Vacuum fluctuations and non-local gravity models

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The accelerating Universe

Dark Energy: effects on the expansion rate of Universe, 3 approaches:

—- Standard candles: measure luminosity distance as function of redshift

—- Standard rulers: angular diam distance & expansion rate as f of redsh

—- Growth of fluctuations: generated at origin of U & amplified by inflation

Both angular diameter and luminosity distances are integrals of (inverse) expansion rate: encode effects of DE

To get competitive constraints on DE need see changes in $H(z)$ at 1% would give statistical errors in DE EoS of $O(10\%)$

—- calibrate the ruler accurately over most of the age of the universe

—- measure the ruler over much of the volume of the universe

—- make ultra-precise measurements of the ruler

On large scales or early times the perturbative treatment is valid: calculations are perfectly under control

Length scales from physics of the early universe are imprinted on the distribution of mass and radiation: they form time-independent rulers

(M White, Berkeley)

The accelerating Universe (II)

Evidence for the acceleration of the Universe expansion:

- distant supernovae
  A.G Riess et al. ’98, S. Perlmutter et al. ’99
- the cosmic microwave background
  S. Dunkley et al., ’99, E. Komatsu et al. ’99, B.D. Sherwin et al. ’11, A. van Engelen et al. ’12
- baryon acoustic oscillations BAO Martin White *mwhite/bao/*
- the galaxy distribution AG Sanchez ea 12, Gaztanaga ea
- correlations of galaxy distribs R Scranton ea ’03,

Multiple sets of evidence: no systematics affect the conclusion that $\ddot{a} > 0$, a scale factor of the Universe

Different types of models

- In General Relativity (GR): Gravity leads to deceleration

  But pressure also influences geometry: R Tolman '32

  negative pressure can drive acceleration

  Cosmological evidence could be explained by an undiscovered substance with negative pressure, so-called dark energy

  J.A. Frieman ea '95, K. Coble ea '97, R. Caldwell ea '97, B. Ratra, P.J.E. Peebles '98, C. Wetterich '98, D. Huterer, M.S. Turner '99

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- Another possib: GR is (wrong!) not accurate enough at large scales

  S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner '04, Capozziello '04

  GR was developed using information and intuition at Solar System scales, it is (almost) only checked there, it could need to be modified on large scales

  A. Starobinsky arrived at same conclusion, through very different arguments based on quantum corrections to ordinary GR: lead to terms of second order in R, and higher

Several problems

- Do not have simple guidelines, gedanken experiment, reasons of elegance and simplicity, as those of Einstein in constructing GR

  Besides that, even if beauty is abandoned, a modification of gravity must still confront three additional problems (Park & Dodelson)

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- Another problem: modified gravity models should comply with the successes of GR in the Solar System

  These constraints already doomed one of the first most promising modified gravity models introduced to explain acceleration and still place tight constraints on many models

Models with non-local interactions

- One class of modified gravity models that overcomes most of these problems contains non-local interactions

  S. Deser and R. Woodard, Phys.Rev.Lett. 99, 111301 (2007), 0706.2151

  Deser and Woodard consider terms that are functionals of $\Box^{-1}R$

  $\Box$ the d'Alembertian and R the Ricci scalar

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- At cosmological scales, $\Box^{-1}R$ grows very slowly:
  - as $(t/teq)^{1/2}$ in the radiation dominated era
  - logarithmically in the matter dominated era
  - So, at the time of Nucleosynthesis $\Box^{-1}R$ is about 10^{-6}
    and at matter-radiation equilibrium it is only order 1

  In a natural way, these terms are irrelevant at early times and begin to affect the dynamics of the Universe only after the matter-radiation transition
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- Since $\Box^{-1}R$ is dimless, the functional $\times R$ has no new mass parameter

- Finally, because $\Box^{-1}R$ is extremely small in the Solar System, these models easily pass local tests of gravity

Sufficient theoretical motivation

- In string theory, $R\,\Box^{-1}R$ is precisely the term generated by quantum corrections (away from the critical dimension) as first pointed out by Polyakov

  A.M. Polyakov, Phys. Lett. B103, 207 (1981)

- It may be possible to rewrite these models in terms of local models with one or more auxiliary scalar field

  S. Capozziello, E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Lett. B671, 193 (2009), 0809.1535

  N. Koshelev, Grav. Cosmol. 15, 220 (2009), 0809.4927

- Also, non-local interactions could as well describe the early epoch of inflation

  N. Tsamis, R. Woodard, Annals Phys. 267, 145 (1998), hep-ph/9712331

  S. Nojiri, S.D. Odintsov, Phys. Lett. B659, 821 (2008), 0708.0924

- A realistic model for acceleration, with an arbitrary function of $R\,\Box^{-1}R$, reproducing the expansion history of ΛCDM

  C. Deffayet, R. Woodard, JCAP 0908, 023 (2009), 0904.0961

  Used by S. Park & S. Dodelson, arXiv:1209.0836, to discuss structure formation in a nonlocally modified gravity

Additional considerations

- Anyway, the very first problem still remains: a completely arbitrary free function can be chosen at will to fit the expansion history
- But a very interesting aspect of modified gravity models in general is that, even if they are constructed to reproduce the expansion history equivalent to that of a given dark energy model (such as ΛCDM), perturbations will often evolve differently than in a model with standard GR plus dark energy
- Indeed, the way to distinguish DE models from modified gravity is to measure the growth of structure in the Universe

  The deviations from dark energy models are at the 10 to 30$\%$ level and have a characteristic signature as a function of redshift, which suggests that the class of models could be tested by upcoming surveys

Our model of type $f(\Box^{-1}R)$

E. Elizalde, E.O. Pozdeeva, S.Yu. Vernov, Y.-l. Zhang, JCAP07, 034 (2013)

- Consider a nonlocal gravity which contains a function of the $\Box^{-1}$ operator, thus not assuming the existence of a new dimensional parameter in the action. We focus on the study of cosmological solutions both in Jordan and Einstein frames, including matter in the last case
- Observationally, dark energy EoS parameter is now very close to -1, with tendency to stay below this value, what is intriguing. This number, if final, would lead back to GR with a cosmological constant (and nothing else). But a small deviation cannot be excluded by the most accurate astronomical data
- The sign ($+$ or $-$) or the tendency (e.g., the derivative) of this deviation cannot be determined at present. This makes room for a number of theoretical models, derived from quite different fundamental theories, which can accommodate such situation
- Several models are able to reproduce observations, as quintom models, which involve two fields: a phantom and an ordinary scalar

  E Elizalde, S Nojiri, S Odintsov, Phys.Rev.D70(2004)043539, hep-th/0405034

  W Zhao and Y Zhang, Phys.Rev. D73 (2006) 123509, arXiv:astro-ph/0604460

Our model (cont.)

- The nonlocal model we will consider is usually studied in the Jordan frame, but recently its behavior in the Einstein frame, for the case without matter, has been explored too

  K Bamba, S Nojiri, S Odintsov, M Sasaki, GRG44(2012)1321,arXiv:1104.2692

- We focus our effort on the study of
  - cosmological solutions of this model, both in Jordan and Einstein frames
  - including the case with matter for the last one
- We consider gravity models with a cosmological constant Λ and including a perfect fluid, and study in detail their cosmological solutions with a power-law cosmic scalar factor: $a \propto t^n$. The solutions thus obtained are proven to generalize solutions found by Odintsov ea & Bamba ea
- In the Jordan frame we obtain an exhaustive class of power-law solutions (we prove that other power-law solutions cannot exist)
- We analyze the correspondence between solutions got in different frames and prove explicitly how knowledge of power-law solutions in Jordan's frame can be used to get power-law solutions in Einstein's one

The action

- Consider a class of nonlocal gravities, with action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} \left[R \left(1 + f(\Box^{-1} R) \right) - 2\Lambda \right] + \mathcal{L}_{\rm m} \right\}$$

  where $\kappa^2 = 8\pi G = 8\pi / {M_{\rm Pl}}^2$, the Planck mass being $M_{\rm Pl} = G^{-1/2} = 1.2 \times 10^{19}$ GeV, f differentiable function (characterizes nature of nonlocality), $\Box^{-1}$ inverse of d'Alembertian operator, Λ cosmological constant, and $\mathcal{L}_{\rm m}$ matter Lagrangian. For definiteness, we assume that matter is a perfect fluid. We use the signature $(-,+,+,+)$, g determinant of $g_{\mu\nu}$

- Introducing two scalar fields: $\psi = \Box^{-1} R$ & Lagrange multiplier ξ

$$S_{loc} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} \left[R \left(1 + f(\psi) \right) + \xi \left(R - \Box\psi \right) - 2\Lambda \right] + \mathcal{L}_{\rm m} \right\}$$

  Original non-local action is recast as a local action in the Jordan frame. Varying this action with respect to ξ and ψ, one resp gets the field eqs

$$\Box\psi = R \,, \qquad \Box\xi = f_{,\psi}(\psi) R \,,$$

  where $f_{,\psi}(\psi) \equiv {\rm d}f / {\rm d}\psi$

The action (II)

- The corresponding Einstein equations are obtained by variation of the local action wrt the metric tensor

$$\frac{g_{\mu\nu}}{2}\left[R\Psi + \partial_\rho\xi\partial^\rho\psi - 2(\Lambda + \Box\Psi)\right] - R_{\mu\nu}\Psi - \frac{1}{2}\left(\partial_\mu\xi\partial_\nu\psi + \partial_\mu\psi\partial_\nu\xi\right) + \nabla_\mu\partial_\nu\Psi = -\kappa^2 T$$

where $\Psi \equiv 1 + f(\psi) + \xi$, and $T_{(\mathrm{m})\,\mu\nu}$ energy-momentum tensor of matter sector

$$T_{(\mathrm{m})\,\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\mathrm{m}}\right)}{\delta g^{\mu\nu}}$$

Note the system of equations here does not include the function ψ itself, but instead $f(\psi)$ and $f_{,\psi}(\psi)$, together with time derivatives of ψ. Also, $f(\psi)$ can only be determined up to a constant: one may add a constant to $f(\psi)$ and subtract the same constant from ξ without changing eqs

- Here, we assume a spatially flat FLRW universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

and consider the case where the scalar fields $\psi(t)$ and $\xi(t)$ are functions of the cosmological time t only

The action (III)

- Thus, the system of Eqs. reduces to

$$
\begin{aligned}
3H^2\Psi &= -\frac{1}{2}\dot{\xi}\dot{\psi} - 3H\dot{\Psi} + \Lambda + \kappa^2\rho_{\rm m} \\
\left(2\dot{H} + 3H^2\right)\Psi &= \frac{1}{2}\dot{\xi}\dot{\psi} - \ddot{\Psi} - 2H\dot{\Psi} + \Lambda - \kappa^2 P_{\rm m} \\
\ddot{\psi} &= -3H\dot{\psi} - 6\left(\dot{H} + 2H^2\right) \\
\ddot{\xi} &= -3H\dot{\xi} - 6\left(\dot{H} + 2H^2\right) f_{,\psi}(\psi)
\end{aligned}
$$

dot means differentiation with respect to time, t, in the Jordan frame: $\dot{A}(t) \equiv {\rm d}A(t)/{\rm d}t$, $H = \dot{a}/a$ Hubble parameter

For a perfect matter fluid, $T_{({\rm m})\,00} = \rho_{\rm m}$ and $T_{({\rm m})\,ij} = P_{\rm m} g_{ij}$

- The continuity equation is

$$
\dot{\rho}_{\rm m} = -3H(P_{\rm m} + \rho_{\rm m})
$$

Adding them we obtain a 2nd order linear differential equation for Ψ

$$
\ddot{\Psi} + 5H\dot{\Psi} + \left(2\dot{H} + 6H^2\right)\Psi - 2\Lambda + \kappa^2(P_{\rm m} - \rho_{\rm m}) = 0
$$

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Renormalization-group improved inflationary scalar electrodynamics and $SU(5)$ scenarios confronted with Planck 2013 and BICEP2 results

E. Elizalde,[1,*] S. D. Odintsov,[1,2,3,†] E. O. Pozdeeva,[4,‡] and S. Yu. Vernov[4,§]

The possibility to construct inflationary models for RG improved potentials corresponding to scalar electrodynamics and to SU(2) and SU(5) models is investigated.

The tree-level potential, which corresponds to the cosmological constant in the Einstein frame, is seen to be nonsuitable for inflation.

Rather than adding the Hilbert-Einstein term to the action, quantum corrections to the potential, coming from the RG equation, are included.

For the finite SU(2) model and SU(2) gauge model, there are no de Sitter solutions suitable for inflation, unless exit from it occurs according to some weird, nonstandard scenarios.

Inflation is realized both for scalar electrodynamics and for SU(5) RG-improved potentials, and the corresponding values of the coupling function are seen to be positive.

For reasonable values of the parameters, the inflationary models obtained are in good agreement with recent observations data from thePLANCK 2013 and BICEP2 surveys.

Quasi-matter domination parameters in bouncing cosmologies

E. Elizalde, J. Haro, S.D. Odintsov   arXiv:1411.3475

For bouncing cosmologies, a fine set of parameters is introduced in order to describe the nearly matter dominated phase, and which play the same role that the usual slow-roll parameters play in inflationary cosmology.

It is shown that, as in the inflation case, the spectral index and the running parameter for scalar perturbations in bouncing cosmologies can be best expressed in terms of them.

Further, they explicitly exhibit the duality which exists between a nearly matter dominated Universe in its contracting phase and the quasi de Sitter regime in the expanding one.

The results obtained also confirm and extend the known evidence that the spectral index for a matter dominated Universe in the contracting phase is, in fact, the same as the spectral index for an exact Sitter regime in the expanding phase.

In both the inflationary and the matter bounce scenarios, the theoretical values of the spectral index and of the running parameter are compared with their experimental counterparts, obtained from the most recent PLANCK data, with the result that the bouncing models here discussed do fit well accurate astronomical observations.

Slow-roll inflationary models are generically less favored by observational data, due to the rather small value of the running parameter predicted, as compared with bounce theories.

Thank You