ICE CSIC IEEC

Repulsive Vacuum Forces and GR Alternative Cosmologies

EMILIO ELIZALDE

Higher Research Council of Spain

ICE/CSIC & IEEC, UAB, Barcelona

International Workshop “Beyond the Concordance model”

Stellenbosch I.A.S., August 23-27, 2010

Outline

- On Einstein's Cosmological Constant

Outline

- On Einstein's Cosmological Constant
- CE and Accelerated Expansion (Dark Energy)

Outline

- On Einstein's Cosmological Constant
- CE and Accelerated Expansion (Dark Energy)
- The Sign of the Vacuum Forces

Outline

- On Einstein's Cosmological Constant
- CE and Accelerated Expansion (Dark Energy)
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs

Outline

- On Einstein's Cosmological Constant
- CE and Accelerated Expansion (Dark Energy)
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs
- Emergent GR and Effective Cosmologies

Outline

- On Einstein's Cosmological Constant
- CE and Accelerated Expansion (Dark Energy)
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs
- Emergent GR and Effective Cosmologies
- With THANKS to:

  S Carloni, G Cognola, J Haro, S Nojiri, S Odintsov, D Sáez-Gómez, A Saharian, P Silva, S Zerbini, and several more ...

On Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cc

- For cosmologists and general relativists: a great mistake (Einstein)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc Λ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density μ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g}\, \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g}\, \lambda$$

In other words: two contributions on the same footing (Pauli 20s, Zel'dovich '68)

$$\frac{\Lambda c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar c}{2} \sum_i \omega_i$$

Einstein Eqs, FLRW Sol, Hubble Const

Einstein Equations (1915-17): $G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu}$

Geometry = Energy-Matter

$G_{\mu\nu}$ linear combination of the metric $g_{\mu\nu}$ and 1st & 2nd derivatives

$T_{\mu\nu}$ energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

Schwarzschild solution (1916) r, θ, φ *comoving co*

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Friedmann-Lemaître-Robertson-Walker (1935-36) solut (A. Friedmann 1922)

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right)$$

gen fam: homog + isotrop, k par $\pm 1, 0$ Keeler/Slipher/Campbell 1918, Hubble ea 1923-29

One field eq looks like Newtonian eq for the gravit pot: $\nabla^2 \phi = 4\pi G \left(\rho + 3p/c^2\right)$

density & pressure contribute to the gravit pot $\lambda = 8\pi G \rho_{vac}$, $p_{vac} = -\rho_{vac} c^2$

From the FRW metric and Einstein Eqs, an “equation of motion” of the universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

From GR to Cosmology

With the definitions:

$$\Omega_m \equiv \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \qquad \Omega_k \equiv -\frac{k}{H^2}$$

The equation of motion becomes

$$\left(\frac{da}{dt}\right)^2 = H_0^2\left[\frac{\Omega_m^{(0)}}{a} + a^2\Omega_\lambda^{(0)} + \Omega_k^{(0)}\right]$$

(the superscript (o) represents quantities measured at the present time)

In other terms, Friedmann equation in Cosmology:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_{NR}\left(\frac{a_0}{a}\right)^3 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\lambda\right]$$

Ω_R relativistic matter ($p_R = \frac{1}{3}\rho_R;\ \ \rho_R \propto a^{-4}$) Mach's princ [Wilczek PT Ap04]

Ω_{NR} nonrelativistic matter ($p_{NR} = 0;\ \ \rho_{NR} \propto a^{-3}$)

Ω_λ cosmological constant ($p_\lambda = -\rho_\lambda;\ \ \rho_\lambda = \mathrm{const}$)

$\Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda$ total energy density (cosmic triangle)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0 | H | 0 \rangle = \frac{\hbar \, c}{2} \sum_n \lambda_n = \frac{1}{2} \, \mathrm{tr} \, H = \frac{1}{2} \, \zeta_H^\mu (-1)$$

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2}\right)\lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{\hbar\, c}{2}\sum_n \lambda_n = \frac{1}{2}\,\mathrm{tr}\, H = \frac{1}{2}\,\zeta_H^\mu(-1)$$

gives ∞ physical meaning?

Zero point energy

QFT vacuum to vacuum transition: $\langle 0 | H | 0 \rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2} \right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0 | H | 0 \rangle = \frac{\hbar \, c}{2} \sum_n \lambda_n = \frac{1}{2} \, \text{tr} \, H = \frac{1}{2} \, \zeta_H^\mu (-1)$$

gives ∞ physical meaning?

Regularization $+$ Renormalization (cut-off, dim, ζ)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left(n + \frac{1}{2}\right) \lambda_n \, a_n \, a_n^\dagger$$

$$\langle 0|H|0\rangle = \frac{\hbar\, c}{2} \sum_n \lambda_n = \frac{1}{2} \, \mathrm{tr}\, H = \frac{1}{2} \, \zeta_H^\mu(-1)$$

gives ∞ physical meaning?

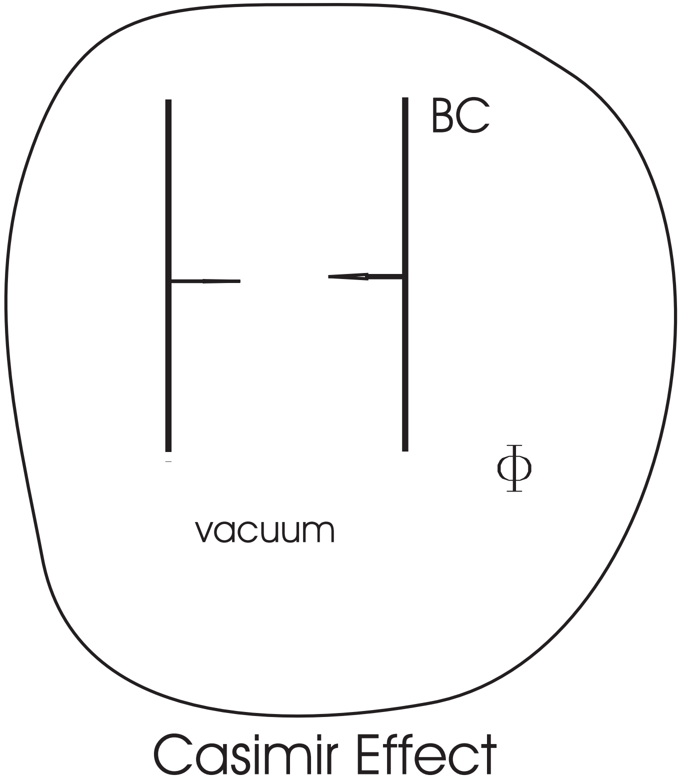
Regularization + Renormalization (cut-off, dim, ζ)

Even then: Has the final value real sense ?

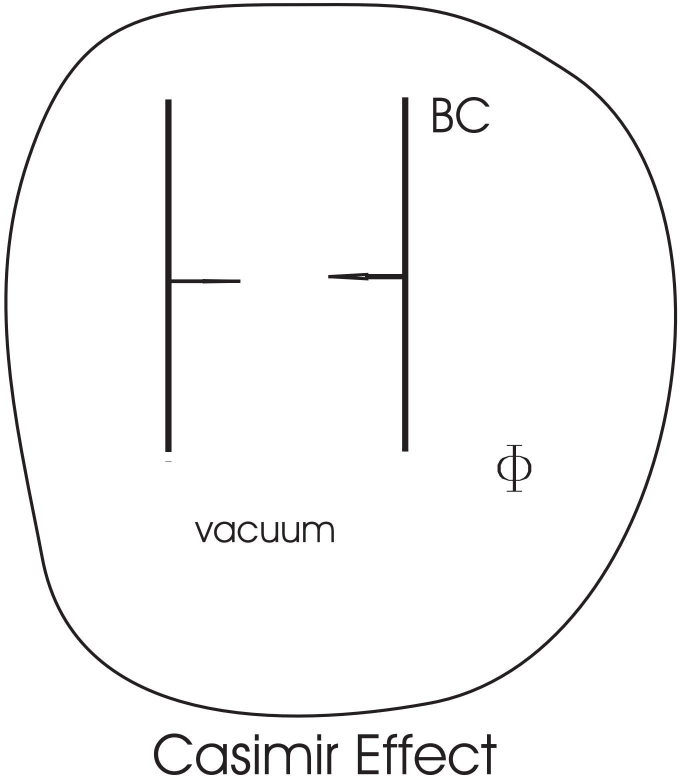
The Casimir Effect

The Casimir Effect

BC e.g. periodic



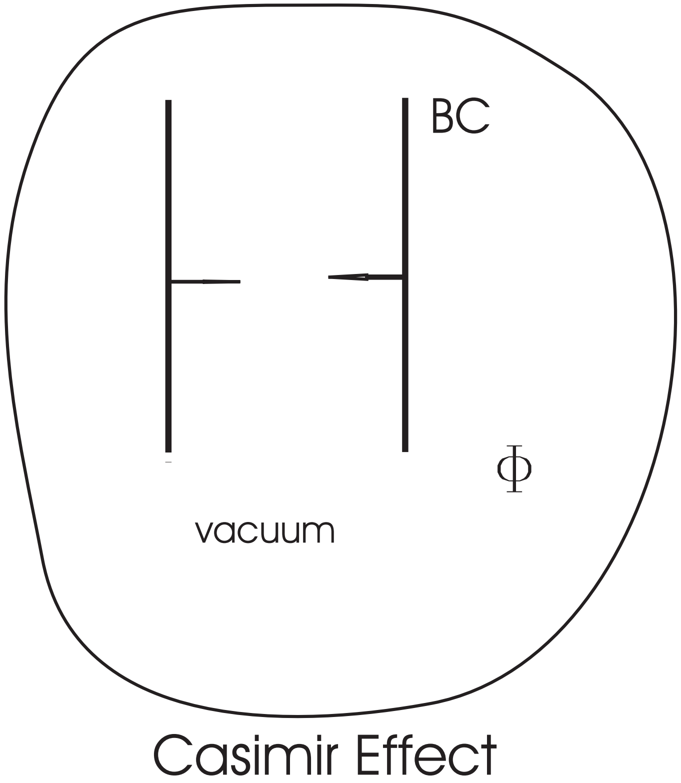
The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

The Casimir Effect

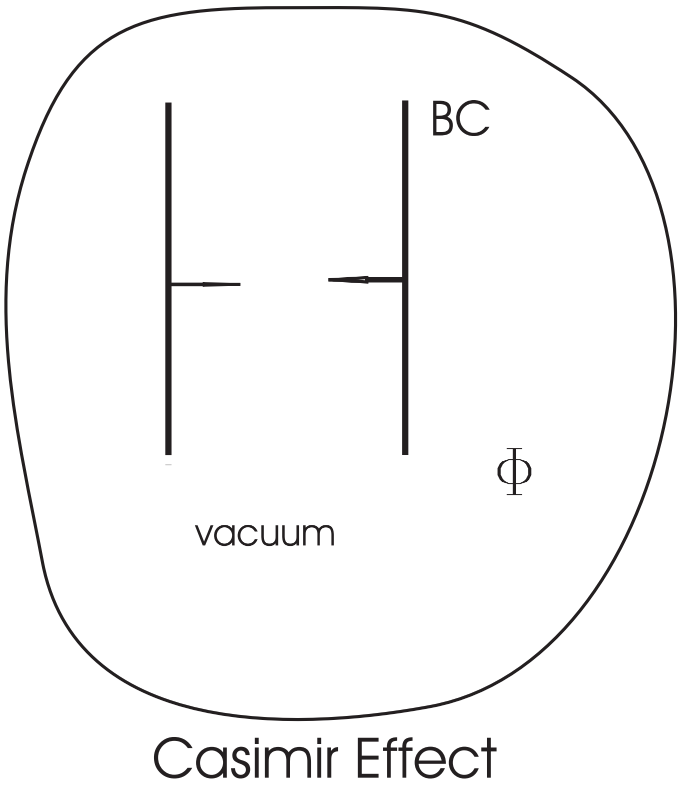


BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

The Casimir Effect



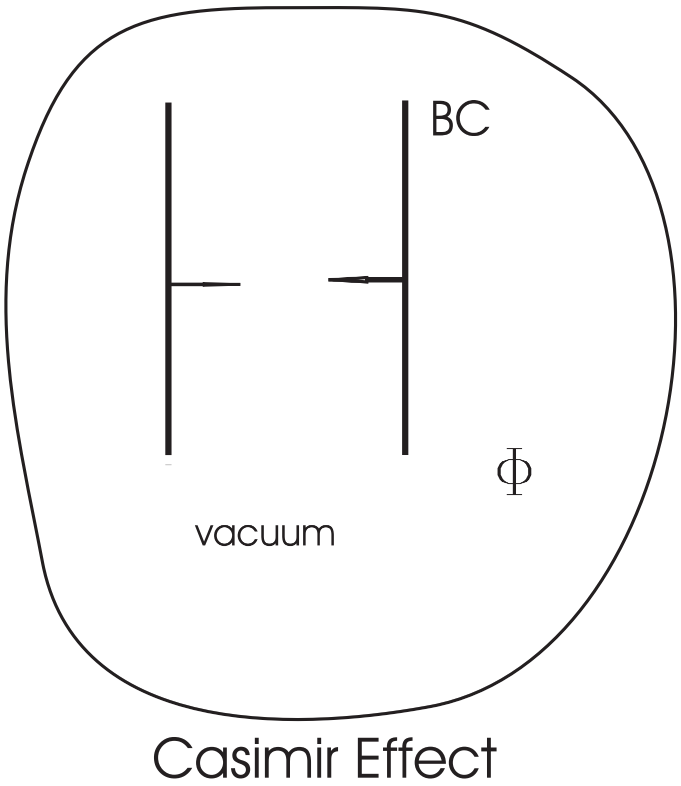
BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

The Casimir Effect



BC e.g. periodic

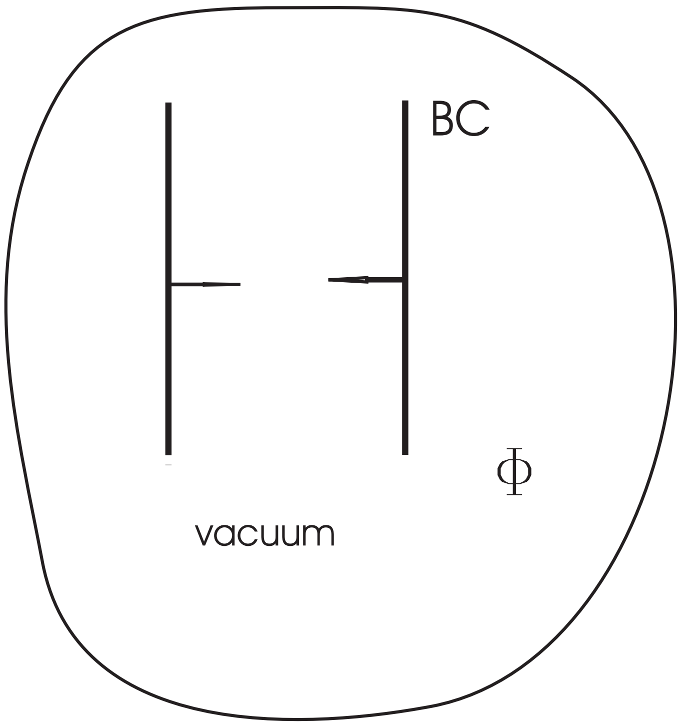
$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

The Casimir Effect



Casimir Effect

BC e.g. periodic

$\Longrightarrow$ all kind of fields

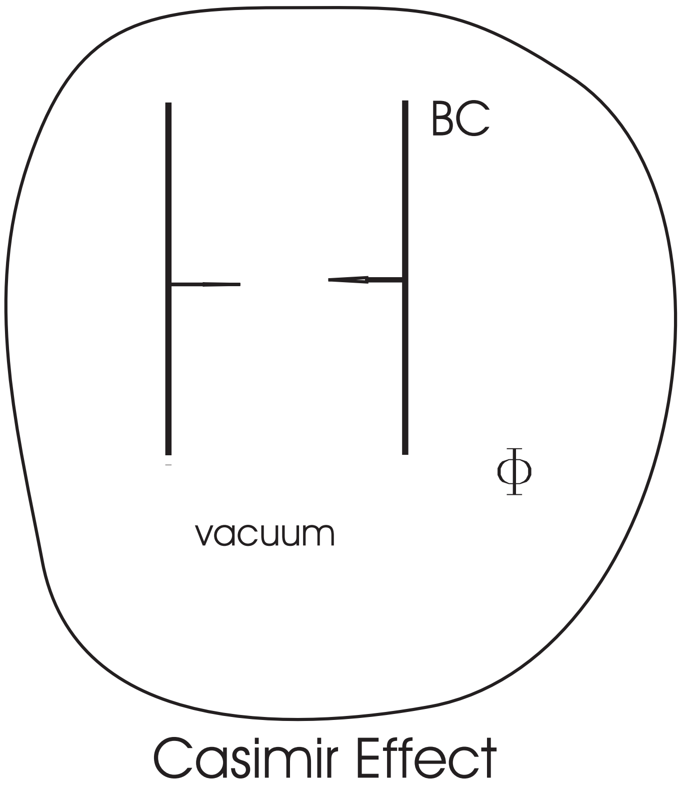
$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lifschitz theory

The Casimir Effect



BC e.g. periodic

$\Longrightarrow$ all kind of fields

$\Longrightarrow$ curvature or topology

Universal process:

- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lifschitz theory

- Dynamical CE
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological const $\Lambda = 8\pi G \mathcal{E}$, $\rho_c = \frac{3H^2}{8\pi G}$

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological const $\Lambda = 8\pi G\mathcal{E}$, $\rho_c = \frac{3H^2}{8\pi G}$

- Observations: M. Tegmark et al. [SDSS Collab.] PRD 2004

$$\Lambda \ = \ (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \quad \sim \ 4.32 \times 10^{-9} \text{ erg/cm}^3$$

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

- Appears on the rhs of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G (\widetilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu})$$

  It affects cosmology: $\widetilde{T}_{\mu\nu}$ excitations above the vacuum

- Equivalent to a cosmological const $\Lambda = 8\pi G \mathcal{E}$, $\rho_c = \frac{3H^2}{8\pi G}$

- Observations: M. Tegmark et al. [SDSS Collab.] PRD 2004

$$\Lambda \;=\; (2.14 \pm 0.13 \times 10^{-3} \text{ eV})^4 \quad \sim \; 4.32 \times 10^{-9} \text{ erg/cm}^3$$

- Idea: zero point fluctuations can contribute to the cosmological constant Pauli 20s, Zeldovich '68

CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\lambda$$

CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar\, c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \;\; k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$M_P/M_{ew} \sim 10^{16}, \; M_P/M_{cc} \sim 10^{31}, \quad \rho \sim \frac{\hbar\, k_{Planck}^4}{16\pi^2} \sim 10^{123}\rho_{obs}$$

a thick aether! Caldwell, Carroll but Gómez, Dvali: species $\downarrow 10^{30}$

CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar\, c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \;\; k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$M_P/M_{ew} \sim 10^{16}, \; M_P/M_{cc} \sim 10^{31}, \quad \rho \sim \frac{\hbar\, k^4_{Planck}}{16\pi^2} \sim 10^{123}\rho_{obs}$$

a thick aether! Caldwell, Carroll but Gómez, Dvali: species $\downarrow 10^{30}$

- Observational tests see nothing (or very little) of it:

$\Longrightarrow$ (new) cosmological constant problem

CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar\, c}{2} \sum_n \omega_n , \qquad \omega = k^2 + m^2/\hbar^2 , \quad k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$M_P/M_{ew} \sim 10^{16}, \; M_P/M_{cc} \sim 10^{31}, \quad \rho \sim \frac{\hbar\, k^4_{Planck}}{16\pi^2} \sim 10^{123}\rho_{obs}$$

a thick aether! Caldwell, Carroll but Gómez, Dvali: species $\downarrow 10^{30}$

- Observational tests see nothing (or very little) of it:

$\Longrightarrow$ (new) cosmological constant problem

- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

CC Problem

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \quad k = 2\pi/\lambda$$

- Evaluating in a box and putting a cut-off at maximum k_{max} corresp'ng to QFT physics (e.g., Planck energy)

$$M_P/M_{ew} \sim 10^{16}, \; M_P/M_{cc} \sim 10^{31}, \quad \rho \sim \frac{\hbar \, k_{Planck}^4}{16\pi^2} \sim 10^{123} \rho_{obs}$$

  a thick aether! Caldwell, Carroll but Gómez, Dvali: species $\downarrow 10^{30}$

- Observational tests see nothing (or very little) of it:

  $\Longrightarrow$ (new) cosmological constant problem

- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmo-Topol Casimir Eff’t & Alternat’s

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998]
$\longrightarrow$ left with this incremental value coming from the topology or BCs

Cosmo-Topol Casimir Eff't & Alternat's

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998] $\longrightarrow$ left with this incremental value coming from the topology or BCs
- We have shown (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:

Cosmo-Topol Casimir Eff't & Alternat's

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998] $\longrightarrow$ left with this incremental value coming from the topology or BCs
- We have shown (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales JPA39(06)6299

Cosmo-Topol Casimir Eff't & Alternat's

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998] $\longrightarrow$ left with this incremental value coming from the topology or BCs
- We have shown (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales JPA39(06)6299
  - (b) dS & AdS worldbranes hep-th/0209242

Cosmo-Topol Casimir Eff't & Alternat's

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998]
  $\longrightarrow$ left with this incremental value coming from the topology or BCs

- We have shown (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales JPA39(06)6299
  - (b) dS & AdS worldbranes hep-th/0209242
  - (c) supergraviton theo's (discret dims, deconstr) hep-th/0312269

Cosmo-Topol Casimir Eff't & Alternat's

- A. Assuming one is able to prove that the ground value of the cc is zero [Dolgov 1983; Ford 1987, 2002; Tsamis & Woodard 1998] $\longrightarrow$ left with this incremental value coming from the topology or BCs
- We have shown (with different examples) that this value acquires the correct order of magnitude —corresponding to the one coming from the observed acceleration in the expansion of our universe— in some reasonable models involving:
  - (a) small and large compactified scales JPA39(06)6299
  - (b) dS & AdS worldbranes hep-th/0209242
  - (c) supergraviton theo's (discret dims, deconstr) hep-th/0312269
- B. Other alternatives: (i) L Faddeev 0911.0282 (Adler '82) Newton const in E-H Lag has dim of mass $\longrightarrow$ non-renormalizability Describe gravity by vector field (as Higgs mechanism) (ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in euIR sector and a mechanism to relax the cc. Also Shapiro+Sola, ...

More recent alternatives (a sample)

- (iii) E Mottola 1006.3567 Effective field theory approach
  – Casimir effect in flat s-t and large quantum backreaction are effects at the horizon scale of cosmological s-t
  – imply the cosmological VE is dynamical
  – its value depends on macroscopic BCs at the cosm horizon scale, rather than on the extreme ultraviolet Planck scale [we, on both BCs]

More recent alternatives (a sample)

- (iii) E Mottola 1006.3567 Effective field theory approach
  – Casimir effect in flat s-t and large quantum backreaction are effects at the horizon scale of cosmological s-t
  – imply the cosmological VE is dynamical
  – its value depends on macroscopic BCs at the cosm horizon scale, rather than on the extreme ultraviolet Planck scale [we, on both BCs]

- (iv) T Padmanabhan Ad Sci Lett 2 74 09 cc problem and explaining DE independent issues: first find mechanism to make the cc vanish
  – new degrees of freedom, kind of 'gauge freedom' to absorb any λ while maintaining general covariance
  – could succeed in making gravity decouple from the bulk VE
  – emergent gravity approach: thermodynamic description is far more general than just Einstein theory
  – observed cc should be a relic of quantum gravitational physics and arise from degrees of freedom which scale as the surface area
  – numerics: $L_\Lambda / L_P \sim \exp\sqrt{2}\,\pi^4 \sim 10^{60}$ (hierarchy squared) $\sim 10^{61}$

- (v) Shao & Chen 1005.1920   no attempt at explaining the old cc prob
  – an extremely small quantum correction can in fact be produced quite naturally from a massive bulk field, introducing a massive bulk fermion
  – naturally as superpartner of the radion field in a SUSY theory (especially the string theory realization) of brane-world scenario
  – in particle physics Grossman and Neubert used a massive bulk fermion to understand the neutrino mass hierarchy
  – use Goldberger-Wise mechanism where massive bulk scalar field with brane self-interaction induces stabilizing potential
  – could overwhelm the small fermionic Casimir energy & sign ?

- (v) Shao & Chen 1005.1920 no attempt at explaining the old cc prob
  – an extremely small quantum correction can in fact be produced quite naturally from a massive bulk field, introducing a massive bulk fermion
  – naturally as superpartner of the radion field in a SUSY theory (especially the string theory realization) of brane-world scenario
  – in particle physics Grossman and Neubert used a massive bulk fermion to understand the neutrino mass hierarchy
  – use Goldberger-Wise mechanism where massive bulk scalar field with brane self-interaction induces stabilizing potential
  – could overwhelm the small fermionic Casimir energy & sign ?
- (vi) JA Dixon 1006.2334 CyberSUSY solves the cc problem
  – a new mechanism for SUSY breaking
  – its realization mixes elementary and composite states
  – SUSY anomalies present, generates spectrum for SUSY breaking consistent with known particles
  – no cc generated, because SUSY is not spontaneously broken ...

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem
2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds [A Flachi]

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, Nojiri, Odintsov, Ogushi, PRD67(2003)063515, hep-th/0209242 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, Odintsov, Saharian PRD79(2009)065023, 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account
- Casimir calculation: attractive force

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account
- Casimir calculation: attractive force
- Boyer got repulsion [TH, Phys Rev, 174 (1968)] for a spherical shell. It is a special case requiring stringent material properties of the sphere and a perfect geometry and BC

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account
- Casimir calculation: attractive force
- Boyer got repulsion [TH, Phys Rev, 174 (1968)] for a spherical shell. It is a special case requiring stringent material properties of the sphere and a perfect geometry and BC
- Systematic calculation, for different fields, BCs, and dimensions
  J Ambjørn, S Wolfram, Ann Phys NY 147, 1 (1983) attract, repuls

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account
- Casimir calculation: attractive force
- Boyer got repulsion [TH, Phys Rev, 174 (1968)] for a spherical shell. It is a special case requiring stringent material properties of the sphere and a perfect geometry and BC
- Systematic calculation, for different fields, BCs, and dimensions
  J Ambjørn, S Wolfram, Ann Phys NY 147, 1 (1983) attract, repuls
- Possibly not relevant at lab scales, but very important for cosmological models

The Sign of the Casimir Force

- Many papers dealing on this issue: here just short account
- Casimir calculation: attractive force
- Boyer got repulsion [TH, Phys Rev, 174 (1968)] for a spherical shell. It is a special case requiring stringent material properties of the sphere and a perfect geometry and BC
- Systematic calculation, for different fields, BCs, and dimensions
  J Ambjørn, S Wolfram, Ann Phys NY 147, 1 (1983) attract, repuls
- Possibly not relevant at lab scales, but very important for cosmological models
- More general results: Kenneth, Klich, PRL 97, 160401 (2006)
  a mirror pair of dielectric bodies always attract each other
  CP Bachas, J Phys A40, 9089 (2007) from a general property of Euclidean QFT 'reflection positivity' (Osterwalder - Schrader 73, 75):
  $\exists$ of positive Hilbert space and self-adjoint non-negative Hamiltonian

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a mathematically singular operation (which introduces divergent edge contributions)

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a mathematically singular operation (which introduces divergent edge contributions)
- Theorem does not apply for

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a mathematically singular operation (which introduces divergent edge contributions)
- Theorem does not apply for
  - mirror probes in a Fermi sea (chemical-potential term), eg when electron-gas fluctuations become important

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a mathematically singular operation (which introduces divergent edge contributions)
- Theorem does not apply for
  - mirror probes in a Fermi sea (chemical-potential term), eg when electron-gas fluctuations become important
  - periodic BCs for fermions

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
  Θ reflection with respect to a 3-dim hyperplane in R^4
  the action of Θ on f is anti-unitary $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator Θ is a consequence of unitarity only, and makes no assumptions about the discrete C, P, T symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a mathematically singular operation (which introduces divergent edge contributions)
- Theorem does not apply for
  - mirror probes in a Fermi sea (chemical-potential term), eg when electron-gas fluctuations become important
  - periodic BCs for fermions
  - Robin BCs in general $\Leftarrow$

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space
- Most general case: constants in the BCs different for the two plates
  It is shown that Robin BCs with different coefficients are necessary
  to obtain repulsive Casimir forces

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space
- Most general case: constants in the BCs different for the two plates
  It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces
- Robin type BCs are an extension of Dirichlet and Neumann's
  $\Longrightarrow$ most suitable to describe physically realistic situations

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space
- Most general case: constants in the BCs different for the two plates
  It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces
- Robin type BCs are an extension of Dirichlet and Neumann's
  $\Longrightarrow$ most suitable to describe physically realistic situations
- Genuinely appear in: $\rightarrow$ vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83],
  $\rightarrow$ spinor and gauge field theories,
  $\rightarrow$ quantum gravity and supergravity [Luckock ea 91]
  Can be made conformally invariant, purely-Neumann conditions cannot
  $\Longrightarrow$ needed for conformally invariant theories with BC, to preserve cf invar

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space
- Most general case: constants in the BCs different for the two plates
  It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces
- Robin type BCs are an extension of Dirichlet and Neumann's
  $\Longrightarrow$ most suitable to describe physically realistic situations
- Genuinely appear in: $\rightarrow$ vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83],
  $\rightarrow$ spinor and gauge field theories,
  $\rightarrow$ quantum gravity and supergravity [Luckock ea 91]
  Can be made conformally invariant, purely-Neumann conditions cannot
  $\Longrightarrow$ needed for conformally invariant theories with BC, to preserve cf invar
- Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein's entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]

- Robin BCs can model the finite penetration of the field through the boundary: the 'skin-depth' param related to Robin coefficient [Mostep ea 85,Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]

- Robin BCs can model the finite penetration of the field through the boundary: the 'skin-depth' param related to Robin coefficient [Mostep ea 85,Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]
- Naturally arise for scalar and fermion bulk fields in the Randall-Sundrum model

- Robin BCs can model the finite penetration of the field through the boundary: the 'skin-depth' param related to Robin coefficient [Mostep ea 85,Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]
- Naturally arise for scalar and fermion bulk fields in the Randall-Sundrum model

For arbitrary internal space, interaction part of the Casimir energy given by

$$\Delta E_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^{\infty} dx\, x(x^2 - m_\beta^2)^{D_1/2-1}$$

$$\times \ln\left[1 - \frac{(\beta_1 x + 1)(\beta_2 x + 1)}{(\beta_1 x - 1)(\beta_2 x - 1)} e^{-2ax}\right] \qquad (*)$$

- Robin BCs can model the finite penetration of the field through the boundary: the 'skin-depth' param related to Robin coefficient [Mostep ea 85,Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]
- Naturally arise for scalar and fermion bulk fields in the Randall-Sundrum model

For arbitrary internal space, interaction part of the Casimir energy given by

$$\Delta E_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^\infty dx\, x(x^2 - m_\beta^2)^{\,D_1/2-1}$$

$$\times \ln\left[1 - \frac{(\beta_1 x + 1)(\beta_2 x + 1)}{(\beta_1 x - 1)(\beta_2 x - 1)} e^{-2ax}\right] \qquad (*)$$

For Dirichlet and Neumann BCs on both plates this leads to

$$\Delta E^{(\mathrm{J},\mathrm{J})}_{[a_1,a_2]} = -\frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{n^{D_1+1}}$$

with $f_\nu(z) = z^\nu K_\nu(z)$ $\longrightarrow$ energy always negative

For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

$$\Delta E^{(\mathrm{D},\mathrm{N})}_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2} a}{\Gamma(D_1/2+1)} \sum_\beta \int_{m_\beta}^\infty dx \, \frac{\left(x^2 - m_\beta^2\right)^{D_1/2}}{e^{2ax}+1}$$

$$= \frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{(-1)^{n+1} n^{D_1+1}}$$

positive for all values of the inter-plate distance

For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

$$\Delta E^{(\mathrm{D,N})}_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}a}{\Gamma(D_1/2+1)} \sum_\beta \int_{m_\beta}^\infty dx\, \frac{\left(x^2-m_\beta^2\right)^{D_1/2}}{e^{2ax}+1}$$

$$= \frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{(-1)^{n+1}n^{D_1+1}}$$

positive for all values of the inter-plate distance

In the case of a conformally coupled massless field on the background of a spacetime conformally related to the one described by the line element

$$ds^2 = g_{MN}dx^M dx^N = \eta_{\mu\nu}dx^\mu dx^\nu - \gamma_{il}dX^i dX^l$$

$\eta_{\mu\nu} = \mathrm{diag}(1,-1,\ldots,-1)$ metric of (D_1+1)-dim Minkowski st and X^i coordinates of Σ, with the conformal factor $\Omega^2(x^{D_1})$. Interaction part of Casimir energy is given (*), with coeffs β_j related to coeffs of the Robin BCs

$$(1+\overline{\beta}_j n^M \nabla_M)\overline{\varphi}(x) = [1+(-1)^{j-1}\Omega_j^{-1}\overline{\beta}_j\partial_{D_1}]\overline{\varphi}(x) = 0,\ \Omega_j = \Omega(x_j^{D_1})$$

& conformal factor $\beta_j = \left[\Omega_j + (-1)^j \frac{D-1}{2\Omega_j}\overline{\beta}_j\Omega_j'\right]^{-1}\overline{\beta}_j,\ \Omega_j' = \Omega_j'(x_j^{D_1})$

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\overline{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

The vacuum energy can have a minimum, for the stable equilibrium point

Can be used in braneworld models for the stabilization of the radion field

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\overline{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

The vacuum energy can have a minimum, for the stable equilibrium point

Can be used in braneworld models for the stabilization of the radion field

We have considered a piston-like geometry, introducing a third plate (then this plate is sent to infinity) Casimir force

$$P = -\frac{2(4\pi)^{-D_1/2}}{V_\Sigma \Gamma(D_1/2) a^{D_1+1}} \sum_\beta \int_{am_\beta}^\infty dx \, \frac{x^2(x^2 - a^2 m_\beta^2)^{D_1/2-1}}{\frac{(b_1x-1)(b_2x-1)}{(b_1x+1)(b_2x+1)} e^{2x} - 1}$$

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\bar{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

The vacuum energy can have a minimum, for the stable equilibrium point

Can be used in braneworld models for the stabilization of the radion field

We have considered a piston-like geometry, introducing a third plate (then this plate is sent to infinity) Casimir force

$$P = -\frac{2(4\pi)^{-D_1/2}}{V_\Sigma \Gamma(D_1/2) a^{D_1+1}} \sum_\beta \int_{am_\beta}^{\infty} dx \, \frac{x^2 (x^2 - a^2 m_\beta^2)^{D_1/2-1}}{\frac{(b_1 x - 1)(b_2 x - 1)}{(b_1 x + 1)(b_2 x + 1)} e^{2x} - 1}$$

With independence of the geometry of the internal space, the force is attractive for Dirichlet or Neumann boundary conditions on both plates

$$P^{(\mathrm{J,J})} = -\frac{2(4\pi)^{-D_1/2}}{V_\Sigma \Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^{\infty} dx \, x^2 \frac{(x^2 - m_\beta^2)^{D_1/2-1}}{e^{2ax} - 1}$$
$$= \frac{2a^{-D_1-1}}{(8\pi)^{(D_1+1)/2} V_\Sigma} \sum_\beta \sum_{n=1}^{\infty} \frac{1}{n^{D_1+1}} \left[f_{(D_1+1)/2}(2nam_\beta) - f_{(D_1+3)/2}(2nam_\beta) \right]$$

$\mathrm{J} = \mathrm{D}, \mathrm{N}$, and repulsive for Dirichlet BC on one plate and Neumann on the other, a monotonic function of the distance

⟹ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

$\Longrightarrow$ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

$\Longrightarrow$ For small values of the size of internal space, in models with zero modes along the internal space, main contribution to Casimir force comes from the zero modes: contributions of non-zero modes are exponentially suppressed

$\Longrightarrow$ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

$\Longrightarrow$ For small values of the size of internal space, in models with zero modes along the internal space, main contribution to Casimir force comes from the zero modes: contributions of non-zero modes are exponentially suppressed

$\Longrightarrow$ In this limit, to leading order we recover the standard result for the Casimir force between two plates in $(D_1 + 1)$-dim Minkowski spacetime

$\Longrightarrow$ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

$\Longrightarrow$ For small values of the size of internal space, in models with zero modes along the internal space, main contribution to Casimir force comes from the zero modes: contributions of non-zero modes are exponentially suppressed

$\Longrightarrow$ In this limit, to leading order we recover the standard result for the Casimir force between two plates in (D_1+1)-dim Minkowski spacetime

$\Longrightarrow$ In absence of zero modes (case of twisted BCs along compactified dimens): Casimir forces exponentially suppressed in limit of small size of internal space For small values of the inter-plate distance Casimir forces generically attractive, except for Dirichlet BCs on one plate and non-Dirichlet BCs on the other: then Casimir force is repulsive at small distances. When separation is large, the sign depends not only on BCs, but also on geometry of transversal dimens

$\Longrightarrow$ For general Robin BCs the Casimir force can be either attractive (negative P) or repulsive (positive P), depending on the Robin coefficients and distance between plates [also L P Teo arXiv:0907.2989 arXiv:0907.5258]

$\Longrightarrow$ For small values of the size of internal space, in models with zero modes along the internal space, main contribution to Casimir force comes from the zero modes: contributions of non-zero modes are exponentially suppressed

$\Longrightarrow$ In this limit, to leading order we recover the standard result for the Casimir force between two plates in $(D_1 + 1)$-dim Minkowski spacetime

$\Longrightarrow$ In absence of zero modes (case of twisted BCs along compactified dimens): Casimir forces exponentially suppressed in limit of small size of internal space
For small values of the inter-plate distance Casimir forces generically attractive, except for Dirichlet BCs on one plate and non-Dirichlet BCs on the other: then Casimir force is repulsive at small distances. When separation is large, the sign depends not only on BCs, but also on geometry of transversal dimens

$\Longrightarrow$ Remarks: (i) This property could be used in the proposal of a Casimir experiment with the purpose to carry out an explicit detailed observation of 'large' extra dimensions as allowed by some models of particle physics
(ii) Possible laboratory verification (Robin BCs model skin depth of material)

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an “integration constant”

  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...

  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow

- Ted Jacobson [PRL1995] obtained Einstein's equations from local thermodynamics arguments only

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an “integration constant”
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow

- Ted Jacobson [PRL1995] obtained Einstein’s equations from local thermodynamics arguments only

- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an “integration constant”
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow
- Ted Jacobson [PRL1995] obtained Einstein’s equations from local thermodynamics arguments only
- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer
- This strongly suggests, in a fundamental context:
  Einstein’s Eqs are to be viewed as EoS

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow
- Ted Jacobson [PRL1995] obtained Einstein's equations from local thermodynamics arguments only
- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer
- This strongly suggests, in a fundamental context:
  Einstein's Eqs are to be viewed as EoS
- Should, probably, not be taken as basic for quantizing gravity

Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity Also I Shapiro, J Solà,... cc RG flow
- Ted Jacobson [PRL1995] obtained Einstein's equations from local thermodynamics arguments only
- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer
- This strongly suggests, in a fundamental context:
  Einstein's Eqs are to be viewed as EoS
- Should, probably, not be taken as basic for quantizing gravity
- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  - entropy proport to variation of the horizon area: $\delta S = \eta \, \delta \mathcal{A}$
  - local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$
  - functional dependence of S wrt energy and size of system

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T\delta S$$

  – entropy proport to variation of the horizon area: $\delta S = \eta\, \delta \mathcal{A}$
  – local temperature T defined as Unruh temp: $T = \hbar k/2\pi$
  – functional dependence of S wrt energy and size of system

- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_\Sigma E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta \left(\eta_e A \right)$$

η_e is a function of the metric and its deriv's to a given order

$$\eta_e = \eta_e \left(g_{ab}, R_{cdef}, \nabla^{(l)} R_{pqrs} \right)$$

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T\delta S$$

– entropy proport to variation of the horizon area: $\delta S = \eta\, \delta \mathcal{A}$
– local temperature T defined as Unruh temp: $T = \hbar k/2\pi$
– functional dependence of S wrt energy and size of system

- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_{\Sigma} E_R^{pqrs} \epsilon_{pq}\epsilon_{rs}, \qquad \delta S = \delta\left(\eta_e A\right)$$

η_e is a function of the metric and its deriv's to a given order

$$\eta_e = \eta_e\left(g_{ab}, R_{cdef}, \nabla^{(l)} R_{pqrs}\right)$$

- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\frac{1}{8\pi G_{eff}} = E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs}$$

$$= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad S = \frac{A}{4\,G_{eff}}$$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\frac{1}{8\pi G_{eff}} = E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs}$$

$$= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad S = \frac{A}{4\,G_{eff}}$$

- For these theories, the different polarizations of the gravitons only enter in the definition of the effective Newton constant through the metric itself

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\frac{1}{8\pi G_{eff}} = E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs}$$
$$= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad S = \frac{A}{4\,G_{eff}}$$

- For these theories, the different polarizations of the gravitons only enter in the definition of the effective Newton constant through the metric itself

- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics

  EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
  EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2
- By means of a more general definition of local entropy, using Wald's definition of dynamic BH entropy
  RM Wald PRD1993; V Iyer, RM Wald PRD1994

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2
- By means of a more general definition of local entropy, using Wald's definition of dynamic BH entropy
RM Wald PRD1993; V Iyer, RM Wald PRD1994
- And also the concept of an effective Newton constant for graviton exchange (effective propagator)
R. Brustein, D. Gorbonos, M. Hadad, arXiv:0712.3206

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
  EE, P. Silva, Phys Rev D78, 061501(R) (2008), arXiv:0804.3721v2
- By means of a more general definition of local entropy, using Wald's definition of dynamic BH entropy
  RM Wald PRD1993; V Iyer, RM Wald PRD1994
- And also the concept of an effective Newton constant for graviton exchange (effective propagator)
  R. Brustein, D. Gorbonos, M. Hadad, arXiv:0712.3206
- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
  T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
  C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]