ICE CSIC IEEC

Repulsive Casimir Forces from Additional Dimensions

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Outline

- On Einstein's Cosmological Constant

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- Casimir Effect & the ζ Function Method

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- Gravity Equations as Equations of State

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- With THANKS to:
  S. Carloni, G. Cognola, J. Haro, S.D. Odintsov,
  A. Saharian, P.J. Silva, S. Zerbini, ...

On Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cosmological constant

- For cosmologists and general relativists: a great mistake (Einstein)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc Λ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density μ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g}\, \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g}\, \lambda$$

In other words: two contributions on the same footing (Zel'dovich, 68)

$$\frac{\Lambda\, c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar\, c}{2} \sum_i \omega_i$$

Einstein Eqs, FLRW Sol, Hubble Const

Einstein Equations (1915-17): $G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu}$

Geometry = Energy-Matter

$G_{\mu\nu}$ linear combination of the metric $g_{\mu\nu}$ and 1st & 2nd derivatives

$T_{\mu\nu}$ energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

Schwarzschild solution (1916) r, θ, φ *comoving co*

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Friedmann-Lemaître-Robertson-Walker (1935-36) sol (A. Friedmann 1922)

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

gen fam: homog + *isotrop*, k *par* $\pm 1, 0$ *Hubble ea 1923-29, Keeler Slipher Campbell 1918*

One field eq looks like Newtonian eq for the gravit pot: $\nabla^2 \phi = 4\pi G \left(\rho + 3p/c^2\right)$

density & pressure contribute to the gravit pot $\lambda = 8\pi G \rho_{vac}$, $p_{vac} = -\rho_{vac} c^2$

From the FRW metric and Einstein Eqs, an "equation of motion" of the universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

From GR to Cosmology

With the definitions:

$$\Omega_m \equiv \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \qquad \Omega_k \equiv -\frac{k}{H^2}$$

The equation of motion becomes

$$\left(\frac{da}{dt}\right)^2 = H_0^2\left[\frac{\Omega_m^{(0)}}{a} + a^2\Omega_\lambda^{(0)} + \Omega_k^{(0)}\right]$$

(the superscript (o) represents quantities measured at the present time)

In other terms, Friedmann equation in Cosmology:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_{NR}\left(\frac{a_0}{a}\right)^3 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\lambda\right]$$

Ω_R relativistic matter ($p_R = \frac{1}{3}\rho_R;\ \ \rho_R \propto a^{-4}$) Mach's princ [Wilczek]

Ω_{NR} nonrelativistic matter ($p_{NR} = 0;\ \ \rho_{NR} \propto a^{-3}$)

Ω_λ cosmological constant ($p_\lambda = -\rho_\lambda;\ \ \rho_\lambda = \mathrm{const}$)

$\Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda$ total energy density (cosmic triangle)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

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Spectrum, normal ordering (harm oscill):

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Even then: Has the final value real sense ?

Existence of ζ_A for A a ΨDO

1. A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}^+$
2. A acts on the space of smooth sections of
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(a) The zeta function is defined as:

$$\zeta_A(s) = \text{tr}\, A^{-s} = \sum_j \lambda_j^{-s}, \quad \text{Re}\, s > \frac{n}{m} := s_0$$

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(b) $\zeta_A(s)$ has a meromorphic continuation to the whole complex plane $\mathbb{C}$ (regular at $s = 0$), provided the principal symbol of A, $a_m(x, \xi)$, admits a spectral cut: $L_\theta = \{\lambda \in \mathbb{C}; \text{Arg } \lambda = \theta, \theta_1 < \theta < \theta_2\}$, $\text{Spec } A \cap L_\theta = \emptyset$ (the Agmon-Nirenberg condition)

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(d) The only possible singularities of $\zeta_A(s)$ are poles at

$$s_j = (n - j)/m, \qquad j = 0, 1, 2, \ldots, n-1, n+1, \ldots$$

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Definition: zeta function of H $\zeta_H(s) = \sum_{i \in I} \lambda_i^{-s} = \text{tr}\, H^{-s}$

As Mellin transform: $\zeta_H(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} dt\, t^{s-1}\, \text{tr}\, e^{-tH}, \; \text{Re}\, s > s_0$

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C. Soulé et al, Lectures on Arakelov Geometry, CUP 1992; A. Voros,...

Properties

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- A zeta function (and corresponding determinant) with the same meromorphic structure in the complex s-plane and extending the ordinary definition to operators of complex order $m \in \mathbb{C}\backslash\mathbb{Z}$ (they do not admit spectral cuts), has been obtained [Kontsevich and Vishik]

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- Asymptotic expansion for the heat kernel:

$$\text{tr}\, e^{-tA} = {\sum}'_{\lambda \in \text{Spec}\, A} e^{-t\lambda}$$

$$\sim \alpha_n(A) + \sum_{n \neq j \geq 0} \alpha_j(A) t^{-s_j} + \sum_{k \geq 1} \beta_k(A) t^k \ln t, \quad t \downarrow 0$$

$$\alpha_n(A) = \zeta_A(0), \quad \alpha_j(A) = \Gamma(s_j)\, \text{Res}_{s=s_j} \zeta_A(s), \; s_j \notin -\mathbb{N}$$

$$\alpha_j(A) = \frac{(-1)^k}{k!} \left[\text{PP}\, \zeta_A(-k) + \psi(k+1)\, \text{Res}_{s=-k} \zeta_A(s) \right], \quad s_j = -k, \; k \in \mathbb{N}$$

$$\beta_k(A) = \frac{(-1)^{k+1}}{k!} \text{Res}_{s=-k} \zeta_A(s), \quad k \in \mathbb{N}\backslash\{0\}$$

$$\text{PP}\, \phi := \lim_{s \to p} \left[\phi(s) - \frac{\text{Res}_{s=p}\, \phi(s)}{s-p} \right]$$

The Chowla-Selberg Expansion Formula: Basics

- Jacobi's identity for the $\theta-$function

$$\theta_3(z,\tau) := 1 + 2\sum_{n=1}^\infty q^{n^2}\cos(2nz), \qquad q := e^{i\pi\tau},\ \tau\in\mathbb{C}$$

$$\theta_3(z,\tau) = \frac{1}{\sqrt{-i\tau}}\, e^{z^2/i\pi\tau}\,\theta_3\left(\frac{z}{\tau}\Big|\frac{-1}{\tau}\right)$$

equivalently:

$$\sum_{n=-\infty}^{\infty} e^{-(n+z)^2 t} = \sqrt{\frac{\pi}{t}}\sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2}{t}}\cos(2\pi n z), \qquad z,t\in\mathbb{C},\ \text{Re}\,t>0$$

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- Higher dimensions: Poisson summ formula (Riemann)

$$\sum_{\vec{n}\in\mathbb{Z}^p} f(\vec{n}) = \sum_{\vec{m}\in\mathbb{Z}^p} \widetilde{f}(\vec{m})$$

$\widetilde{f}$ Fourier transform

[Gelbart + Miller, BAMS '03, Iwaniec, Morgan, ICM '06]

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- Truncated sums $\longrightarrow$ asymptotic series

Extended CS Formulas (ECS)

- Consider the zeta function (Re$s > p/2, A > 0,$ Re$q > 0$)

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[\frac{1}{2}(\vec{n}+\vec{c})^T A(\vec{n}+\vec{c}) + q\right]^{-s} = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[Q(\vec{n}+\vec{c}) + q\right]^{-s}$$

prime: point $\vec{n} = \vec{0}$ to be excluded from the sum

(inescapable condition when $c_1 = \cdots = c_p = q = 0$)

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- Case $q \neq 0$ (Re$q > 0$)

$$\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2} q^{p/2-s}}{\sqrt{\det A}} \frac{\Gamma(s-p/2)}{\Gamma(s)} + \frac{2^{s/2+p/4+2}\pi^s q^{-s/2+p/4}}{\sqrt{\det A}\,\Gamma(s)}$$

$$\times \sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c}) \left(\vec{m}^T A^{-1}\vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

[ECS1]

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[ECS1]

- Pole: $s = p/2$ Residue:

$$\text{Res}_{s=p/2}\,\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2}}{\Gamma(p/2)} (\det A)^{-1/2}$$

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- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbb{Z}^p_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbb{Z}^p$ participate in the sum. E.g., if we take an $\vec{m} \in \mathbb{Z}^p$ we must then exclude $-\vec{m}$ [simple criterion: one may select those vectors in $\mathbb{Z}^p \backslash \{\vec{0}\}$ whose first non-zero component is positive]

- Gives (analytic cont of) multidimensional zeta function in terms of an exponentially convergent multiseries, valid in the whole complex plane
- Exhibits singularities (simple poles) of the meromorphic continuation —with the corresponding residua— explicitly
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- Case $c_1 = \cdots = c_p = q = 0$ [true extens of CS, diag subcase]

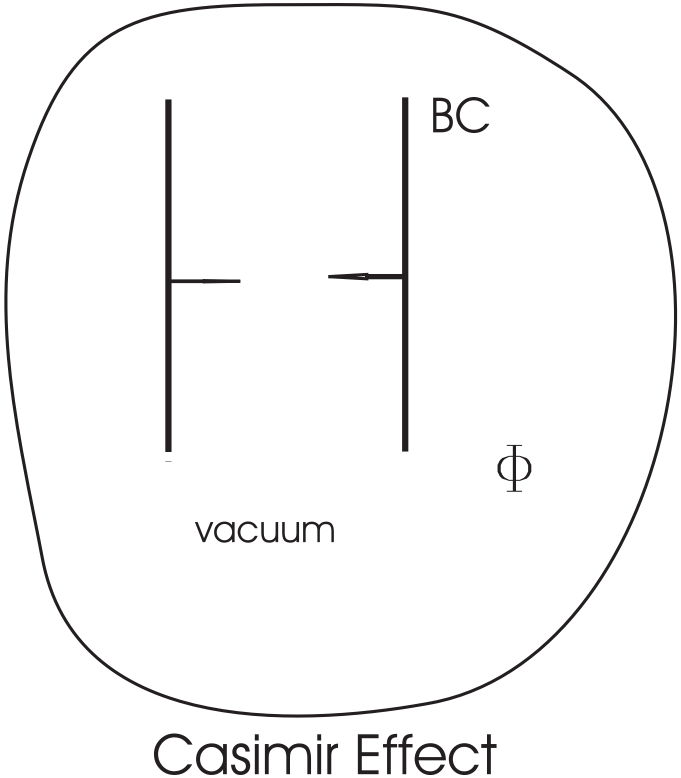
$$\zeta_{A_p}(s) = \frac{2^{1+s}}{\Gamma(s)} \sum_{j=0}^{p-1} (\det A_j)^{-1/2} \left[\pi^{j/2} a_{p-j}^{j/2-s}\, \Gamma\left(s - \frac{j}{2}\right) \zeta_R(2s-j) + 4\pi^s a_{p-j}^{\frac{j}{4}-\frac{s}{2}} \sum_{n=1}^{\infty} \sum_{\vec{m}_j \in \mathbb{Z}^j}{}' n^{j/2-s} \left(\vec{m}_j^t A_j^{-1} \vec{m}_j\right)^{s/2-j/4} K_{j/2-s}\left(2\pi n \sqrt{a_{p-j} \vec{m}_j^t A_j^{-1} \vec{m}_j}\right)\right]$$

[ECS3d]

The Casimir Effect

The Casimir Effect

BC e.g. periodic



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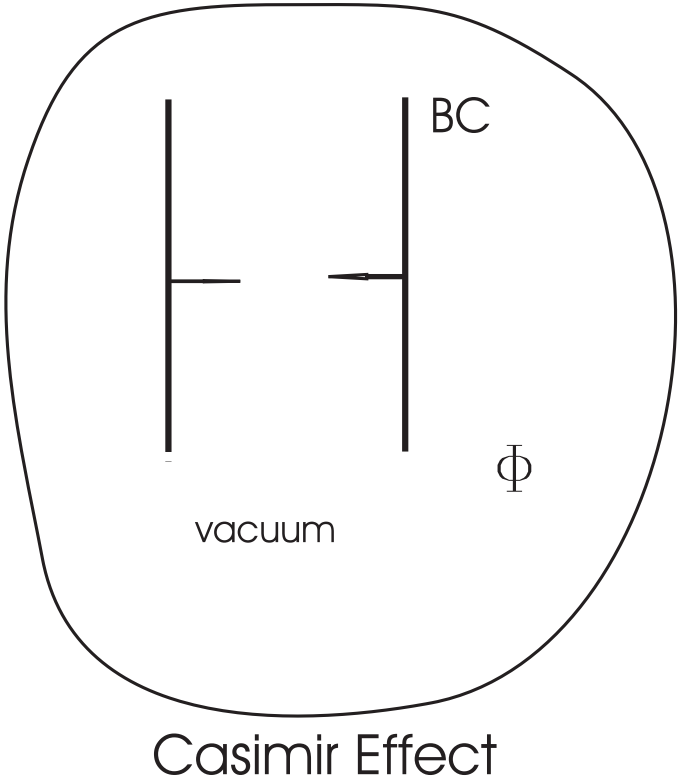

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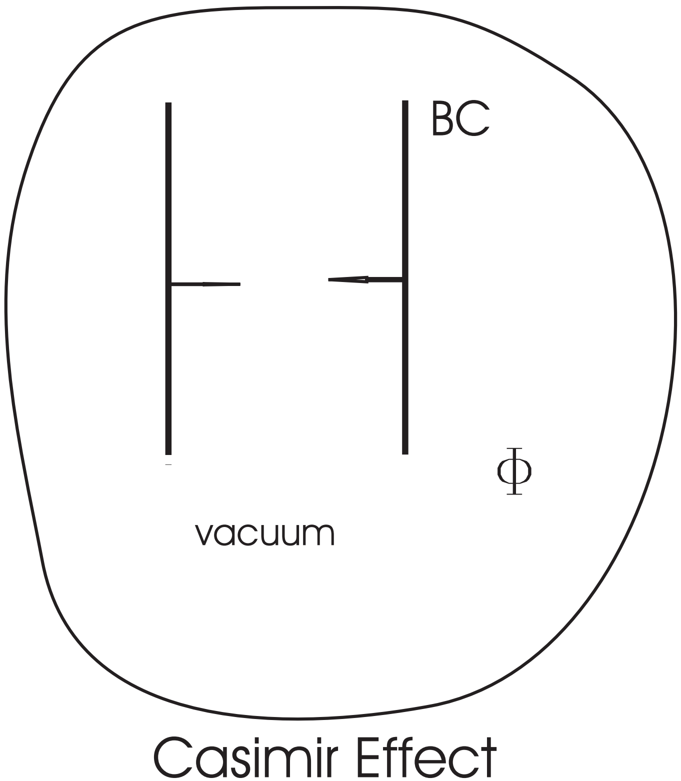

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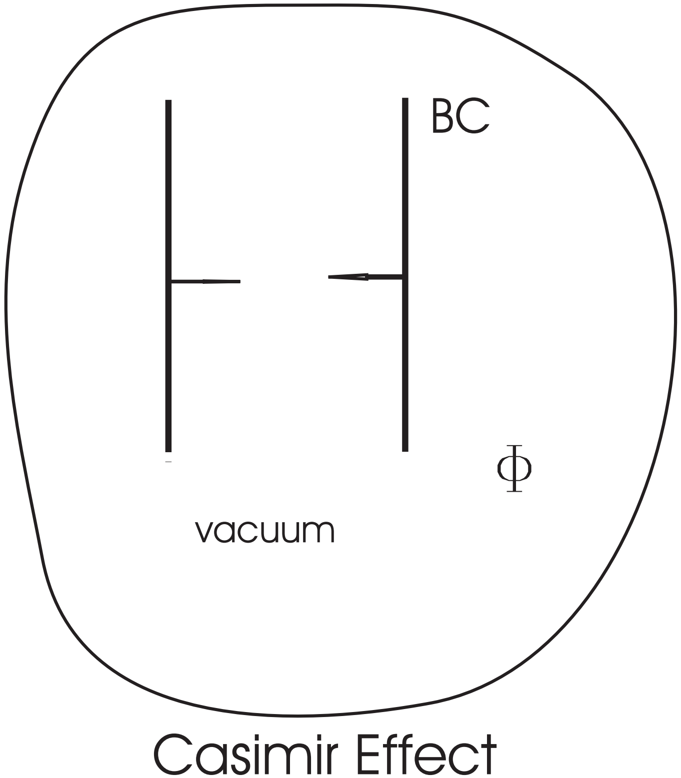

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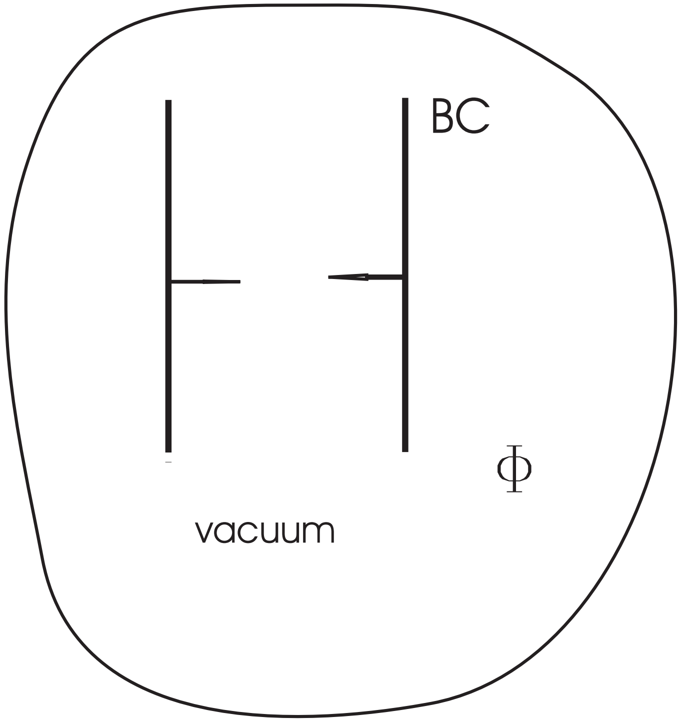

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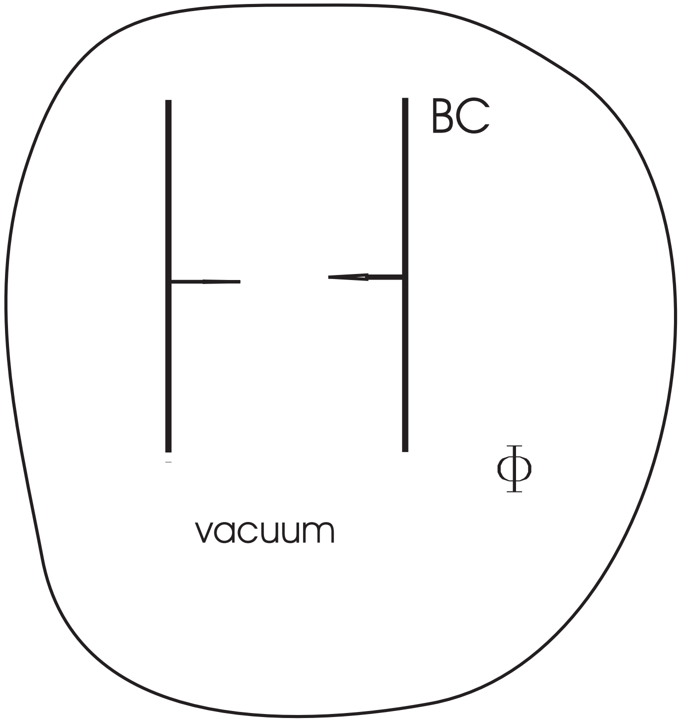

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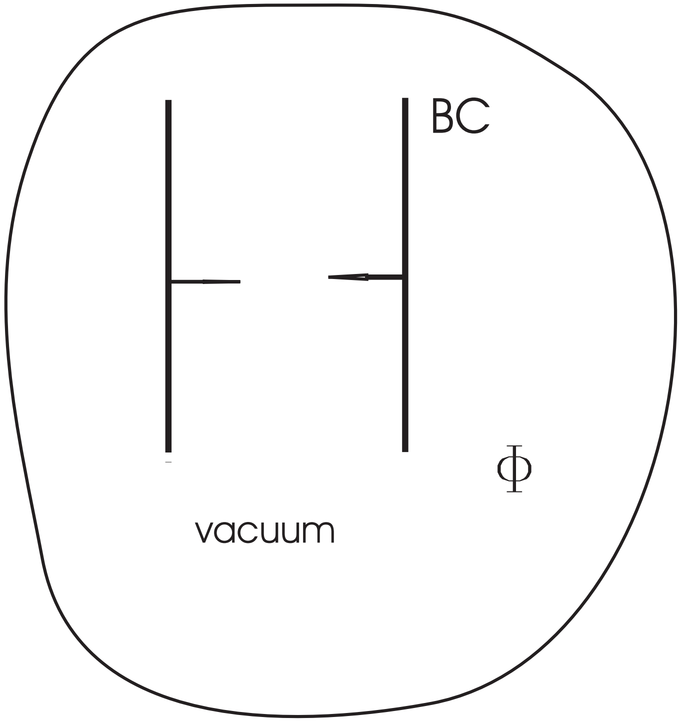

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- Dynamical CE ⟸
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant ⟸

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al;
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Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al;
Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

SOME DETAILS OF THE METHOD

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

(of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

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renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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Consider, instead, a partially transmitting mirror, transparent to high frequencies (math. implementation of a physical plate).

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Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)\, e^{-2i\omega L} \\ r(\omega)\, e^{2i\omega L} & s(\omega) \end{pmatrix}$$

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$ [S Reynaud]

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

RESULTS ARE REWARDING:

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In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t)\rangle = -\frac{\epsilon}{2\pi^2}\int_0^\infty\int_0^\infty \frac{d\omega d\omega'\omega\omega'}{\omega+\omega'}\,\text{Re}\left[e^{-i(\omega+\omega')t}\,\widehat{\dot{g}\theta_t}(\omega+\omega')\right]$$
$$\times\left[|r(\omega)+r^*(\omega')|^2+|s(\omega)-s^*(\omega')|^2\right]+\mathcal{O}(\epsilon^2)$$

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209v2

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

CC Problem

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\lambda$$

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

Cosmolog Imprint of the Casimir Eff't?

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
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- We show (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:

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  - (c) supergraviton theories (discret dims, deconstr)

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem

2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds [A Flachi]

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, S Nojiri, SD Odintsov, S Ogushi, Phys Rev D67 (2003) 063515 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, SD Odintsov, AA Saharian 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

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- More general results: Kenneth, Klich, PRL 97, 160401 (2006)
  a mirror pair of dielectric bodies always attract each other
  CP Bachas, J Phys A40, 9089 (2007) from a general property of Euclidean QFT 'reflection positivity' (Osterwalder - Schrader 73, 75):
  $\exists$ of positive Hilbert space and self-adjoint non-negative Hamiltonian

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
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  - Robin BCs in general

Casimir eff in brworl's w large extra dim

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- Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein's entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]

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For arbitrary internal space, interaction part of the Casimir energy given by

$$\Delta E_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_{\beta} \int_{m_\beta}^{\infty} dx\, x(x^2 - m_\beta^2)^{D_1/2-1}$$

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For Dirichlet and Neumann BCs on both plates this leads to

$$\Delta E_{[a_1,a_2]}^{(\mathrm{J},\mathrm{J})} = -\frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{n^{D_1+1}}$$

with $f_\nu(z) = z^\nu K_\nu(z)$ $\longrightarrow$ energy always negative

For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

$$\Delta E^{(\mathrm{D},\mathrm{N})}_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2} a}{\Gamma(D_1/2+1)} \sum_{\beta} \int_{m_\beta}^{\infty} dx \, \frac{\left(x^2 - m_\beta^2\right)^{D_1/2}}{e^{2ax}+1}$$

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In the case of a conformally coupled massless field on the background of a spacetime conformally related to the one described by the line element

$$ds^2 = g_{MN}dx^M dx^N = \eta_{\mu\nu}dx^\mu dx^\nu - \gamma_{il}dX^i dX^l$$

$\eta_{\mu\nu} = \mathrm{diag}(1,-1,\ldots,-1)$ metric of (D_1+1)-dim Minkowski st and X^i coordinates of Σ, with the conformal factor $\Omega^2(x^{D_1})$. Interaction part of Casimir energy is given (*), with coeffs β_j related to coeffs of the Robin BCs

$$(1+\overline{\beta}_j n^M\nabla_M)\overline{\varphi}(x) = [1+(-1)^{j-1}\Omega_j^{-1}\overline{\beta}_j\partial_{D_1}]\overline{\varphi}(x) = 0,\ \Omega_j = \Omega(x_j^{D_1})$$

& conformal factor $\beta_j = \left[\Omega_j + (-1)^j\frac{D-1}{2\Omega_j}\overline{\beta}_j\Omega_j'\right]^{-1}\overline{\beta}_j,\ \Omega_j' = \Omega_j'(x_j^{D_1})$

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\overline{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

The vacuum energy can have a minimum, for the stable equilibrium point

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With independence of the geometry of the internal space, the force is attractive for Dirichlet or Neumann boundary conditions on both plates

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$$= \frac{2a^{-D_1-1}}{(8\pi)^{(D_1+1)/2} V_\Sigma} \sum_\beta \sum_{n=1}^\infty \frac{1}{n^{D_1+1}} \left[f_{(D_1+1)/2}(2nam_\beta) - f_{(D_1+3)/2}(2nam_\beta) \right]$$

$\mathrm{J} = \mathrm{D}, \mathrm{N}$, and repulsive for Dirichlet BC on one plate and Neumann on the other, a monotonic function of the distance

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⟹ Interesting remark: this property could be used in the proposal of a Casimir experiment with the purpose to carry out an explicit detailed observation of 'large' extra dimensions as allowed by some models of particle physics

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"

  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...

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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  – entropy proport to variation of the horizon area: $\delta S = \eta\, \delta \mathcal{A}$

  – local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$

  – functional dependence of S wrt energy and size of system

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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_{\Sigma} E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta\left(\eta_e A\right)$$

η_e is a function of the metric and its deriv's to a given order

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\frac{1}{8\pi G_{eff}} = E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs}$$

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- For these theories, the different polarizations of the gravitons only enter in the definition of the effective Newton constant through the metric itself

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$$
\begin{aligned}
\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\
&= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad\qquad S = \frac{A}{4\,G_{eff}}
\end{aligned}
$$

- For these theories, the different polarizations of the gravitons only enter in the definition of the effective Newton constant through the metric itself

- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

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- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]

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Thanks for your attention

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