ICE CSIC IEEC

Cosmological Constant, Vacuum Fluctuations & the Equivalence Principle

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ICE/CSIC & IEEC, UAB, Barcelona

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Outline of this presentation

- Einstein's Cosmological Constant

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- The Casimir Effect: Theory, Experiments, Uses

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- CE and Accelerated Expansion (Dark Energy)
- Gravity Eqs as Eqs of State

Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cosmological constant

- For cosmologists and general relativists: a great mistake

  why was it put there, in the first place? (Einstein)

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc Λ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density μ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g}\, \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g}\, \lambda$$

In other words: two contributions on the same footing (Zel'dovich, 68)

$$\frac{\Lambda\, c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar\, c}{2} \sum_i \omega_i$$

Einstein Eqs, FLRW Sol, Hubble Const

Einstein Equations (1915-17): $G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu}$

Geometry = Energy-Matter

$G_{\mu\nu}$ linear combination of the metric $g_{\mu\nu}$ and 1st & 2nd derivatives

$T_{\mu\nu}$ energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

Schwarzschild solution (1916) r, θ, φ *comoving co*

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Friedmann-Lemaître-Robertson-Walker (1935-36) sol (A. Friedmann 1922)

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right)$$

most general family: homogeneous and isotropic, k *parameter* $\pm 1, 0$

One field eq looks like Newtonian eq for the gravit pot: $\nabla^2 \phi = 4\pi G \left(\rho + 3p/c^2\right)$

density & pressure contribute to the gravit pot $\lambda = 8\pi G \rho_{vac}$, $p_{vac} = -\rho_{vac} c^2$

From the FRW metric and Einstein Eqs, an "equation of motion" of the universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

From GR to Cosmology

With the definitions:

$$\Omega_m \equiv \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \qquad \Omega_k \equiv -\frac{k}{H^2}$$

The equation of motion becomes

$$\left(\frac{da}{dt}\right)^2 = H_0^2\left[\frac{\Omega_m^{(0)}}{a} + a^2\Omega_\lambda^{(0)} + \Omega_k^{(0)}\right]$$

(the superscript (o) represents quantities measured at the present time)

In other terms, Friedmann equation in Cosmology:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_{NR}\left(\frac{a_0}{a}\right)^3 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\lambda\right]$$

Ω_R relativistic matter ($p_R = \frac{1}{3}\rho_R;\ \ \rho_R \propto a^{-4}$) Mach's princ

Ω_{NR} nonrelativistic matter ($p_{NR} = 0;\ \ \rho_{NR} \propto a^{-3}$)

Ω_λ cosmological constant ($p_\lambda = -\rho_\lambda;\ \ \rho_\lambda = \mathrm{const}$)

$\Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda$ total energy density (cosmic triangle)

Zero point energy

QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

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Even then: Has the final value real sense ?

The Casimir Effect

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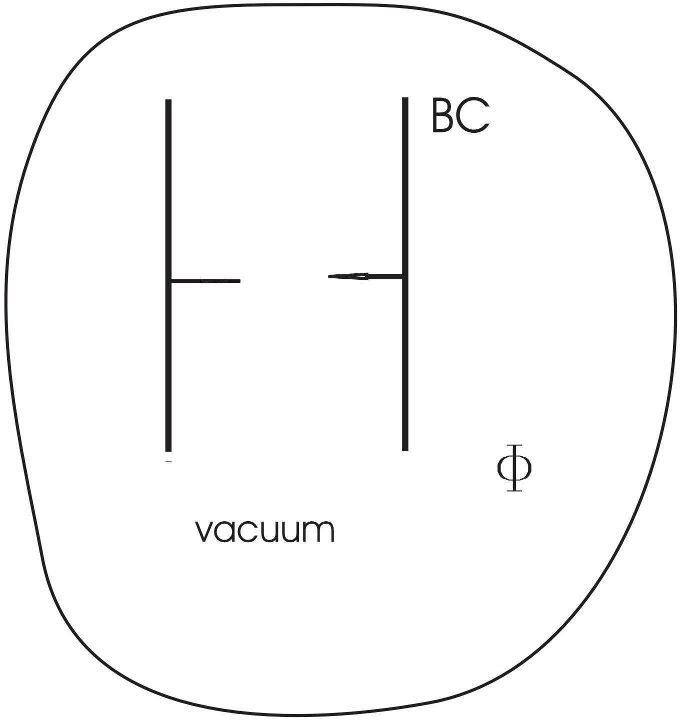
BC e.g. periodic



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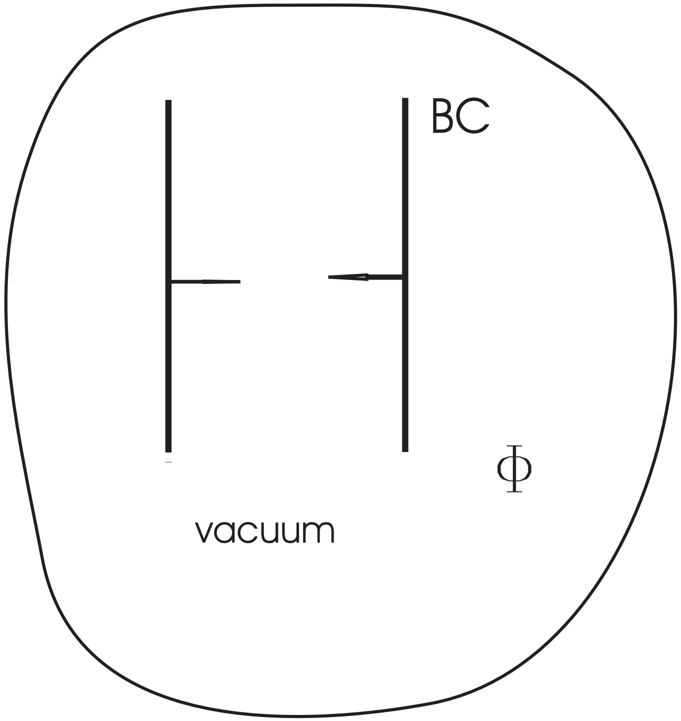

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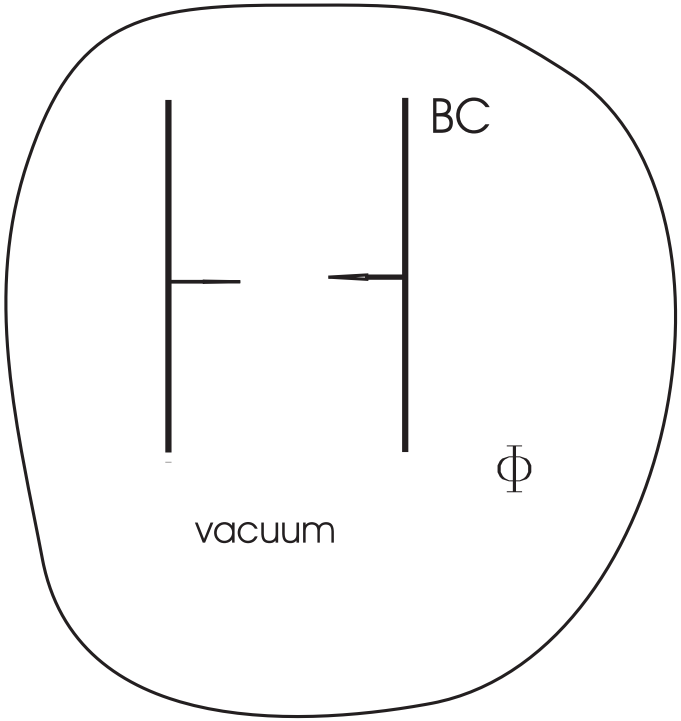

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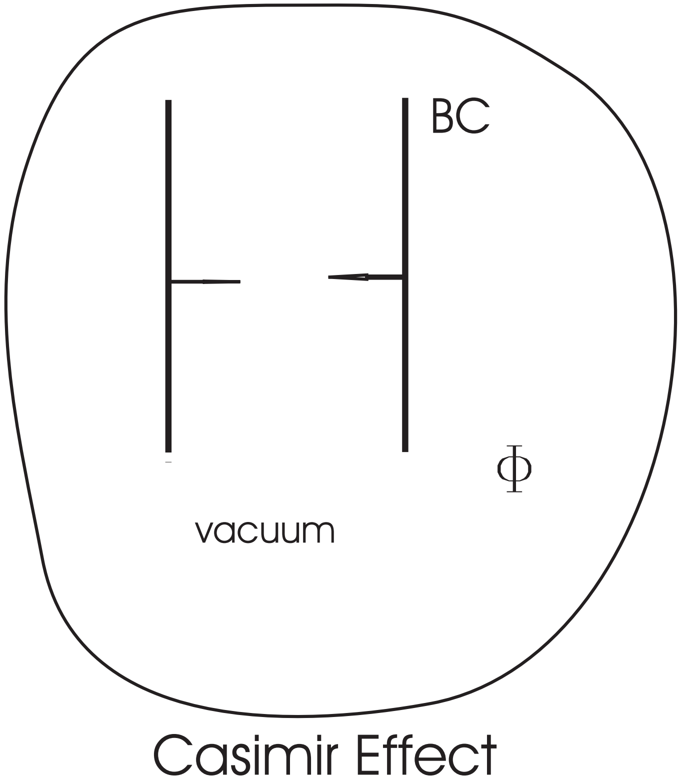

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Universal process:

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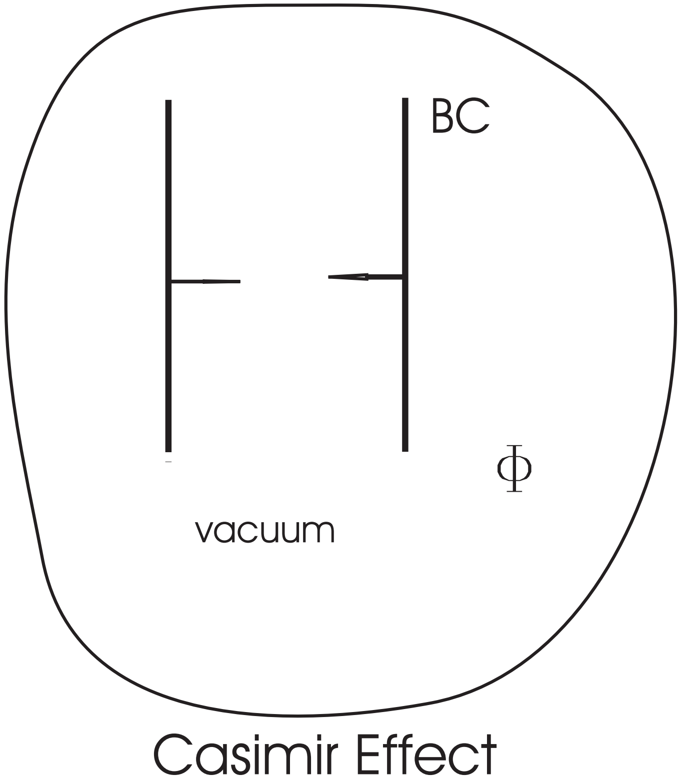

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- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

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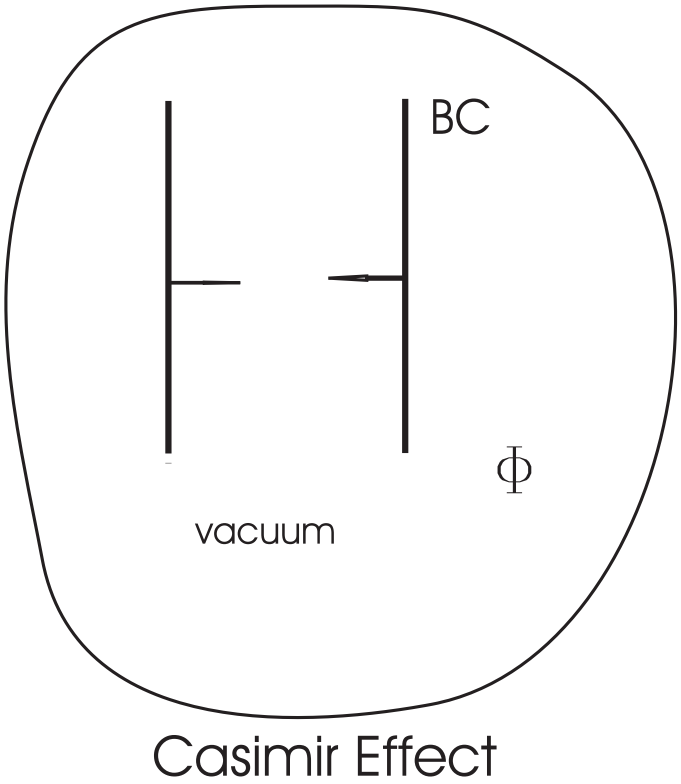

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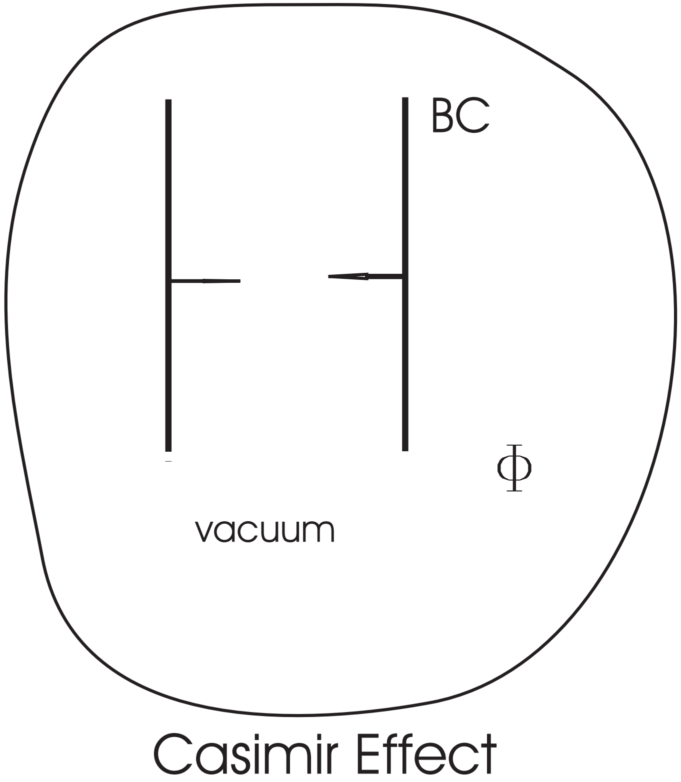

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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

The standard approach

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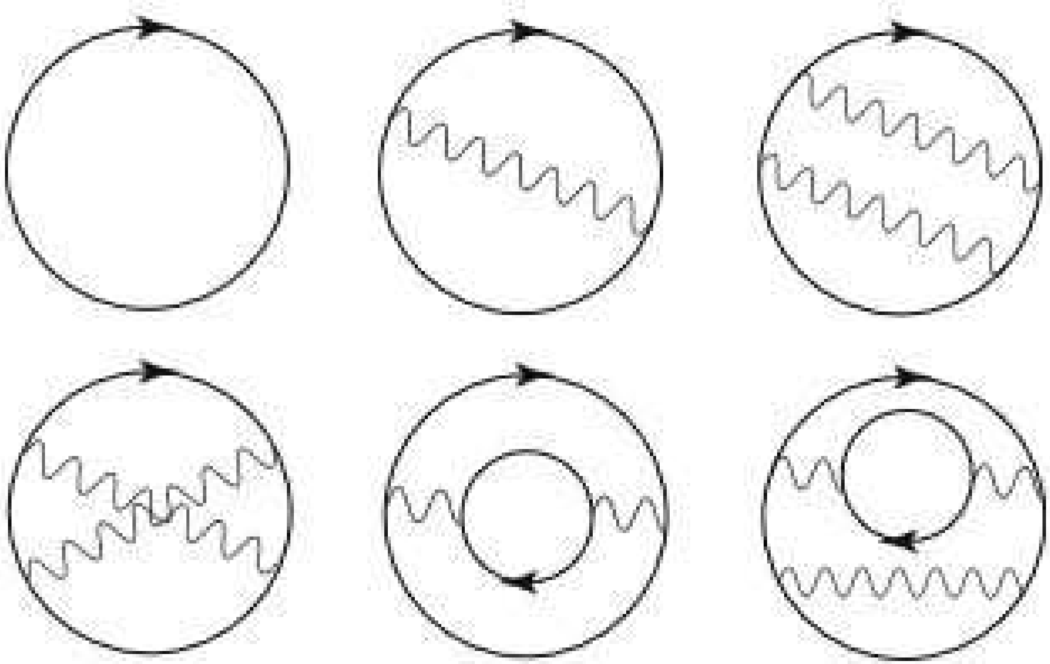

$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

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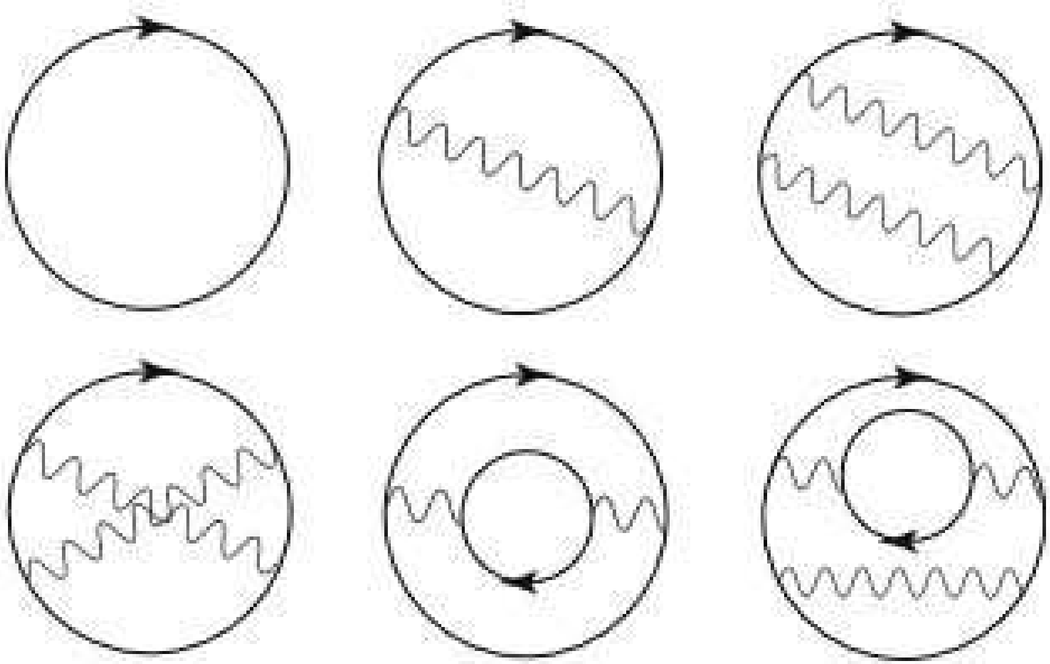

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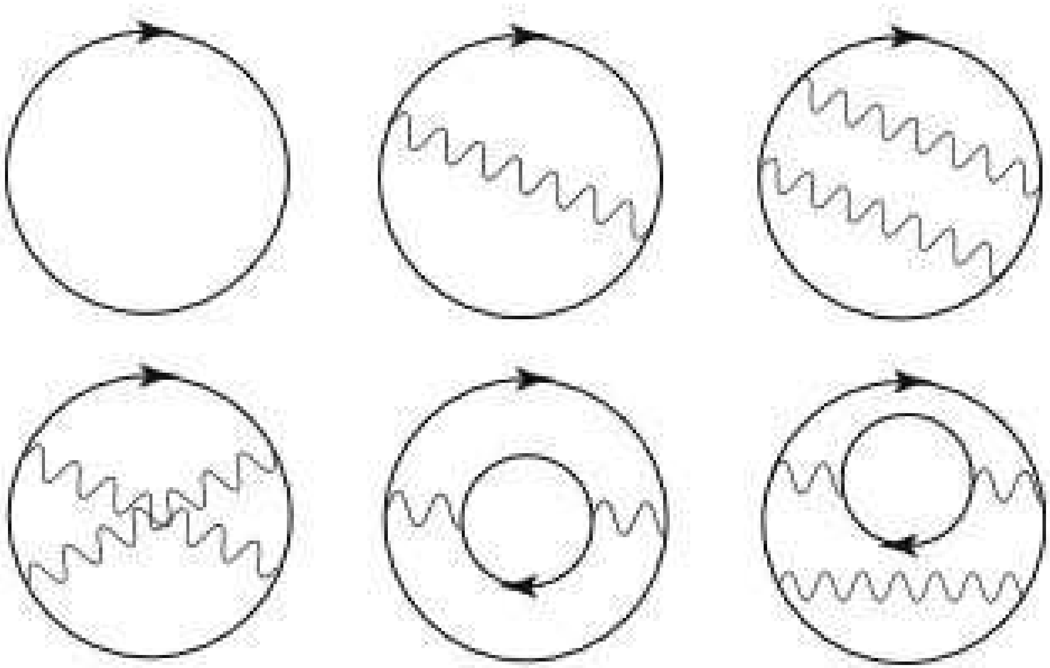

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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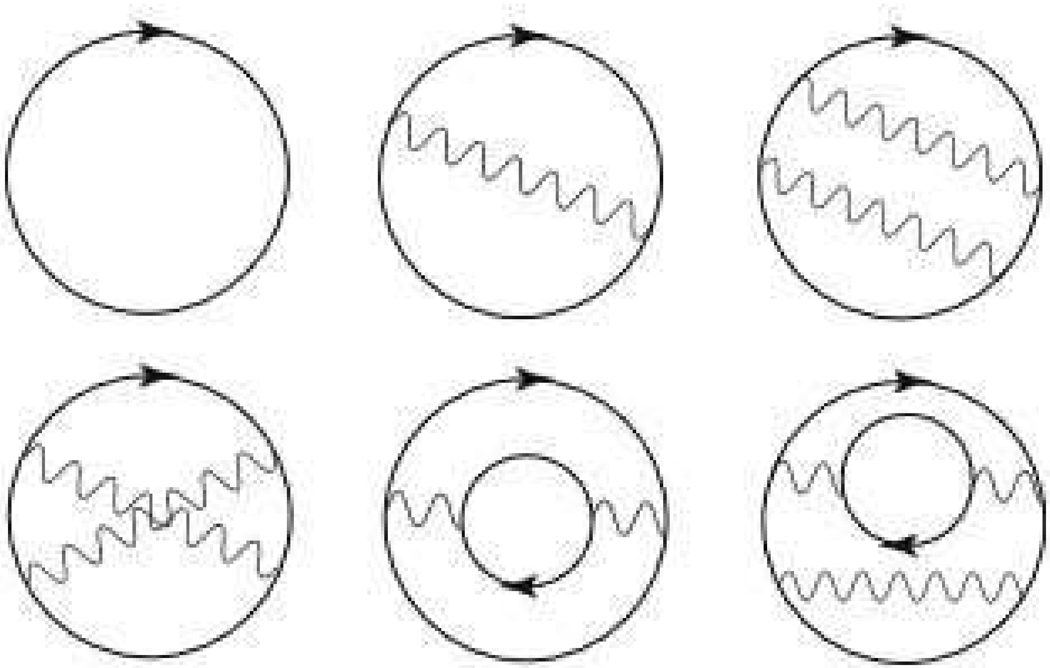

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function   Trace is over spin

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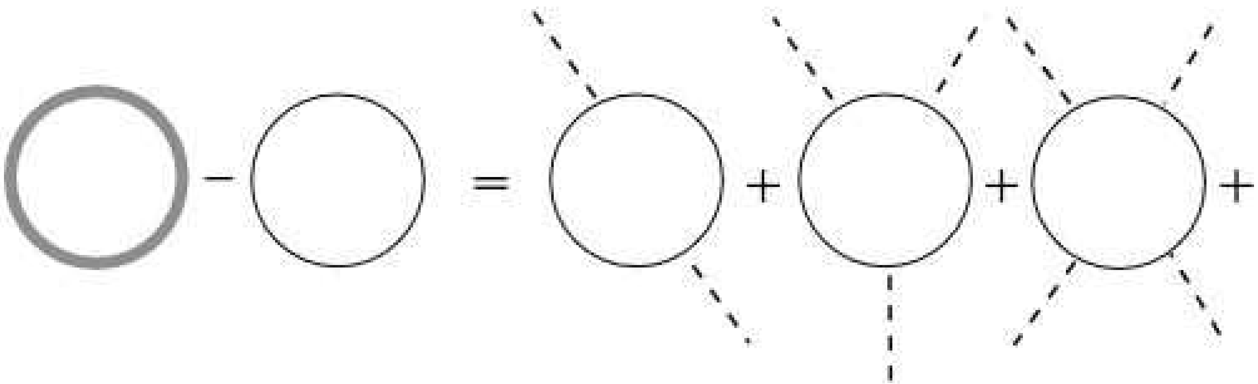

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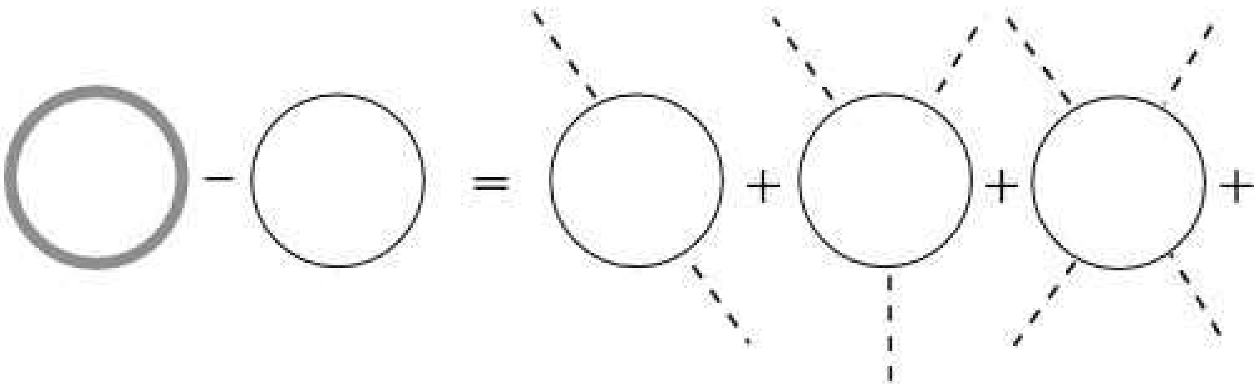

$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al;
Plunien et al; Barton, Eberlein, Calogeracos; Ford, Vilenkin;
Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al;
Dalvit, Maia-Neto et al; Law; Parentani, ...

A CONSISTENT APPROACH:

J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

Some details of the method

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

(of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

Case of a single, partially transmitting mirror

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Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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Consider, instead, a partially transmitting mirror, transparent to high frequencies (math. implementation of a physical plate).

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

RESULTS ARE REWARDING:

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In our Hamiltonian approach

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$$\times\left[|r(\omega)+r^*(\omega')|^2+|s(\omega)-s^*(\omega')|^2\right]+\mathcal{O}(\epsilon^2)$$

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

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- The main issue: S.A. Fulling et. al., hep-th/070209v2

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

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- Casimir stress tensor between pair of parallel perfectly conducting plates, at distance a, transverse dimensions $L >> a$

[Brown and Maclay, Phys. Rev. 184 (1969) 1272]

$$\langle T_{\mu\nu} \rangle = \frac{\mathcal{E}_{\mathcal{C}}}{a} \, \text{diag}(1, -1, -1, 3)$$

third spatial direction is normal to plates, $\mathcal{E}_{\mathcal{C}}$ Casimir energy per unit area

$$\mathcal{E}_{\mathcal{C}} = -\frac{\pi \hbar c}{720 a^3}$$

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- Gravitational interaction of the Casimir apparatus: use gravitational definition of the energy-momentum tensor as variation of matter part of action:

$$\delta W_m = \frac{1}{2} \int \sqrt{-g}\, \delta g_{\mu\nu}\, T^{\mu\nu} \qquad (*)$$

Following Schwinger (note the factor 2 in the definition), for a weak field Fulling et al define: $g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$

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  energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force
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CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

$$E_0 = \frac{\hbar c}{2} \sum_n \omega_n, \qquad \omega = k^2 + m^2/\hbar^2, \;\; k = 2\pi/\lambda$$

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

Cosmolog Imprint of the Casimir Eff't?

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
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  - (c) supergraviton theories (discret dims, deconstr)

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem
2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, S Nojiri, SD Odintsov, S Ogushi, Phys Rev D67 (2003) 063515 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, SD Odintsov, AA Saharian 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

Gravity Eqs as Eqs of State: f(R) Case

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Gravity Eqs as Eqs of State: f(R) Case

- The cosmological constant as an "integration constant"
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
  Unimodular Gravity
- Ted Jacobson [PRL1995] obtained Einstein's equations from local thermodynamics arguments only
- By way of generalizing black hole thermodynamics to space-time thermodynamics as seen by a local observer
- This strongly suggests, in a fundamental context:
  Einstein's Eqs are to be viewed as EoS
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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  - entropy proport to variation of the horizon area: $\delta S = \eta \, \delta \mathcal{A}$
  - local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$
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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_\Sigma E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta \left(\eta_e A \right)$$

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$
\begin{aligned}
\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\
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- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

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- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
  T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
  C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]

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Moltes gràcies!