ICE CSIC IEEC

# Casimir Effect: Dynamical & Cosmological

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# Outline of the talk

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- Some recent developments

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Even then: Has the final value real sense ?

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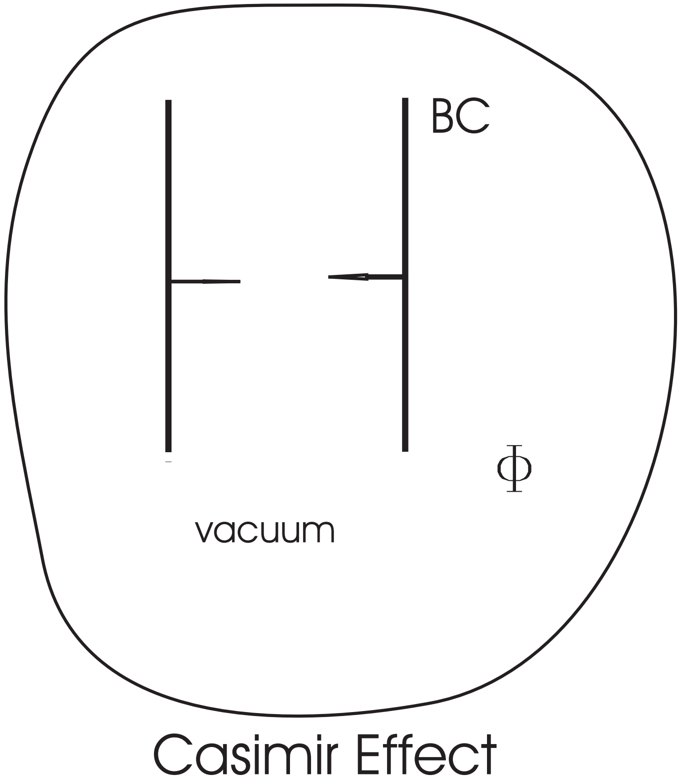

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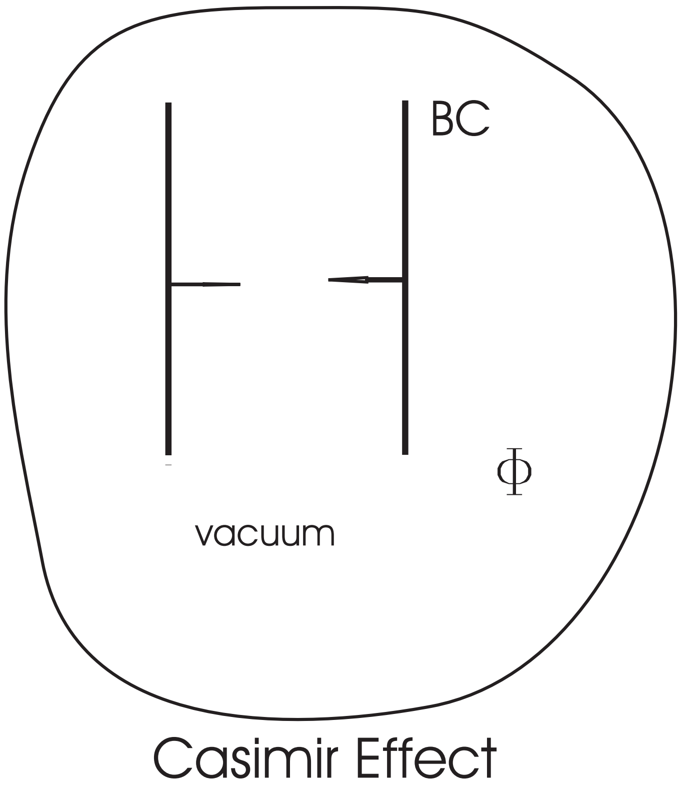


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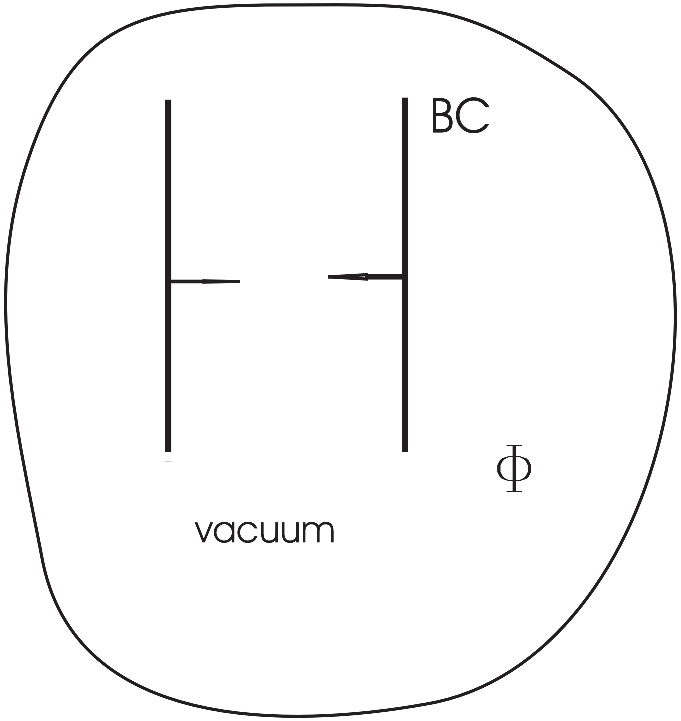


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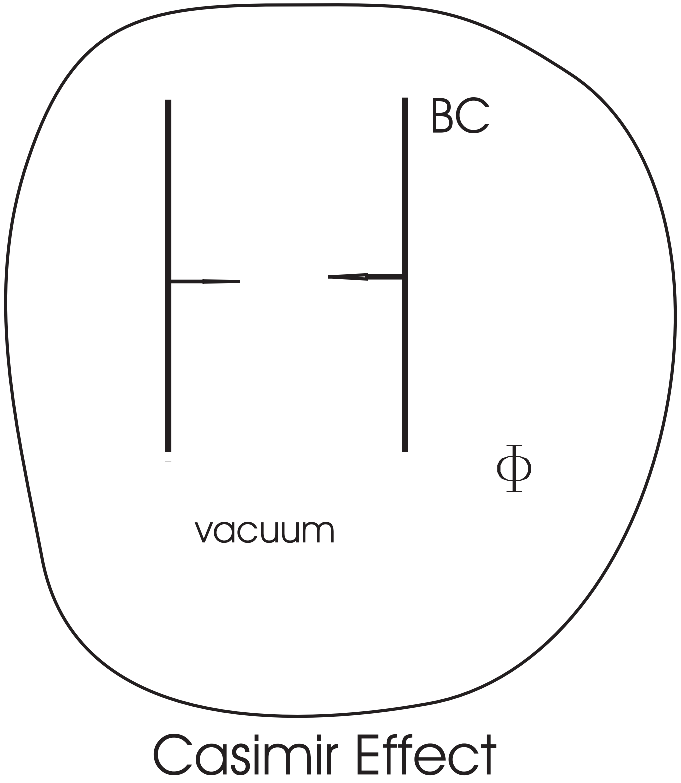


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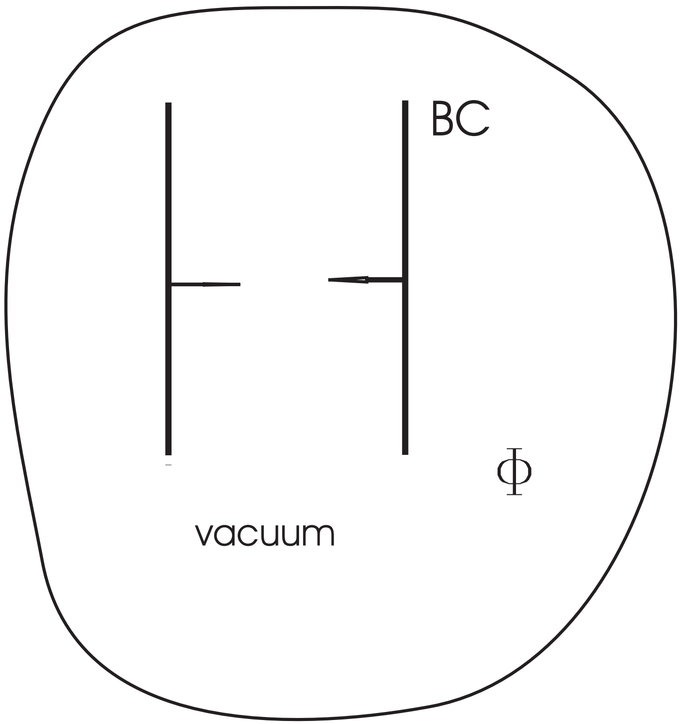


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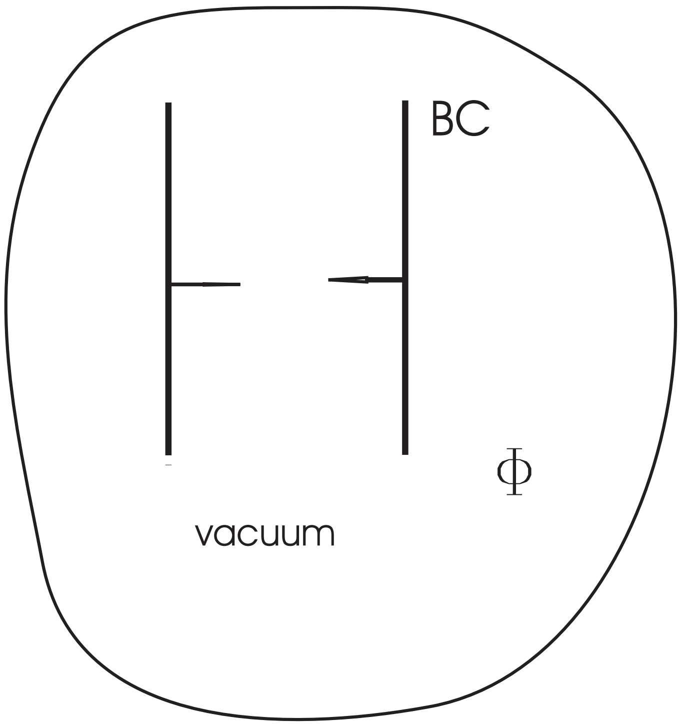

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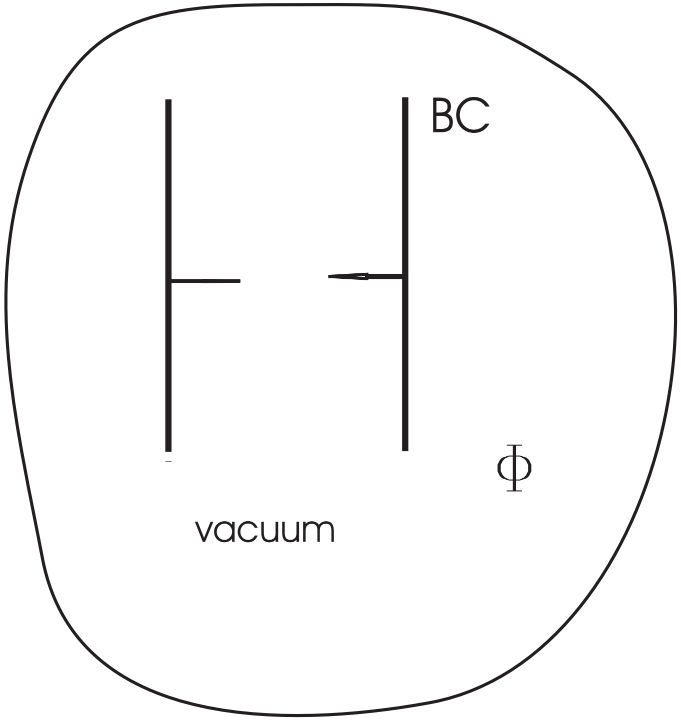


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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Jaeckel, Reynaud, Lambrecht; Ford, Vilenkin; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

# SOME DETAILS OF THE METHOD

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- Hamiltonian method for neutral Klein-Gordon field in a cavity $\Omega_t$, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$ (of order $10^{-8}$ in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)
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  Coordinate change used to convert the moving boundary $\Omega_t$ into a fixed one $\widetilde{\Omega}$: $(t(\bar{t},\mathbf{y}),\mathbf{x}(\bar{t},\mathbf{y})) = \mathcal{R}(\bar{t},\mathbf{y}) = (\bar{t},\mathbf{R}(\bar{t},\mathbf{y}))$

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- Hamiltonian density

$$\widetilde{\mathcal{H}}(\bar{t}, \mathbf{y}) = \frac{1}{2}\left(\widetilde{\xi}^2 + J|\nabla_{\mathbf{x}}\phi|^2\right) + \widetilde{\xi}\left(\partial_{\bar{t}}\widetilde{\phi} - \sqrt{J}\partial_t\phi\right)$$

$\widetilde{\phi}$ field, $\widetilde{\xi}$ conjugate momentum, $J$ Jacobian: $d^3\mathbf{x} \equiv J d^3\mathbf{y}$

# CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

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Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)e^{-2i\omega L} \\ r(\omega)e^{2i\omega L} & s(\omega) \end{pmatrix}$$

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions (material's permitivity and resistivity)

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In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \text{Re}\left[ e^{-i(\omega+\omega')t} \, \widehat{\dot{g}\theta_t}(\omega + \omega') \right]$$
$$\times \left[ |r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 \right] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

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Energy conservation is fulfilled: the dynamical energy at any time $t$ equals, with the opposite sign, the work performed by the reaction force up to that time $t$

$$\langle \hat{E}(t) \rangle = -\epsilon \int_0^t \langle \hat{F}_{Ha}(\tau) \rangle \dot{g}(\tau) d\tau$$

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- Been able to prove that the force coincides with the radiation-reaction force calculated by Jaekel and Reynaud after renormalization:

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- Barton and Calogeracos [95,00]: two important differences with our results. First, to obtain the Schrödinger eq they make a unitary transformation not easily generalizable to the case of two moving mirrors. Second, a mass renormalization is performed to eliminate the reactive part, where the energy of the field is not a positive quantity at all time $t$. Again, the concept of particle is ill-defined during the mirror's displacement

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- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)

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- No one could show that source theory or another $S$-matrix based approach can provide a complete description of QED to all orders

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- Are zero point fluctuations of the vacuum real?
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- Milonni has reformulated all of QED from the point of view of ZPF

# The standard approach

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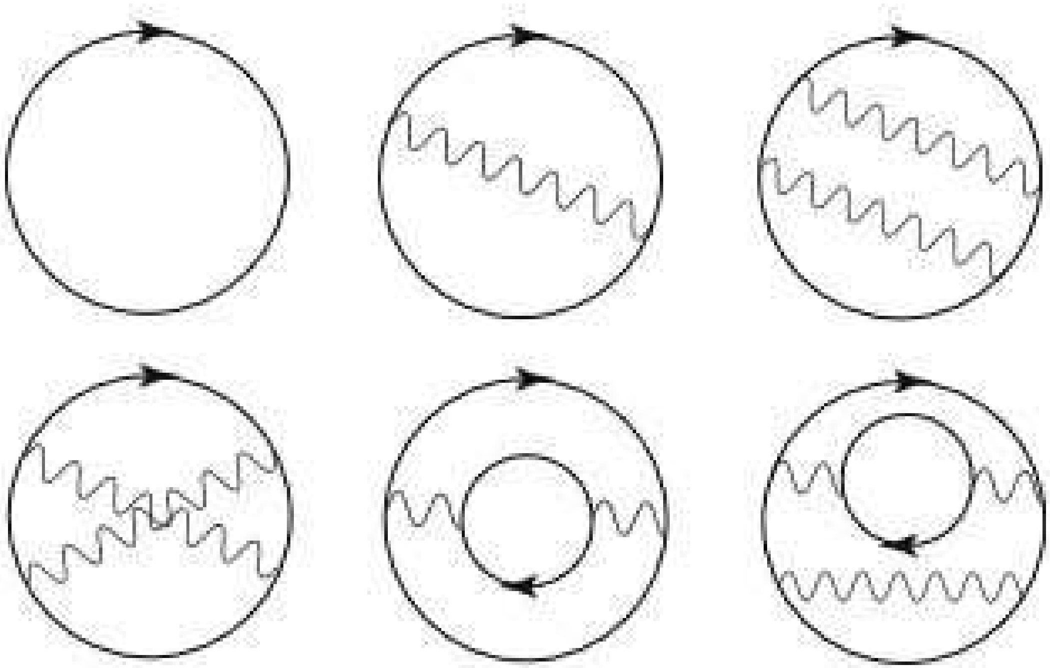

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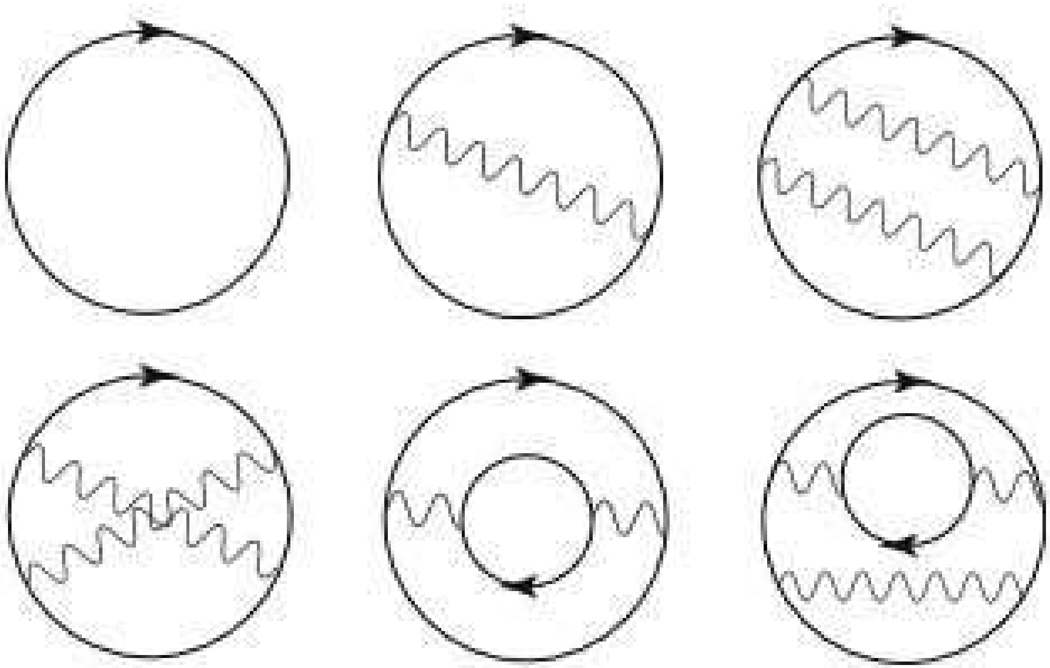

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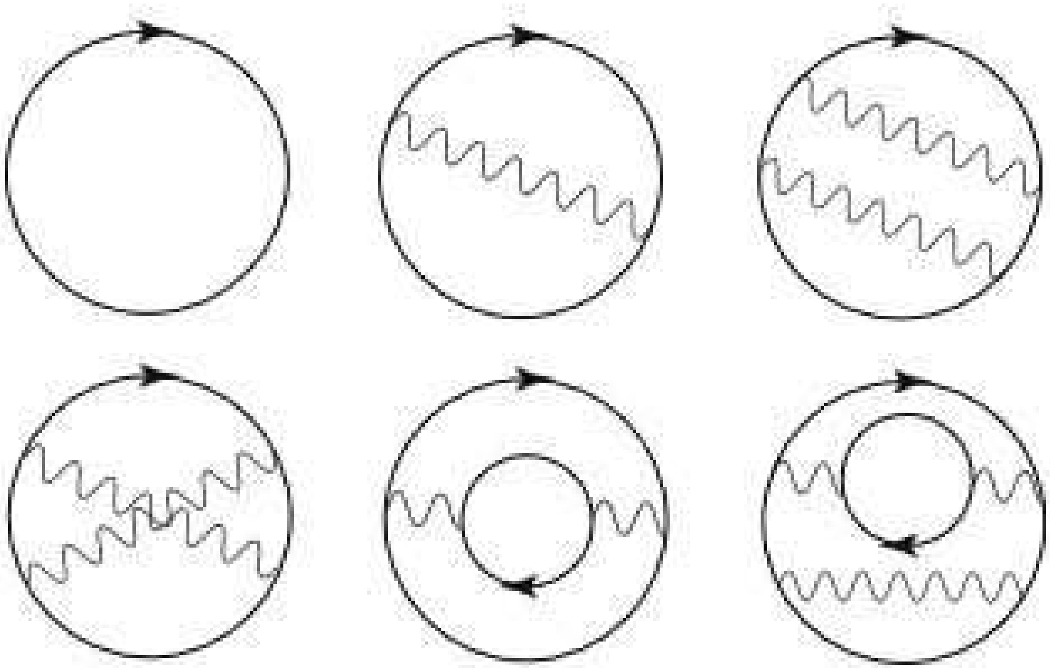

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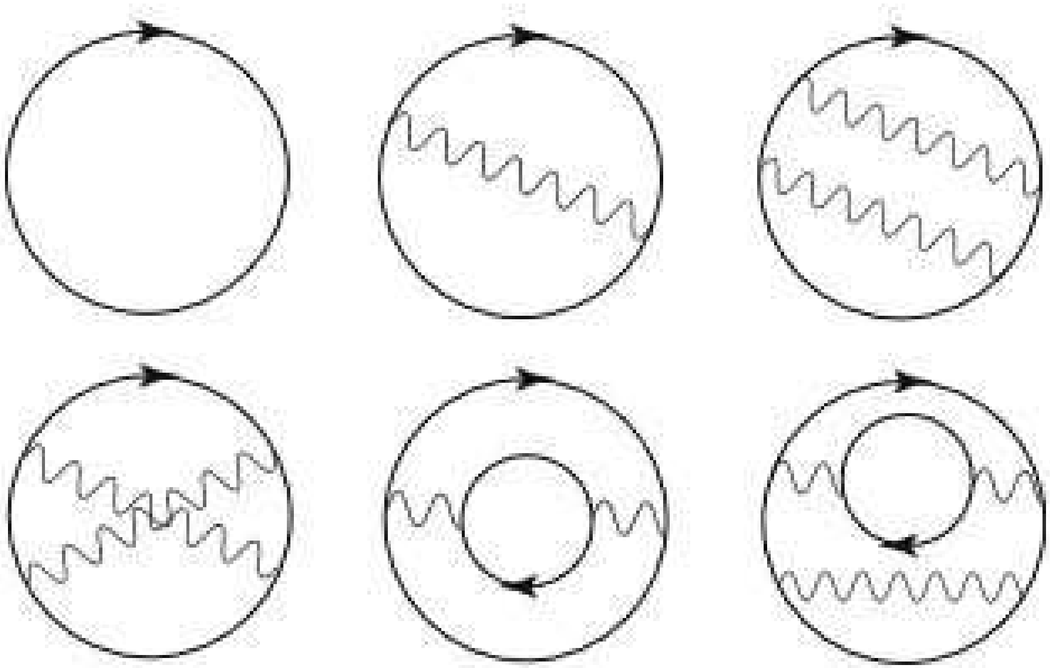

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$\mathcal{G}_0$ free Greens function

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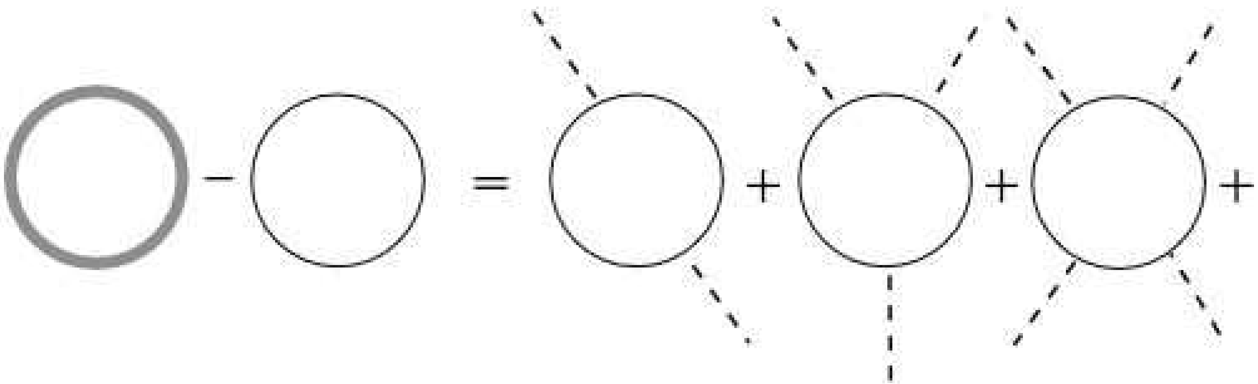

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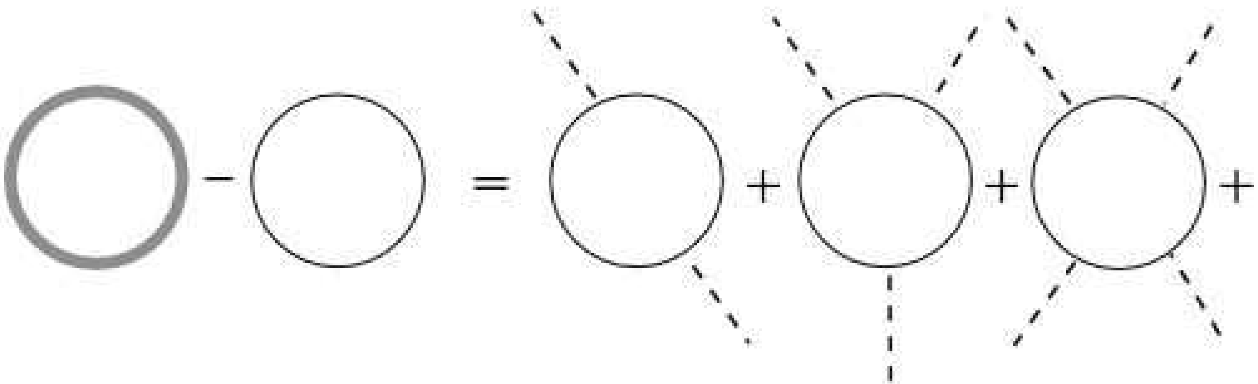

$\Longrightarrow$ Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations

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- Look for direct evidence of vacuum fluctuations!

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

# Cosmo-Topological Casimir Effect

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density

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- $M$ effective mass term, $m$ arbitrarily small
  (a tiny mass for the field cannot be excluded, and fits well)

* L. Parker & A. Raval, PRL86 749 (2001); PRD62, 083503 (2000)

* V.G. Gurzadyan & S.-S. Xue, Mod Phys Lett A18, 561 (2003)

* A. Chodos & E. Myers, '85-'86

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- We produce an exact analysis of the one-loop effective action in the 4D, alternative model [EE-Minamitsuji-Naylor, PRD 2007]

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- Phenomenological effects on the brane (eff mass of modulus, degree of hierarchy between fundamental energy scales on brane): need to know the value of $A_4$
- $A_4$ is not just related to the heat kernel: need to evaluate $\zeta'(0)$
- Accurate evaluation of $A_4$ is crucial for making physical predictions: hierarchy & CC problems

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where we redefine the terms:

$$\hat{\alpha}(n)=\alpha+\frac{bn}{2a}=\frac{1}{2}+\frac{(1+r)|n|}{2\kappa}, \qquad \hat{q}(n)=-\frac{n^2}{\kappa^2}(1-r)^2-1$$

# Extended binomial expansion

$$\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left[a(m+\alpha)^2+b(m+\alpha)n+cn^2+q\right]^{-s+1}$$

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$$\zeta(s)\big|_{n\neq 0}=\frac{g^{2(1-s)}}{\pi}\int d^2x\sum_{j=0}^{\infty}G(j,s)\sum_{n=1}^{\infty}\left[\frac{n^2}{\kappa^2}(1-r)^2+1\right]^j\zeta_H\left(2s+2j-2,\frac{1}{2}+\frac{1+r}{2\kappa}n\right)$$

$$G(j,s)\equiv\frac{2^{-2(j+s-1)}\Gamma(s+j-1)}{\Gamma(s)\,j!}$$

# Analytic continuation of the $\zeta$ function

$$P(s) = \frac{g^{2(1-s)}}{\pi} \int d^2x \sum_{j=0}^{\infty} G(j,s) \sum_{n=1}^{\infty} \left[ \left( \frac{n^2}{\kappa^2} (1-r)^2 + 1 \right)^j \times \zeta_H \left( 2s+2j-2, \frac{1}{2} + \frac{1+r}{2\kappa} n \right) - F(n,j;s) \right]$$

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$\Longrightarrow$ Extend our analysis to the cases of

(1) 6 dimensions

(2) a bulk scalar field with self-interactions and other fields in the multiplets appearing in the supergravity model