}^3$$

- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \;\; k = 2\pi/\lambda$$

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$\rho \sim \frac{\hbar k_{Planck}^4}{16\pi^2} \sim 10^{123} \rho_{obs}$$

kind of a modern (and thick!) aether R. Caldwell, S. Carroll, ...

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$\rho \sim \frac{\hbar\, k_{Planck}^4}{16\pi^2} \sim 10^{123} \rho_{obs}$$

  kind of a modern (and thick!) aether R. Caldwell, S. Carroll, ...

- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar\, c}{2} \sum_n \omega_n , \qquad \omega = k^2 + m^2/\hbar^2 , \quad k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$\rho \sim \frac{\hbar\, k^4_{Planck}}{16\pi^2} \sim 10^{123} \rho_{obs}$$

  kind of a modern (and thick!) aether R. Caldwell, S. Carroll, ...

- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem

- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar\, c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \;\; k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$\rho \sim \frac{\hbar\, k^4_{Planck}}{16\pi^2} \sim 10^{123} \rho_{obs}$$

  kind of a modern (and thick!) aether R. Caldwell, S. Carroll, ...

- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem

- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales
  - (b) dS & AdS worldbranes

Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
  * T. Padmanabhan, gr-qc/0606061: Holographic Perspective, CC is an intg const, no response of gravity to changes in bulk vac energy dens

- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales
  - (b) dS & AdS worldbranes
  - (c) supergraviton theories (discret dims, deconstr)

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$
- Scalar field, ϕ, pervading the universe ($\hbar = c = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + (m^2 + \xi R) \phi \phi^* \right]$$

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$
- Scalar field, ϕ, pervading the universe ($\hbar = c = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + (m^2 + \xi R) \phi \phi^* \right]$$

- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2 \right)^{1/2}$$

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$
- Scalar field, ϕ, pervading the universe ($\hbar = c = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + (m^2 + \xi R) \phi \phi^* \right]$$

- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2 \right)^{1/2}$$

- $\sum_i$ and $\sum_{\mathbf{k}}$ are generalized sums, μ mass-dim parameter

A. Simple model: large & small dim's

- Space-time: $\mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{T}^q, \qquad \mathbb{R}^{d+1} \times \mathbb{T}^p \times \mathbb{S}^q, \quad \cdots$
- Scalar field, ϕ, pervading the universe ($\hbar = c = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi^* + (m^2 + \xi R) \phi \phi^* \right]$$

- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2 \right)^{1/2}$$

- $\sum_i$ and $\sum_{\mathbf{k}}$ are generalized sums, μ mass-dim parameter
- M effective mass term, m arbitrarily small
  (a tiny mass for the field cannot be excluded, and fits well)

* L. Parker & A. Raval, PRL86 749 (2001); PRD62, 083503 (2000)

* V.G. Gurzadyan & S.-S. Xue, Mod Phys Lett A18, 561 (2003)

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^{p} a_j \prod_{h=1}^{q} b_h} \int_0^\infty dk\, k^{d-1}$$

$$\sum_{\mathbf{n}_p=-\infty}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \left[\sum_{j=1}^{p} \left(\frac{2\pi n_j}{a_j}\right)^2 + \sum_{h=1}^{q} \left(\frac{2\pi m_h}{b_h}\right)^2 + \mathbf{k}_d^2 + M^2 \right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q} \sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty} \left(\frac{1}{a^2}\sum_{j=1}^{p} n_j^2 + \frac{1}{b^2}\sum_{h=1}^{q} m_h^2 + M^2 \right)^{\frac{d+1}{2}+1}$$

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j \prod_{h=1}^q b_h} \int_0^\infty dk\, k^{d-1}$$

$$\sum_{\mathbf{n}_p=-\infty}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \left[\sum_{j=1}^p \left(\frac{2\pi n_j}{a_j}\right)^2 + \sum_{h=1}^q \left(\frac{2\pi m_h}{b_h}\right)^2 + \mathbf{k}_d^2 + M^2\right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q} \sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty} \left(\frac{1}{a^2}\sum_{j=1}^p n_j^2 + \frac{1}{b^2}\sum_{h=1}^q m_h^2 + M^2\right)^{\frac{d+1}{2}+1}$$

- For d–open, (p–toroidal, q–spherical) universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j\, b^q} \int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^{\infty} \sum_{l=1}^{\infty} P_{q-1}(l)$$

$$\left[\sum_{j=1}^p \left(\frac{2\pi n_j}{a_j}\right)^2 + \frac{Q_2(l)}{b^2} + \mathbf{k}_d^2 + M^2\right]^{1/2}$$

[$P_{q-1}(l)$ poly in l deg $q-1$]

$$\simeq \frac{1}{a^p b^q} \sum_{\mathbf{n}_p=-\infty}^{\infty} \sum_{l=1}^{\infty} P_{q-1}(l) \left(\frac{4\pi^2}{a^2}\sum_{j=1}^p n_j^2 + \frac{l(l+q)}{b^2} + \frac{M^2}{4\pi^2}\right)^{\frac{d+1}{2}+1}$$

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here
  [E.E., J. Math. Phys. 35, 3308, 6100 (1994)]

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here [E.E., J. Math. Phys. 35, 3308, 6100 (1994)]

- Here spectrum of the Hamiltonian op. known explicitly. Corresponds to generalized Chowla-Selberg expression [E.E., Commun. Math. Phys. 198, 83 (1998) E.E., J. Phys. A30, 2735 (1997)]

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \quad \Longrightarrow \quad \rho_\phi = \zeta(-1)$$

- No further subtraction or renormalization needed here [E.E., J. Math. Phys. 35, 3308, 6100 (1994)]

- Here spectrum of the Hamiltonian op. known explicitly. Corresponds to generalized Chowla-Selberg expression [E.E., Commun. Math. Phys. 198, 83 (1998) E.E., J. Phys. A30, 2735 (1997)]

- For the zeta function (Re $s > p/2$):

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[\frac{1}{2}(\vec{n}+\vec{c})^T A(\vec{n}+\vec{c}) + q\right]^{-s} \equiv \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[Q(\vec{n}+\vec{c}) + q\right]^{-s}$$

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly
- Only condition on matrix A: corresponds to (non negative) quadratic form, Q. Vector $\vec{c}$ arbitrary, while q (for the moment) positive constant

$$\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s-p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2}\pi^s q^{-s/2+p/4}}{\sqrt{\det A}\,\Gamma(s)}$$

$$\sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c}) \left(\vec{m}^T A^{-1}\vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

- Prime means that point $\vec{n} = \vec{0}$ is excluded from sum (not important)
- Gives (analyt cont of) multidim zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of meromorphic continuation —with corresponding residua— explicitly
- Only condition on matrix A: corresponds to (non negative) quadratic form, Q. Vector $\vec{c}$ arbitrary, while q (for the moment) positive constant

$$\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s-p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2}\pi^s q^{-s/2+p/4}}{\sqrt{\det A}\,\Gamma(s)}$$

$$\sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c})\,(\vec{m}^T A^{-1}\vec{m})^{s/2-p/4} \, K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbb{Z}^p_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbb{Z}^p$ are summed over. That is, if we take an $\vec{m} \in \mathbb{Z}^p$ we must then exclude $-\vec{m}$ (as simple criterion one can, for instance, select those vectors in $\mathbb{Z}^p\backslash\{\vec{0}\}$ whose first non-zero component is positive).

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2}\Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

$$\times K_{(s-1)/2} \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2} b^{q-(s-1)/2} \Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

$$\times K_{(s-1)/2} \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Yields the vacuum energy density:

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sqrt{\frac{\sum_{k=1}^{q} m_k^2 + M^2}{\sum_{j=1}^{h} n_j^2}}$$

$$\times K_1 \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

Now, from $$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \right.$$
$$\times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}}\, K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right)$$
$$\left. + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right] \right\}$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \Bigg\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}} K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right) + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right]\Bigg\}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p}\binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M small

$$\begin{aligned}\rho_\phi &= -\frac{1}{a^p b^{q+1}} \left\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \right. \\ &\times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^{h} n_j^2}}\, K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^{h} n_j^2}\right) \\ &\left. + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right]\right\}\end{aligned}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p}\binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

α some finite constant (explicit geometrical sum in the limit M small)

Now, from $K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)(z/2)^{-\nu}, \qquad z \to 0$

when M small

$$\rho_\phi = -\frac{1}{a^p b^{q+1}} \Bigg\{ M\, K_1\left(\frac{2\pi a}{b} M\right) + \sum_{h=0}^{p} \binom{p}{h} 2^h \times \sum_{\mathbf{n}_h=1}^{\infty} \frac{M}{\sqrt{\sum_{j=1}^h n_j^2}}\, K_1\left(\frac{2\pi a}{b} M \sqrt{\sum_{j=1}^h n_j^2}\right) + \mathcal{O}\left[q\sqrt{1+M^2} K_1\left(\frac{2\pi a}{b}\sqrt{1+M^2}\right)\right] \Bigg\}$$

Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q} \left[1 + \sum_{h=0}^{p} \binom{p}{h} 2^h \alpha \right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

α some finite constant (explicit geometrical sum in the limit M small)

$\Longrightarrow$ Sign may change with BC (e.g., Dirichlet): a problem

Matching the obs. results for the CC

$$b \sim l_{P(lanck)} \qquad a \sim R_{U(niverse)} \qquad a/b \sim 10^{60}$$

Matching the obs. results for the CC

$$b \sim l_{P(lanck)} \qquad a \sim R_{U(niverse)} \qquad a/b \sim 10^{60}$$

ρ_ϕ	$p=0$	$p=1$	$p=2$	$p=3$
$b = l_P$	10^{-13}	10^{-6}	1	10^{5}
$b = 10 l_P$	10^{-14}	$[10^{-8}]$	10^{-3}	10
$b = 10^2 l_P$	10^{-15}	(10^{-10})	10^{-6}	10^{-3}
$b = 10^3 l_P$	10^{-16}	10^{-12}	$[10^{-9}]$	(10^{-7})
$b = 10^4 l_P$	10^{-17}	10^{-14}	10^{-12}	10^{-11}
$b = 10^5 l_P$	10^{-18}	10^{-16}	10^{-15}	10^{-15}

Table 2: Vacuum energy density in units of erg/cm^3, for p large compactified dimensions a, and $q = p+1$ small compactified dimensions b, $p = 0, \ldots, 3$, for different values of b, proportional to the Planck length l_P

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

* Both situations correspond to a marginally closed universe

Results of this simple model

* Precise (in absolute value) coincidence with the observational value for the cosmological constant with

$$\rho_\phi \longrightarrow [\quad]$$

* Approximate coincidence $\longrightarrow (\quad)$

* $b \sim$ 10 to 1000 times the Planck length l_P

* $q = p + 1$: (1,2) and (2,3) compactified dimensions, respectively

* Everything dictated by two basic lengths:

  Planck value and radius of observable Universe

* Both situations correspond to a marginally closed universe

$\Longrightarrow$ To examine $\rightarrow$ couplings in GR

$\rightarrow$ alternative theories

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

$\Longrightarrow$ Casimir energy density and effective potential for a de Sitter (dS) brane in a five-dimensional anti-de Sitter (AdS) background

B. Realistic models: dS & AdS BW

Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

- Bulk Casimir effect may play a role in radion stabilization of BW
- Bulk Casimir effect (eff. pot.) for a conformal or massive scalar field
- Bulk: 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for large extra dimension

Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$

$\Longrightarrow$ Casimir energy density and effective potential for a de Sitter (dS) brane in a five-dimensional anti-de Sitter (AdS) background

$\Longrightarrow$ Action for conformally inv massless scalar with scalar-gravit coupling

$$\mathcal{S} = \frac{1}{2}\int d^5x\sqrt{g}\left[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \xi_5 R^{(5)}\phi^2\right]$$

$\xi_5 = -3/16$ $\quad$ $R^{(5)}$ 5-dim scalar curvature

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right) \\
d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2
\end{aligned}
$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2 \right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2 \right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2\xi d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6}\sum_{n,l=1}^{\infty}(l+1)(l+2)(2l+3)\left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2+3l+\frac{9}{4}\right)\right]^{-s}$$

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2 \right)$$
$$d\Omega_4^2 = d\xi^2 + \sin^2 \xi d\Omega_3^2$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6} \sum_{n,l=1}^{\infty} (l+1)(l+2)(2l+3) \left[\left(\frac{\pi n}{L} \right)^2 + \mathcal{R}^{-2} \left(l^2 + 3l + \frac{9}{4} \right) \right]^{-s}$$

The Casimir energy density (pressure) follows (non-trivially)

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z}\left(dz^2 + d\Omega_4^2\right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2\xi d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy (L brane separation, $\mathcal{R}$ brane radius)

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6}\sum_{n,l=1}^{\infty}(l+1)(l+2)(2l+3)\left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2+3l+\frac{9}{4}\right)\right]^{-s}$$

The Casimir energy density (pressure) follows (non-trivially)

$$\mathcal{E}_{\text{Cas}} = \frac{\hbar c}{2L\mathcal{R}^4}\zeta\left(-\frac{1}{2}|L_5\right) = -\frac{\hbar c\pi^3}{36L^6}\left[\frac{\pi^2}{315} - \frac{1}{240}\left(\frac{L}{\mathcal{R}}\right)^2 + \mathcal{O}\left(\frac{L}{\mathcal{R}}\right)^4\right]$$

C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435

$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127

$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91

$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757

$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96

$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)

C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435

$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127

$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91

$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757

$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96

$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)
- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$
  $\Longrightarrow$ shown the induced cosmological constant could be positive due to topological contributions

C. Supergraviton Theories

$\Longrightarrow$ Cognola, Elizalde, Zerbini, PLB624 (2005) 70

$\Longrightarrow$ Cognola, Elizalde, Nojiri, Odintsov, Zerbini, MPLA19 (2004) 1435

$\Longrightarrow$ Boulanger, Damour, Gualtieri, Henneaux, NPB597 (2001) 127

$\Longrightarrow$ Sugamoto, Grav. Cosmol. 9 (2003) 91

$\Longrightarrow$ Arkani-Hamed, Cohen, Georgi, PRL86 (2001) 4757

$\Longrightarrow$ Arkani-Hamed, Georgi, Schwartz, Ann. Phys. (NY) 305 (2003) 96

$\Longrightarrow$ Hill, Pokorski, Wang; Damour, Kogan, Papazoglou; Deffayet, Mourad

- Effective potential for a multi-graviton model with supersymmetry (discretized dim's, deconstruction)
- The bulk is a flat manifold with torus topology $\mathbb{R} \times \mathbb{T}^3$
  $\Longrightarrow$ shown the induced cosmological constant could be positive due to topological contributions
- Previously considered in $\mathbb{R}^4$

⟹ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

⟹ Allow for non-nearest-neighbor couplings

⟹ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

⟹ Lagrangian of the theory: a generalization of N. Kan–K. Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^{\lambda}\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$

$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$

$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]$$

Δ, $\Delta^\dagger$ difference operators

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Lagrangian of the theory: a generalization of N. Kan–K. Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^\lambda \partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$
$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu}\right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu\right)\Big]$$

Δ, $\Delta^\dagger$ difference operators

operate on the indices n as

$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k}$, $\Delta^\dagger\phi_n \equiv \sum_{k=0}^{N-1} a_k\phi_{n-k}$, $\sum_{k=0}^{N-1} a_k = 0$,

a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

$\Longrightarrow$ Allow for non-nearest-neighbor couplings

$\Longrightarrow$ Multi-graviton model defined by taking $N-$copies of fields, with graviton $h_{n\mu\nu}$ and Stückelberg fields $A_{n\mu}$, φ_n

$\Longrightarrow$ Lagrangian of the theory: a generalization of N. Kan–K. Shiraishi

$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^\lambda \partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$
$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2 \right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n \right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n \right)$$
$$-\frac{1}{2}\left(\partial_\mu A_{n\nu} - \partial_\nu A_{n\mu} \right)\left(\partial^\mu A_n^\nu - \partial^\nu A_n^\mu \right) \Big]$$

Δ, $\Delta^\dagger$ difference operators operate on the indices n as

$\Delta\phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n+k}$, $\Delta^\dagger \phi_n \equiv \sum_{k=0}^{N-1} a_k \phi_{n-k}$, $\sum_{k=0}^{N-1} a_k = 0$,

a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

(Δ becomes usual differentiation operator in properly defined continuum limit)

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields
  - They are positive for anti-periodic fields.

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields
  - They are positive for anti-periodic fields.
- Topology provides a natural mechanism which permits to have a positive cc in the multi-supergravity model with anti-periodic fermions

- In the multi-graviton model the induced cosmological constant can be positive only if # of massive gravitons is sufficiently large
- In the supersymmetric case the cosmological constant can be positive if one imposes anti-periodic BC in the fermionic sector
- Topological effects discussed may also be relevant in the study of electroweak symmetry breaking in models with a similar type of non-nearest-neighbour couplings, for the deconstruction issue
- Case of the torus topology: top. contributions to the eff. potential have always a fixed sign, depending on the BC one imposes
  - They are negative for periodic fields
  - They are positive for anti-periodic fields.
- Topology provides a natural mechanism which permits to have a positive cc in the multi-supergravity model with anti-periodic fermions
- The value of the cc is regulated by the corresponding size of the torus (one can most naturally use the minimum number, $N = 3$, of copies of bosons and fermions), and is not far from the observational values

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale
- Zeta functions provide a fine, precise, and powerful tool to perform the calculations

Conclusions

- At terrestrial scales, vacuum energy fluctuations are becoming important (MEMs, NEMs, nanotubes)
- Topology + Vacuum energy may prove to be a fundamental driving force at cosmological scale
- Zeta functions provide a fine, precise, and powerful tool to perform the calculations

Thank You !