ICE CSIC IEEC

Contribution of the Casimir Effect to the Cosmological Constant

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http://www.ieec.fcr.es/english/recerca/ftc/eli/eli.htm

Montpellier, November 29th, 2005

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Even then: Has the final value any meaning??

The Casimir Effect

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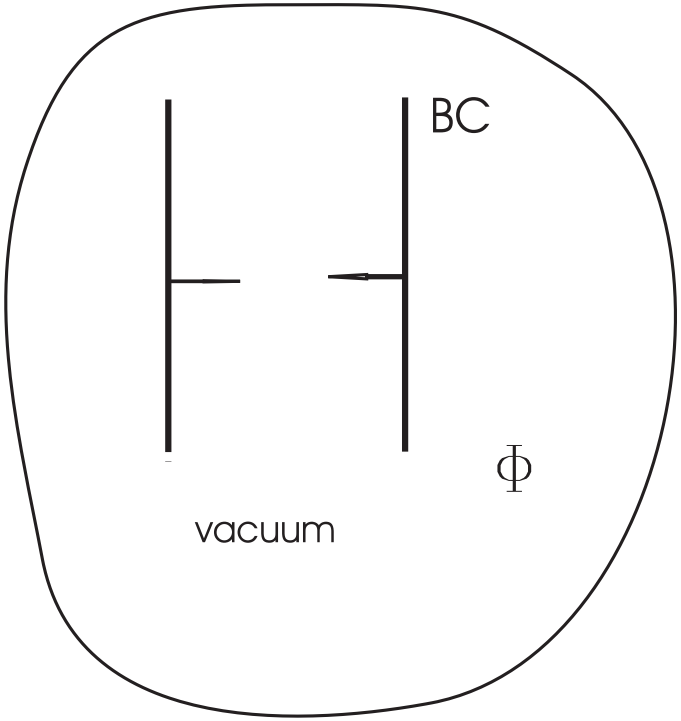

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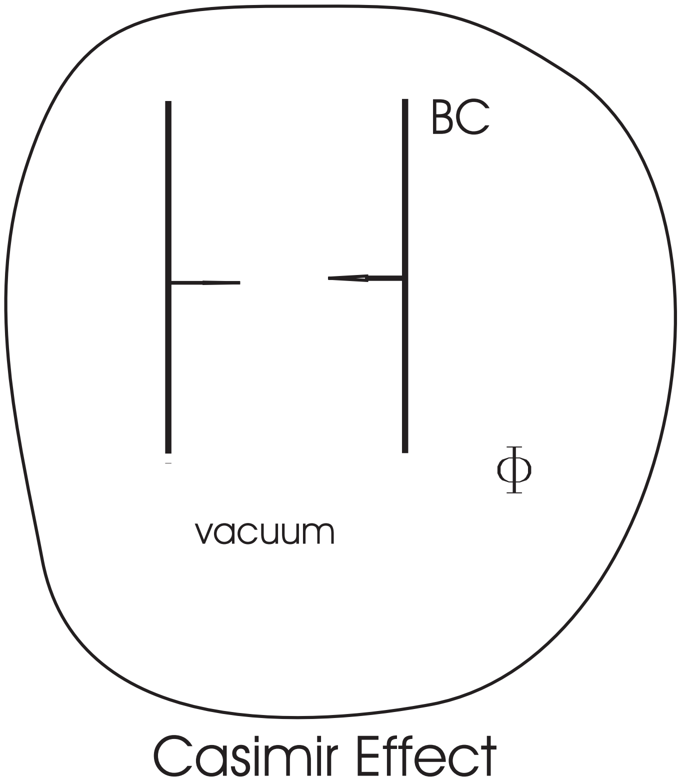

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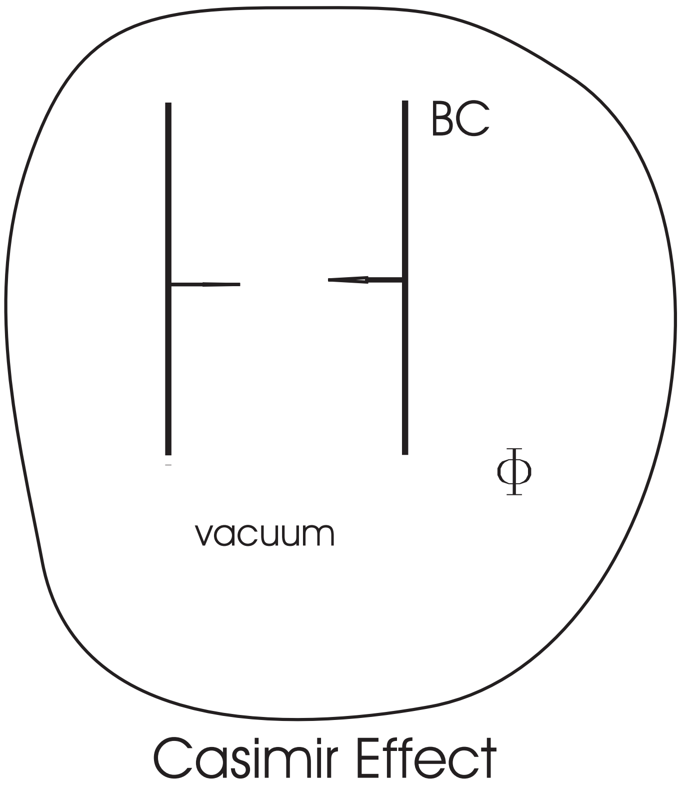

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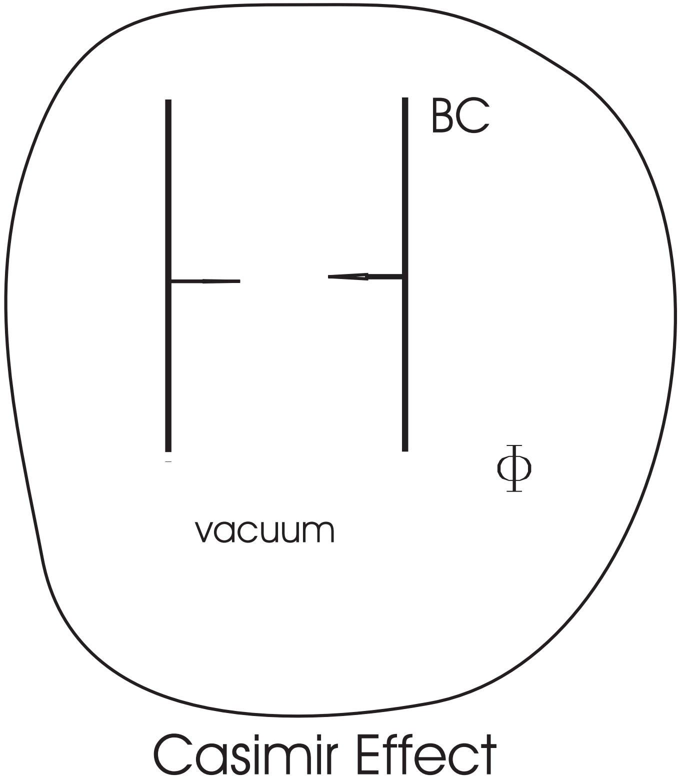

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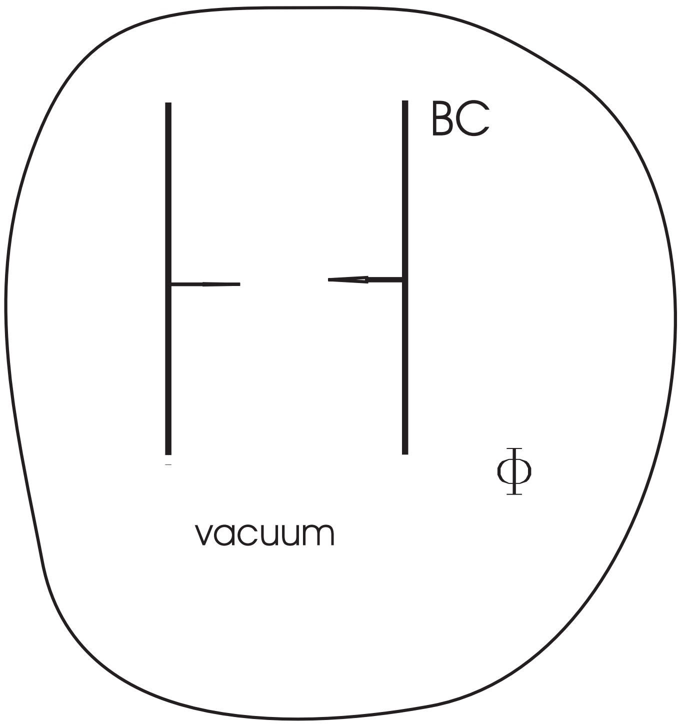

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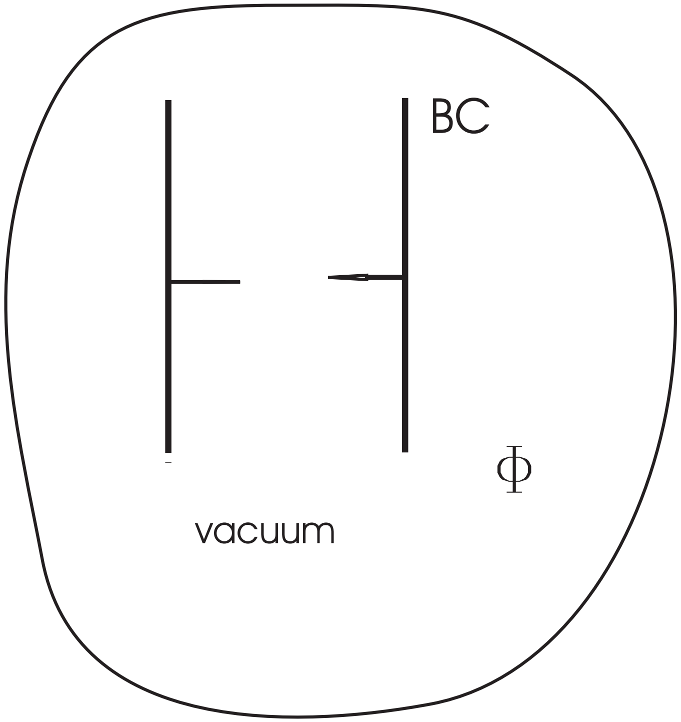

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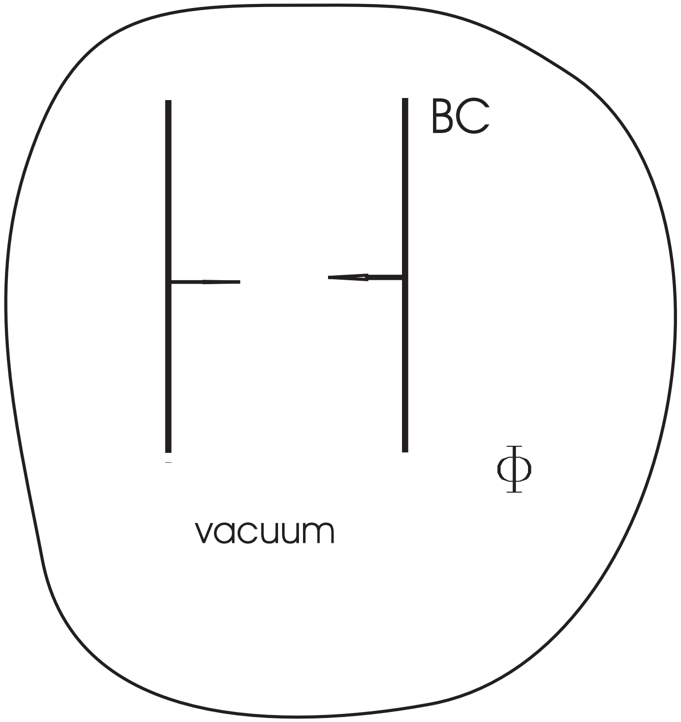

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- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant

On the ‘reality’ of zero point fluctuations

- The Casimir effect gives no more nor less support for the “reality” of the vacuum fluctuations than other one-loop effects in QED (like vacuum polarization contribution to Lamb shift)

    [R. Jaffe, PRD72 (2005) 021301; hep-th/0503158]

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- Milonni has reformulated all of QED from the point of view of ZPF

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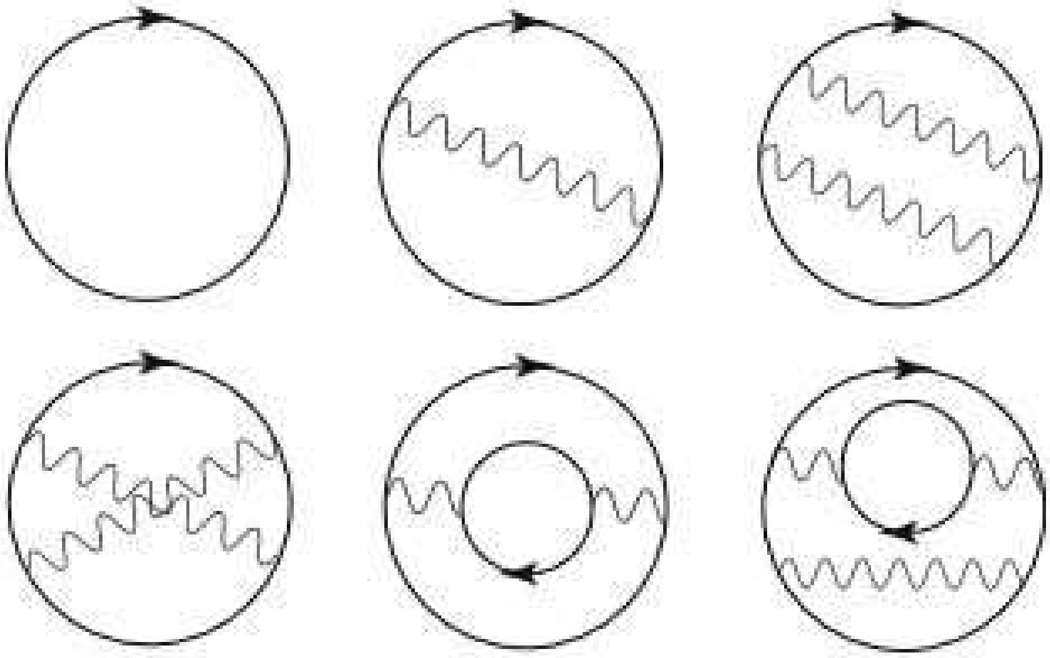

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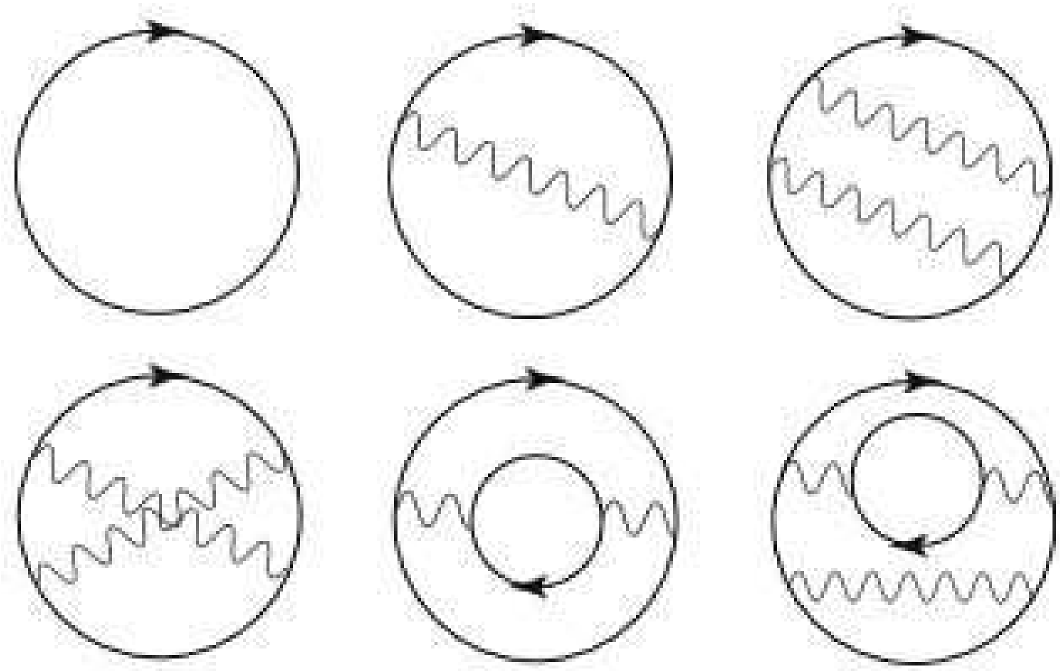

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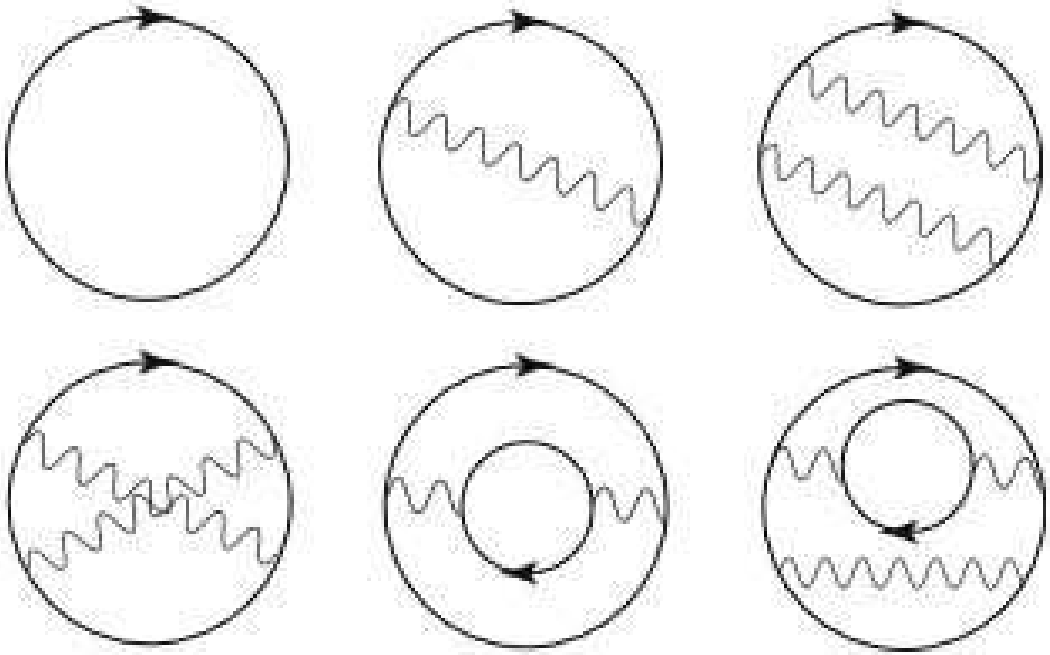

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$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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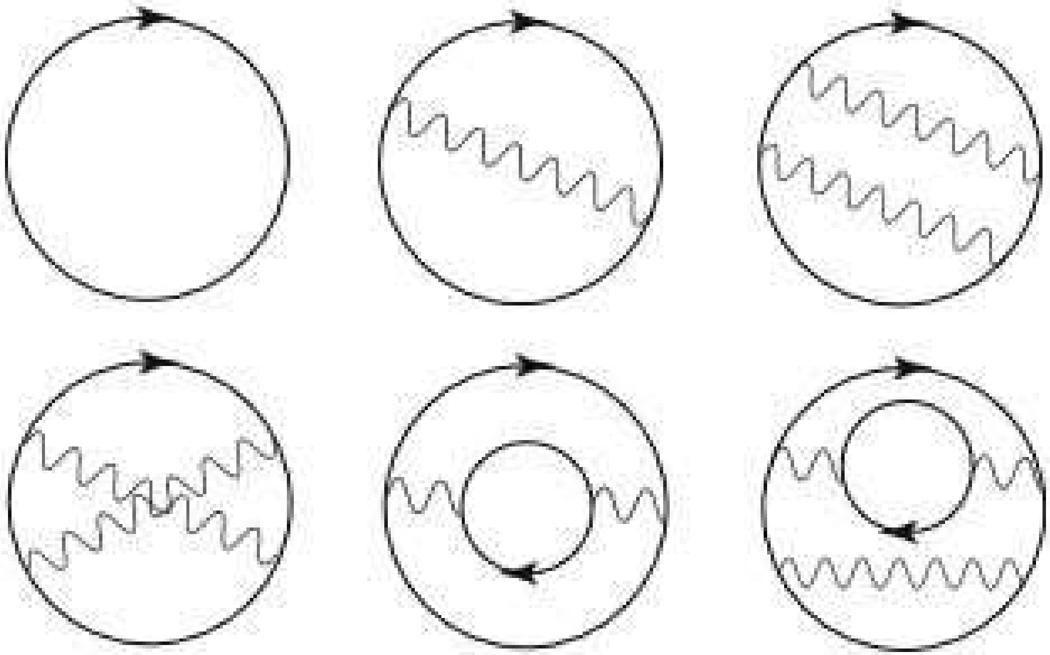

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function   Trace is over spin

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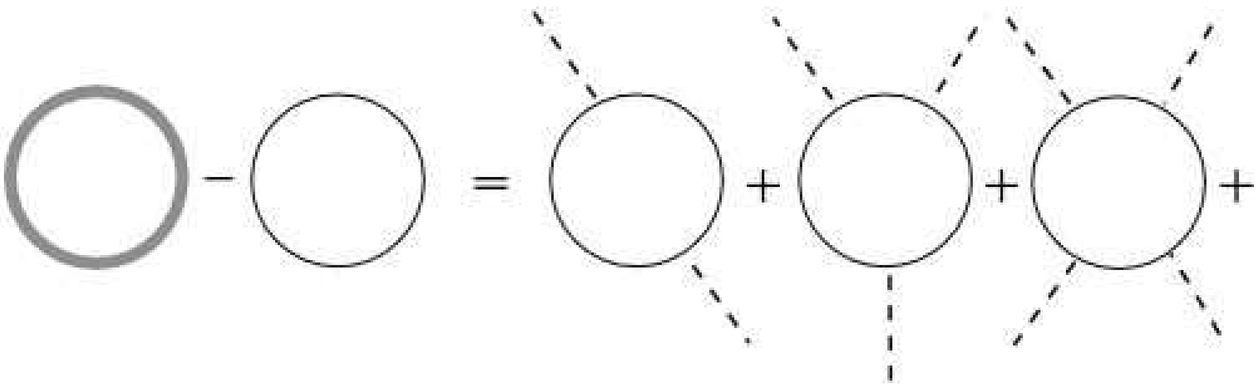

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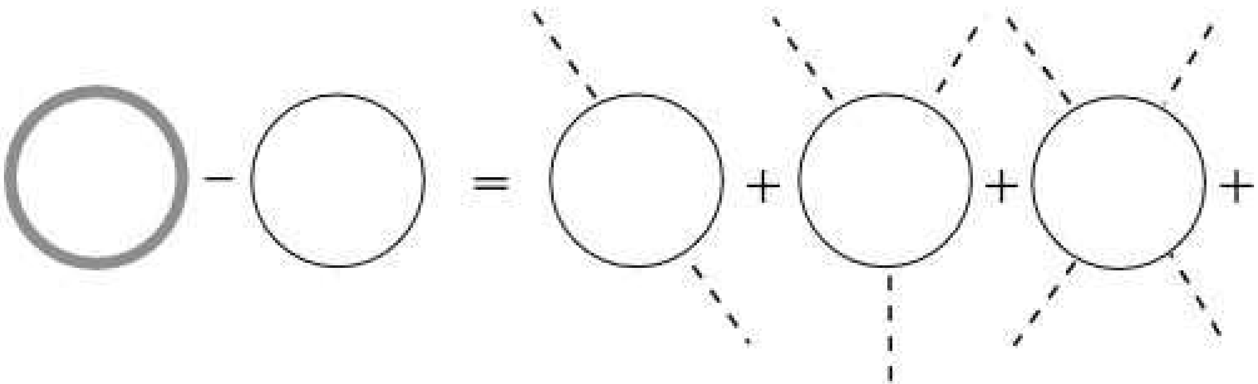

$\Longrightarrow$ Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations

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- Look for direct evidence of vacuum fluctuations!

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- Idea: zero point fluctuations do contribute to the cosmological constant

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- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]
- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:
  $\Longrightarrow$ kind of cosmological Casimir effects

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  - (c) supergravitons

A. Simple model: large & small dim's

- Space-time:

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A. Simple model: large & small dim's

- Space-time:

$$\mathbf{R}^{d+1} \times \mathbf{T}^p \times \mathbf{T}^q, \qquad \mathbf{R}^{d+1} \times \mathbf{T}^p \times \mathbf{S}^q, \qquad \cdots$$

- Scalar field, ϕ, pervading the universe
- ρ_ϕ contribution to ρ_V from this field

$$\rho_\phi = \frac{1}{2} \sum_i \lambda_i = \frac{1}{2} \sum_{\mathbf{k}} \frac{1}{\mu} \left(k^2 + M^2\right)^{1/2}$$

- $\sum_i$ and $\sum_{\mathbf{k}}$ are generalized sums, μ mass-dim parameter, $\hbar = c = 1$
- M mass of the field arbitrarily small (a tiny mass for the field can never be excluded); see L. Parker & A. Raval, PRL86 749 (2001); PRD62 083503 (2000)

- For d–open, (p,q)–toroidal universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^{p} a_j \prod_{h=1}^{q} b_h}\int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{\mathbf{m}_q=-\infty}^{\infty}$$

$$\left[\sum_{j=1}^{p}\left(\frac{2\pi n_j}{a_j}\right)^2 + \sum_{h=1}^{q}\left(\frac{2\pi m_h}{b_h}\right)^2 + \mathbf{k}_d^2 + M^2\right]^{1/2}$$

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p,\mathbf{m}_q=-\infty}^{\infty}\left(\frac{1}{a^2}\sum_{j=1}^{p} n_j^2 + \frac{1}{b^2}\sum_{h=1}^{q} m_h^2 + M^2\right)^{\frac{d+1}{2}+1}$$

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- For d–open, (p–toroidal, q–spherical) universe:

$$\rho_\phi = \frac{\pi^{-d/2}}{2^d\Gamma(d/2)\prod_{j=1}^p a_j\, b^q}\int_0^\infty dk\, k^{d-1} \sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty} P_{q-1}(l)$$

$$\left[\sum_{j=1}^p\left(\frac{2\pi n_j}{a_j}\right)^2 + \frac{Q_2(l)}{b^2} + \mathbf{k}_d^2 + M^2\right]^{1/2}$$

[$P_{q-1}(l)$ poly in l deg $q-1$]

$$\simeq \frac{1}{a^p b^q}\sum_{\mathbf{n}_p=-\infty}^{\infty}\sum_{l=1}^{\infty} P_{q-1}(l)\left(\frac{4\pi^2}{a^2}\sum_{j=1}^p n_j^2 + \frac{l(l+q)}{b^2} + \frac{M^2}{4\pi^2}\right)^{\frac{d+1}{2}+1}$$

- Regularize: zeta function

$$\zeta(s) = \frac{1}{2}\sum_i \lambda_i^{-s} = \frac{1}{2}\sum_{\mathbf{k}} \left(\frac{k^2+M^2}{\mu^2}\right)^{-s/2} \Longrightarrow \rho_\phi = \zeta(-1)$$

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- For zeta function (Re $s > p/2$):

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbf{Z}^p}{}' \left[\frac{1}{2}(\vec{n}+\vec{c})^T A(\vec{n}+\vec{c}) + q\right]^{-s}$$
$$\equiv \sum_{\vec{n}\in\mathbf{Z}^p}{}' \left[Q(\vec{n}+\vec{c}) + q\right]^{-s}$$

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$$\sum_{\vec{m}\in \mathbf{Z}^p_{1/2}}{}' \cos(2\pi \vec{m}\cdot\vec{c}) \left(\vec{m}^T A^{-1} \vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\, \vec{m}^T A^{-1} \vec{m}}\right)$$

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- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbf{Z}^p_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbf{Z}^p$ intervene in the sum. That is, if we take an $\vec{m} \in \mathbf{Z}^p$ we must then exclude $-\vec{m}$ (as simple criterion one can, for instance, select those vectors in $\mathbf{Z}^p \backslash \{\vec{0}\}$ whose first non-zero component is positive).

Calcul. of vacuum energy density

Analytic continuation of the zeta function:

$$\zeta(s) = \frac{2\pi^{s/2+1}}{a^{p-(s+1)/2}b^{q-(s-1)/2}\Gamma(s/2)} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \left(\frac{\sum_{j=1}^{h} n_j^2}{\sum_{k=1}^{q} m_k^2 + M^2} \right)^{\frac{s-1}{4}}$$

$$\times K_{(s-1)/2} \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

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Yields the vacuum energy density:

$$\rho_\phi \quad = \quad -\frac{1}{a^p b^{q+1}} \sum_{h=0}^{p} \binom{p}{h} 2^h \sum_{\mathbf{n}_h=1}^{\infty} \sum_{\mathbf{m}_q=-\infty}^{\infty} \sqrt{\frac{\sum_{k=1}^{q} m_k^2 + M^2}{\sum_{j=1}^{h} n_j^2}}$$

$$\times K_1 \left[\frac{2\pi a}{b} \left(\sum_{j=1}^{h} n_j^2 \right)^{1/2} \left(\sum_{k=1}^{q} m_k^2 + M^2 \right)^{1/2} \right]$$

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Inserting the $\hbar$ and c factors

$$\rho_\phi = -\frac{\hbar c}{2\pi a^{p+1} b^q}\left[1 + \sum_{h=0}^{p} \binom{p}{h} 2^h \alpha\right] + \mathcal{O}\left[q K_1\left(\frac{2\pi a}{b}\right)\right]$$

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Sign may change with BC (e.g., Dirichlet): a problem

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$$b \sim l_{P(lanck)} \qquad a \sim R_{U(niverse)} \qquad a/b \sim 10^{60}$$

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ρ_ϕ	$p=0$	$p=1$	$p=2$	$p=3$
$b = l_P$	10^{-13}	10^{-6}	1	10^{5}
$b = 10 l_P$	10^{-14}	$[10^{-8}]$	10^{-3}	10
$b = 10^2 l_P$	10^{-15}	(10^{-10})	10^{-6}	10^{-3}
$b = 10^3 l_P$	10^{-16}	10^{-12}	$[10^{-9}]$	(10^{-7})
$b = 10^4 l_P$	10^{-17}	10^{-14}	10^{-12}	10^{-11}
$b = 10^5 l_P$	10^{-18}	10^{-16}	10^{-15}	10^{-15}

Table 2: Vacuum energy density in units of erg/cm^3, for p large compactified dimensions a, and $q = p + 1$ small compactified dimensions b, $p = 0, \ldots, 3$, for different values of b, proportional to the Planck length l_P

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$\Longrightarrow$ To examine $\rightarrow$ couplings in GR

$\rightarrow$ alternative theories

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Elizalde, Nojiri, Odintsov, PRD70 (2004) 043539

Elizalde, Nojiri, Odintsov, Ogushi, PRD67 (2003) 063515

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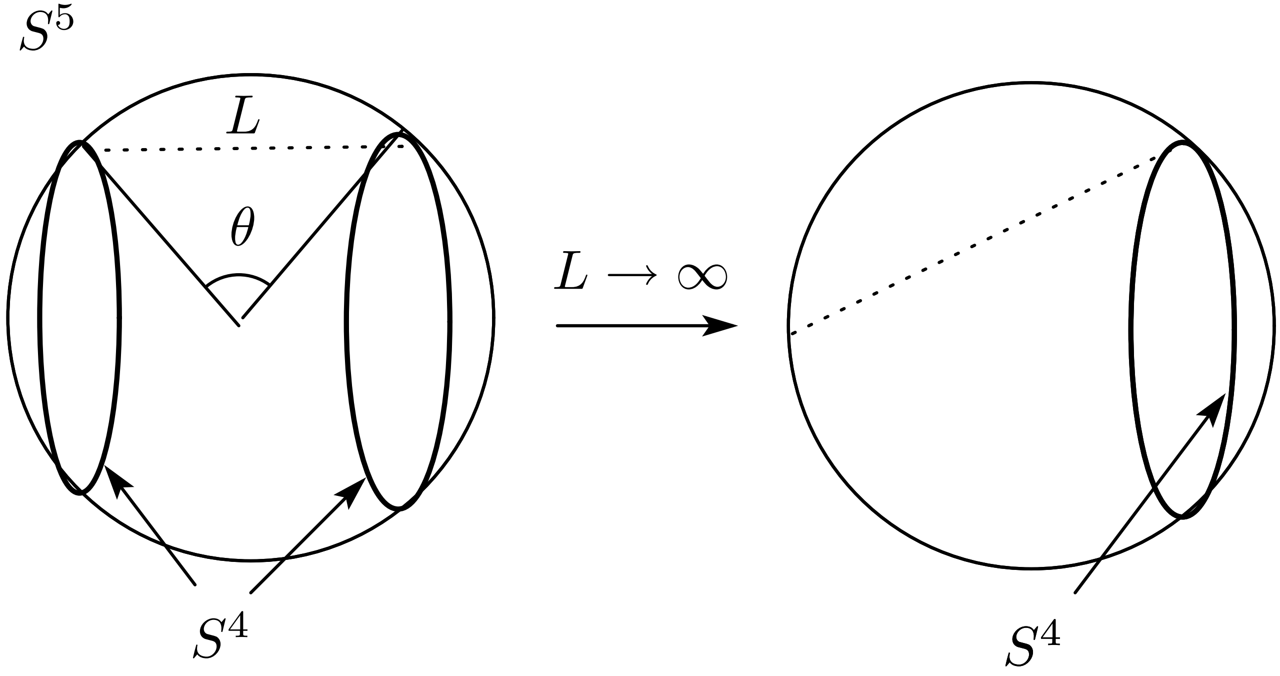
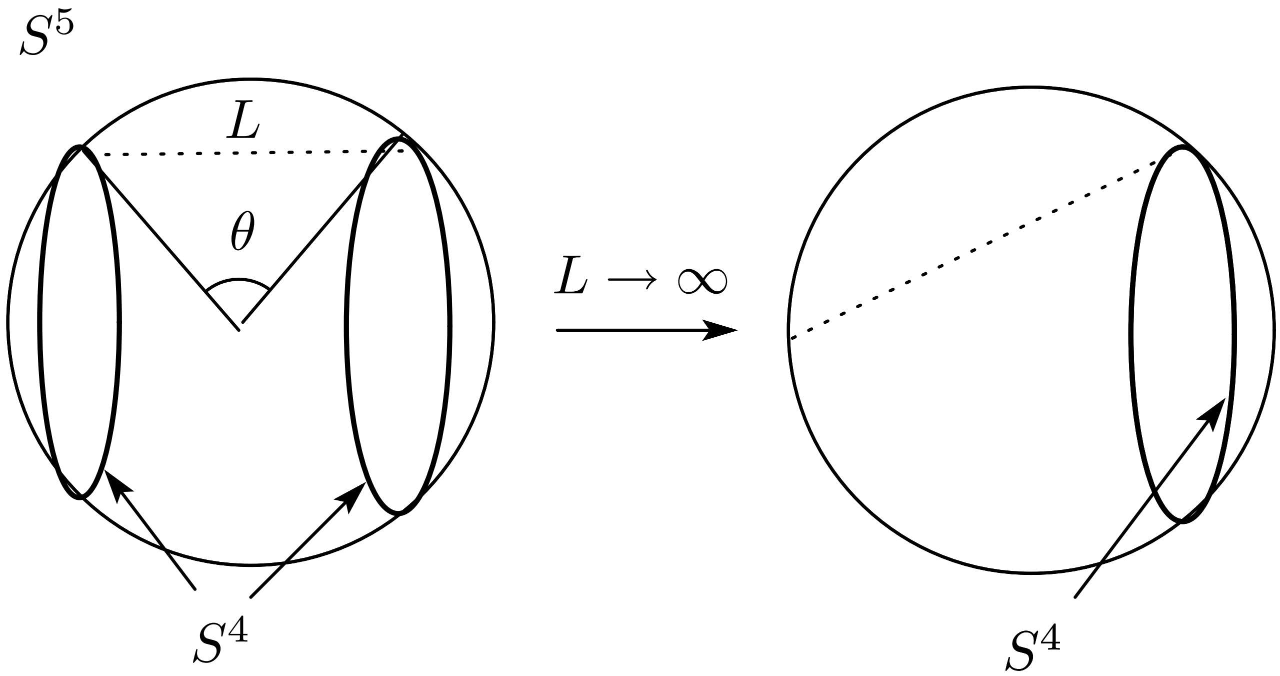
We use zeta regularization at full power



Two dS_4 branes in dS_5 background; becomes a one-brane as $L \to \infty$



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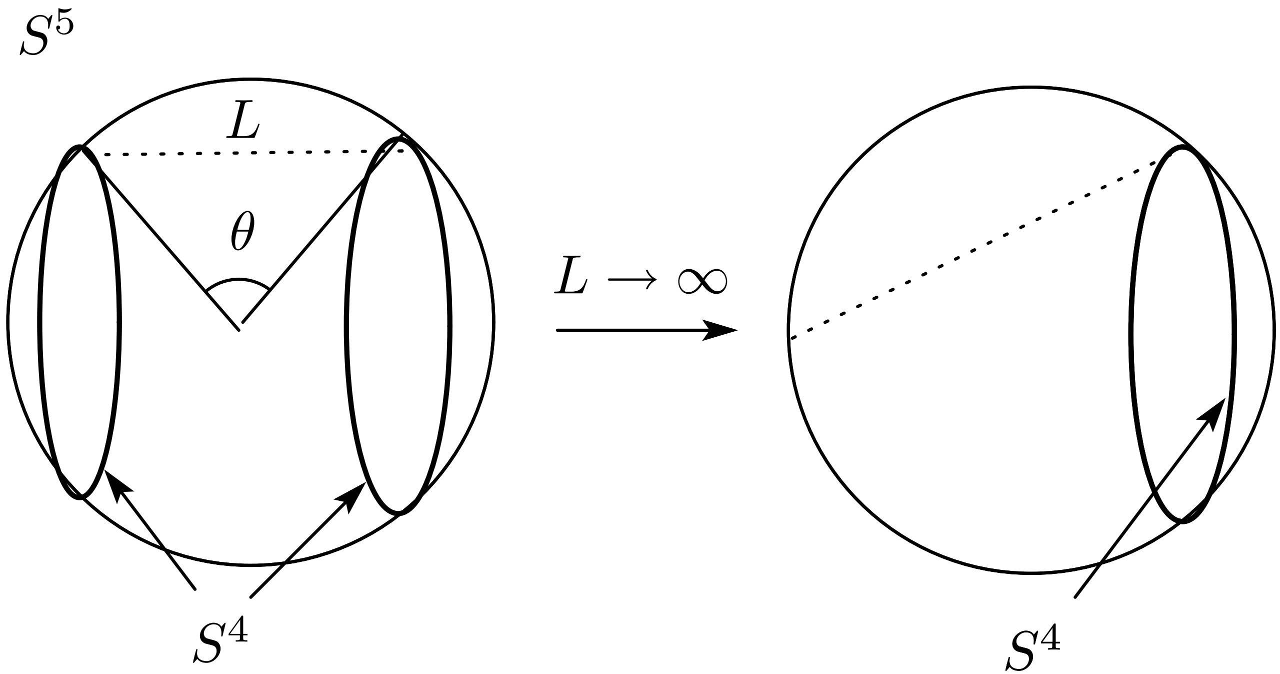

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$\Longrightarrow$ Action for conformally inv massless scalar with scalar-gravit coupling

$$\mathcal{S} = \frac{1}{2} \int d^5x \sqrt{g} \left[-g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi_5 R^{(5)} \phi^2 \right]$$

$\xi_5 = -3/16$ $\quad$ $R^{(5)}$ 5-dim scalar curvature

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = \frac{\alpha^2}{\sinh^2 z} \left(dz^2 + d\Omega_4^2\right) \\ d\Omega_4^2 &= d\xi^2 + \sin^2 \xi d\Omega_3^2 \end{aligned}$$

α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

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α is the AdS radius, related to cc of AdS bulk

$d\Omega_3$ metric on the 3-sphere

CASIMIR ENERGY

$\Longrightarrow$ Euclidean metric of the 5-dim AdS bulk

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CASIMIR ENERGY

(a) One-brane Casimir energy $= 0$

(b) Bulk Casimir energy:

$$\zeta(s|L_5) = \frac{\mu^{-2s}}{6} \sum_{n,l=1}^{\infty} (l+1)(l+2)(2l+3) \times \left[\left(\frac{\pi n}{L}\right)^2 + \mathcal{R}^{-2}\left(l^2 + 3l + \frac{9}{4}\right)\right]^{-s}$$

L brane separation (an imposing zeta function)

Can be written as

$$\zeta(s|L_5) = \frac{\mathcal{R}^{2s}}{6\mu^{2s}} \sum_{n,l=1}^{\infty} 2\left(l+\frac{3}{2}\right)\left\{\left[\left(l+\frac{3}{2}\right)^2+\frac{\pi^2 n^2 \mathcal{R}^2}{L^2}\right]^{1-s} - \left(\frac{\pi^2 n^2}{L^2}+\frac{1}{4}\right)\left[\left(l+\frac{3}{2}\right)^2+\frac{\pi^2 n^2 \mathcal{R}^2}{L^2}\right]^{-s}\right\}$$

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about ten times larger than the ordinary CE

$$\mathcal{E}_{\text{CE}} = -\frac{\hbar c \pi^2}{240\, L^4}$$

which is about 100 dynes / cm^2 at 100 nm

The one-loop effective potential

$$V = \frac{1}{2L\mathrm{Vol}(M_4)} \log\det(L_5/\mu^2) L_5 = -\partial_z^2 - \Delta^{(4)} - \xi_5 R^{(4)} = L_1 + L_4$$

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(a) In the one-brane limit $L \Longrightarrow \infty$

$$\zeta'(0|L_5) = \frac{1}{3\mathcal{R}}\left[\zeta_H\left(-4, \frac{3}{2}\right) - \frac{1}{4}\zeta_H\left(-2, \frac{3}{2}\right)\right] = 0$$

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(b) In the small distance expansion

$$\zeta'(0|L_5) = \frac{\zeta'(-4)\pi^4\mathcal{R}^4}{6L^4} + \frac{\zeta'(-2)\pi^2\mathcal{R}^2}{12L^2} + \frac{1}{24}\left[\zeta'_H(-4,3/2) - \frac{1}{2}\zeta'_H(-2,3/2)\right]\ln\frac{\pi^2\mathcal{R}^2}{L^2}$$
$$+\frac{\zeta'(0)}{6}\left[\zeta'_H(-4,3/2) - \frac{1}{2}\zeta'_H(-2,3/2)\right] + \frac{1}{24}\zeta'_H(-4,3/2)$$
$$+\frac{1}{36}\left[\frac{1}{8}\zeta'_H(-4,3/2) - \frac{1}{3}\zeta'_H(-6,3/2)\right]\frac{L^2}{\mathcal{R}^2} + \mathcal{O}\left(\frac{L^4}{\pi^4\mathcal{R}^4}\right)$$

$$\simeq 0.129652\,\frac{\mathcal{R}^4}{L^4} - 0.025039\,\frac{\mathcal{R}^2}{L^2} - 0.002951\,\ln\frac{\mathcal{R}^2}{L^2} - 0.017956 - 0.000315\,\frac{L^2}{\mathcal{R}^2} + \cdots$$

The massive case

Lagrangian for a massive scalar field with scalar-gravitational coupling in an AdS background

$$\mathcal{L} = \phi\left(\partial_z^2 + \Delta^{(4)} - m^2 l^2 \sinh^{-2} z + \xi_5 R^{(4)}\right)\phi$$

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Effective potential for the massive scalar field

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(a) Small mass limit (with L not large)

eigenvalues

$$\lambda_n^2 \simeq \frac{\pi^2 n^2}{\mu^2 L^2} + m^2 l^2 \frac{\tanh(\mu L/2)}{\mu L/2}.$$

Mass correction: de Sitter brane in AdS bulk

$$
\begin{aligned}
\Delta\zeta'(0|L_5) \quad \simeq \quad & \frac{a\rho + a^2\rho^2}{48} - \frac{\pi^2}{144}\left\{\frac{a\rho^2}{2} + [2\zeta'(-4,3/2) - \zeta'(-2,3/2)]\,\rho\right\} \\
& - \frac{\pi^4}{4370}\,[2\zeta'(-4,3/2) - \zeta'(-2,3/2)]\,\rho^2 + \mathcal{O}(m^6),
\end{aligned}
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Elizalde, Nojiri, Odintsov, Ogushi, PRD 67 (2003) 063515

C. Supergraviton theories

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⟹ Allow for non-nearest-neighbor couplings

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$$\mathcal{L} = \sum_{n=0}^{N-1} \Big[-\frac{1}{2}\partial_\lambda h_{n\mu\nu}\partial^\lambda h_n^{\mu\nu} + \partial_\lambda h_{n\mu}^{\lambda}\partial_\nu h_n^{\mu\nu} - \partial_\mu h_n^{\mu\nu}\partial_\nu h_n + \frac{1}{2}\partial_\lambda h_n \partial^\lambda h_n$$

$$-\frac{1}{2}\left(m^2 \Delta h_{n\mu\nu}\Delta h_n^{\mu\nu} - (\Delta h_n)^2\right) - 2\left(m\Delta^\dagger A_n^\mu + \partial^\mu \varphi_n\right)\left(\partial^\nu h_{n\mu\nu} - \partial_\mu h_n\right)$$

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Δ, $\Delta^\dagger$ difference operators

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a_k are N constants and ϕ_n can be identified with periodic fields on a lattice with N sites if periodic boundary conditions, $\phi_{n+N} = \phi_n$, are imposed

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(Δ becomes usual differentiation operator in properly defined continuum limit)

When anti-periodic boundary conditions are imposed in the fermionic sector, the situation changes completely with respect to the bosonic one, since the fermionic mass spectrum becomes quite different. The one-loop effective potential in the anti-periodic case is calculated to be

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$$V_{eff} = \frac{M_1^4}{4\pi^2}\left(\ln\frac{M_1^2}{\mu_R^2} - \frac{3}{2}\right) - \frac{4M_1^4}{3\pi^2}\int_1^\infty du\, G(M_1 ru)\,(u^2-1)^{3/2}$$
$$-\frac{\tilde{M}_0^4}{4\pi^2}\left(\ln\frac{\tilde{M}_0^2}{\mu_R^2} - \frac{3}{2}\right) + \frac{4\tilde{M}_0^4}{3\pi^2}\int_1^\infty du\, G(\tilde{M}_0 ru)\,(u^2-1)^{3/2}$$
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$$= \quad -\frac{m^4}{36\pi^2}\log\frac{2^{16}}{3^9} + V_T$$

V_T is the sum of all the topological contributions

Note that first term on rhs is always negative

When anti-periodic boundary conditions are imposed in the fermionic sector, the situation changes completely with respect to the bosonic one, since the fermionic mass spectrum becomes quite different. The one-loop effective potential in the anti-periodic case is calculated to be

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$$V_T \sim \frac{1}{8\pi^2 r^4} \implies V_{eff} > 0 \quad \text{for} \quad mr < \left(\frac{2}{9}\log\frac{2^{16}}{3^9}\right)^{-1/4} \sim 1.4$$

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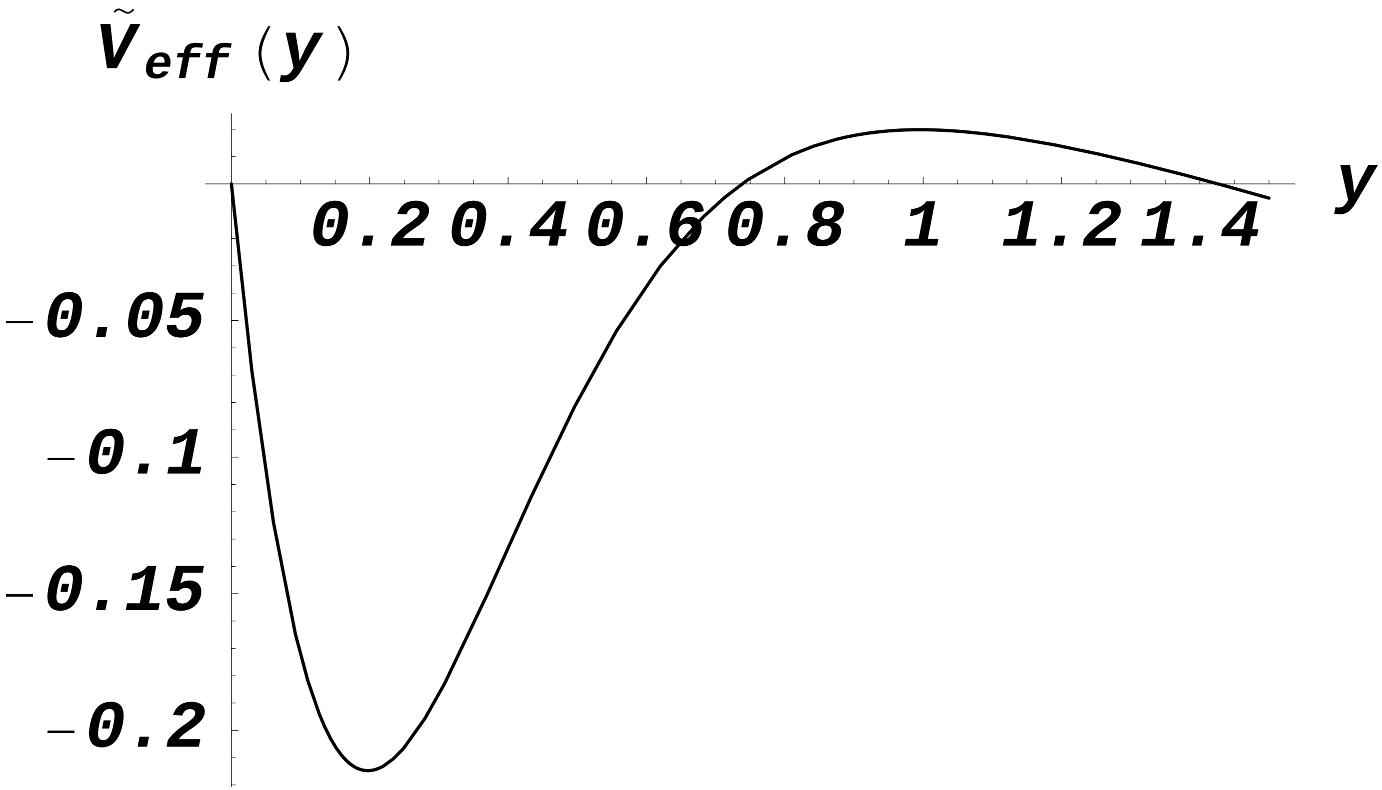

Figure 4: Plot of $\tilde{V}_{eff}(y) \equiv r^4 V_{eff}(r)$ as a function of $y \equiv mr$

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- The value of cc is regulated by the corresponding size of the torus (one can most naturally use the minimum number, N = 3, of copies of bosons and fermions), and can match observational values

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THANKS so much for your attention!