ICE CSIC IEEC

The Casimir Effect: Theoretical, Experimental and Cosmological Issues

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YITP & Ritsumeikan U, Kyoto, Sept-Oct, 2007

Soft introduction

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- Here only: Casimir (experiments), CC (ideas)

Outline of this presentation

- The Casimir Effect: Theory, Experiments, Uses

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- CE and Accelerated Expansion (Dark Energy)

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Even then: Has the final value real sense ?

The Casimir Effect

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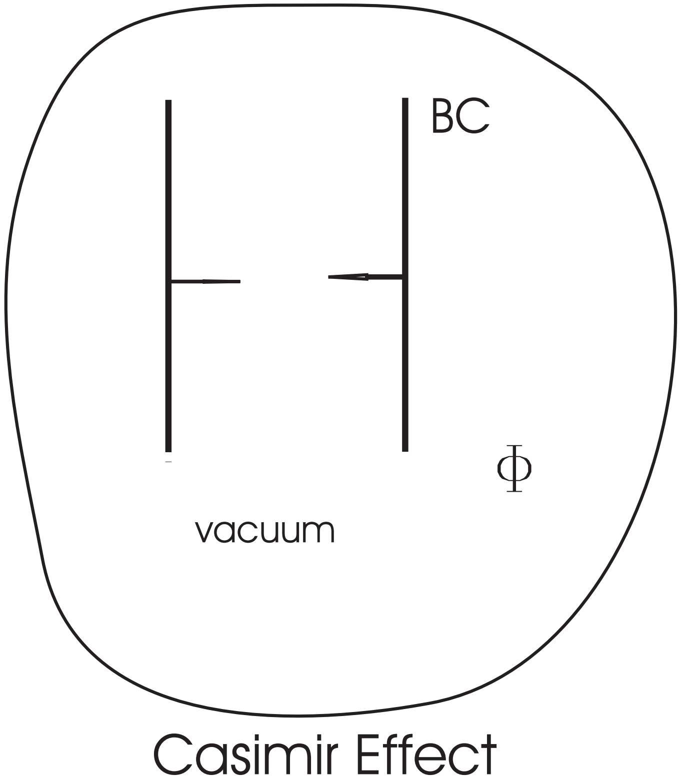
BC e.g. periodic



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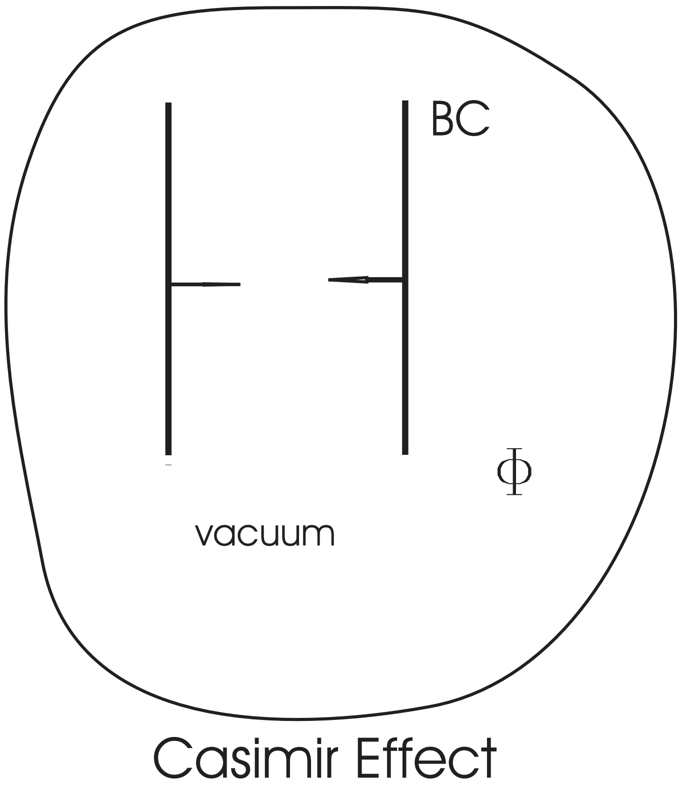

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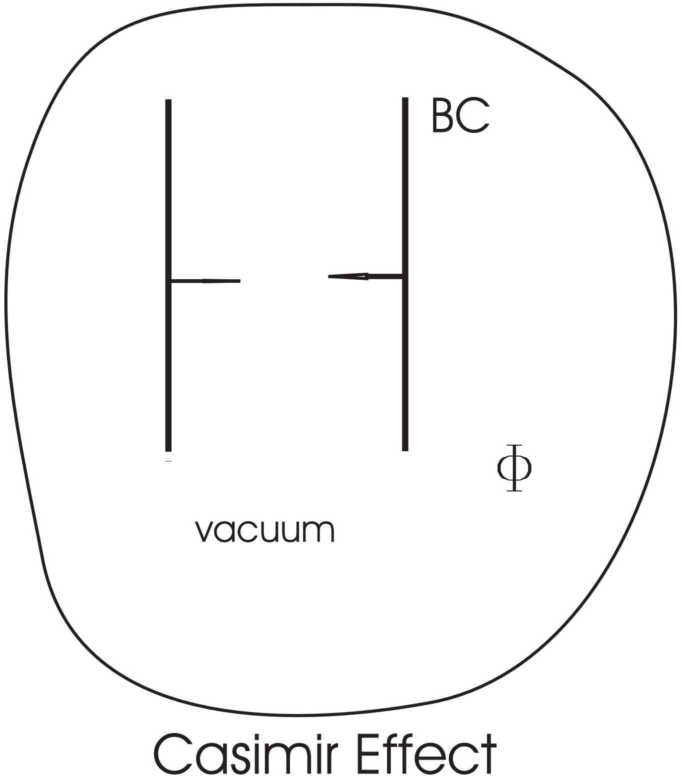

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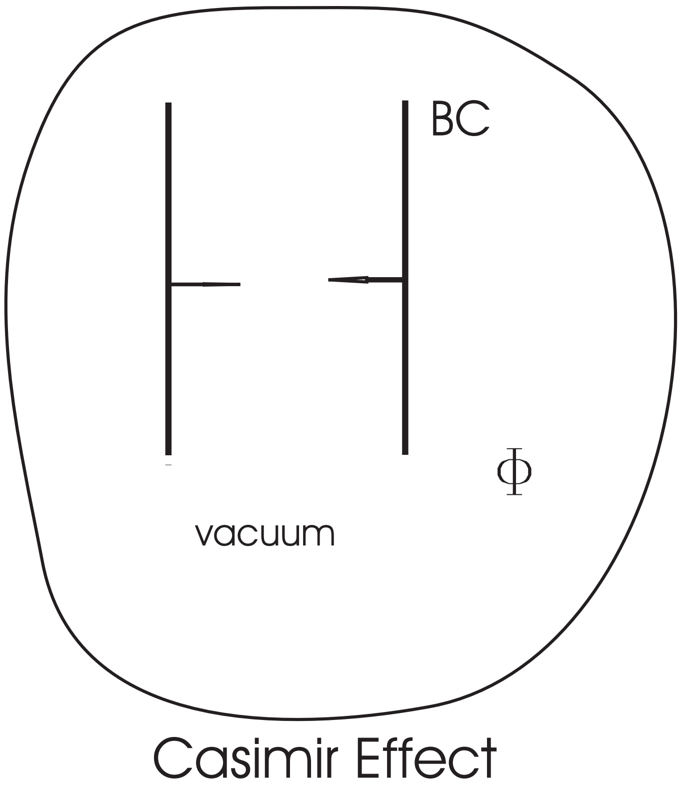

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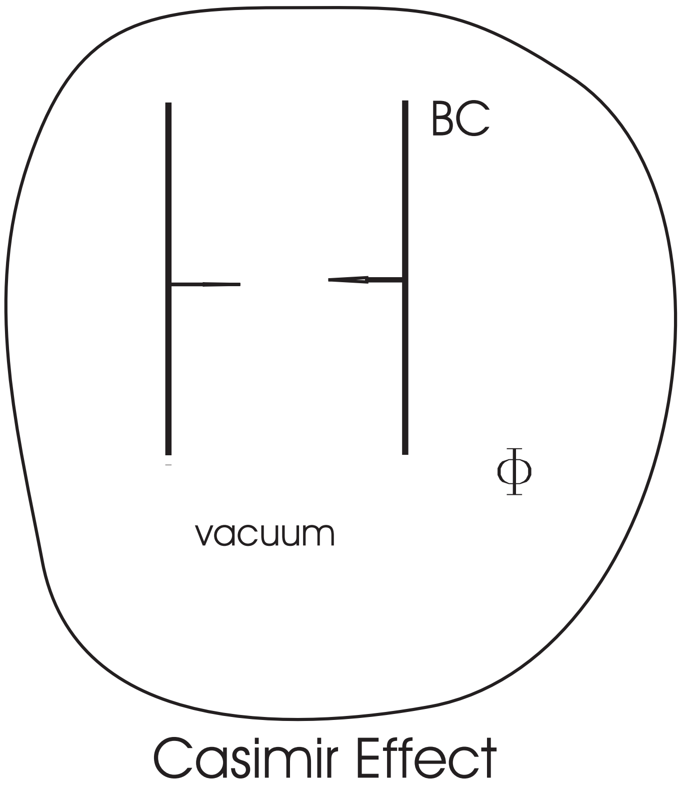

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- Cond. matter (wetting ^{3}He alc.)
- Optical cavities
- Direct experim. confirmation

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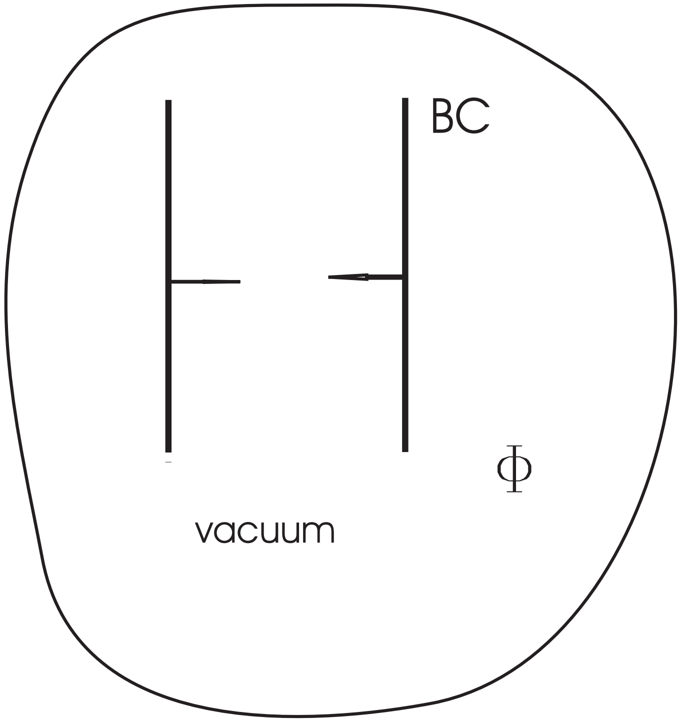

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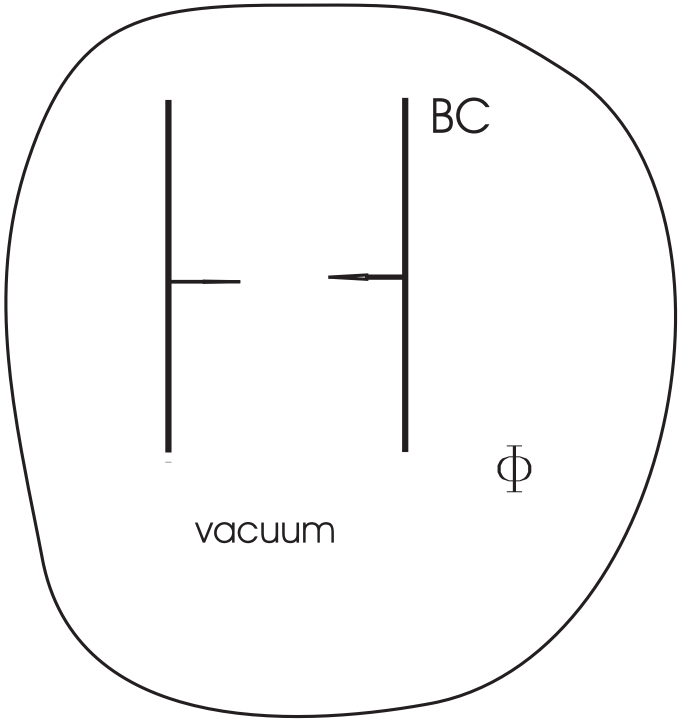

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- Dynamical CE $\Longleftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Longleftarrow$

The standard approach

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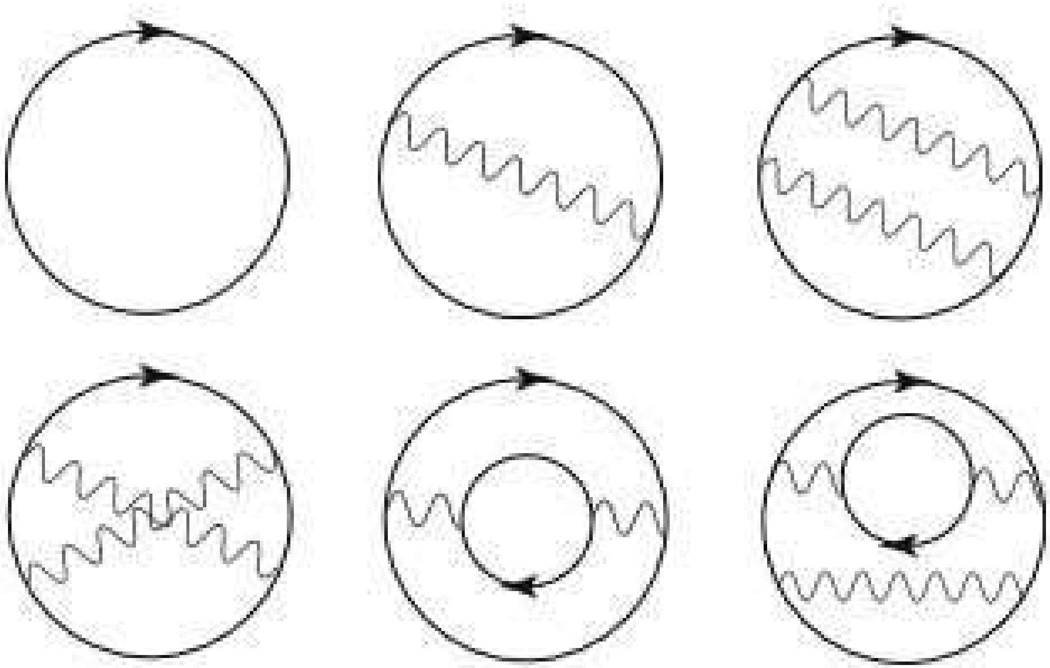

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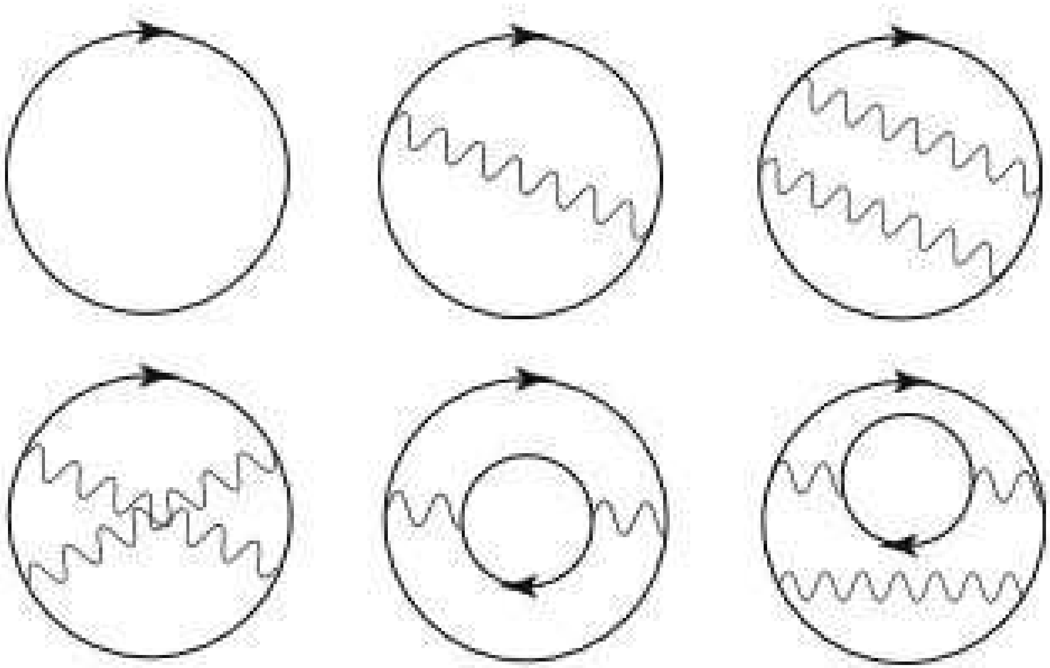

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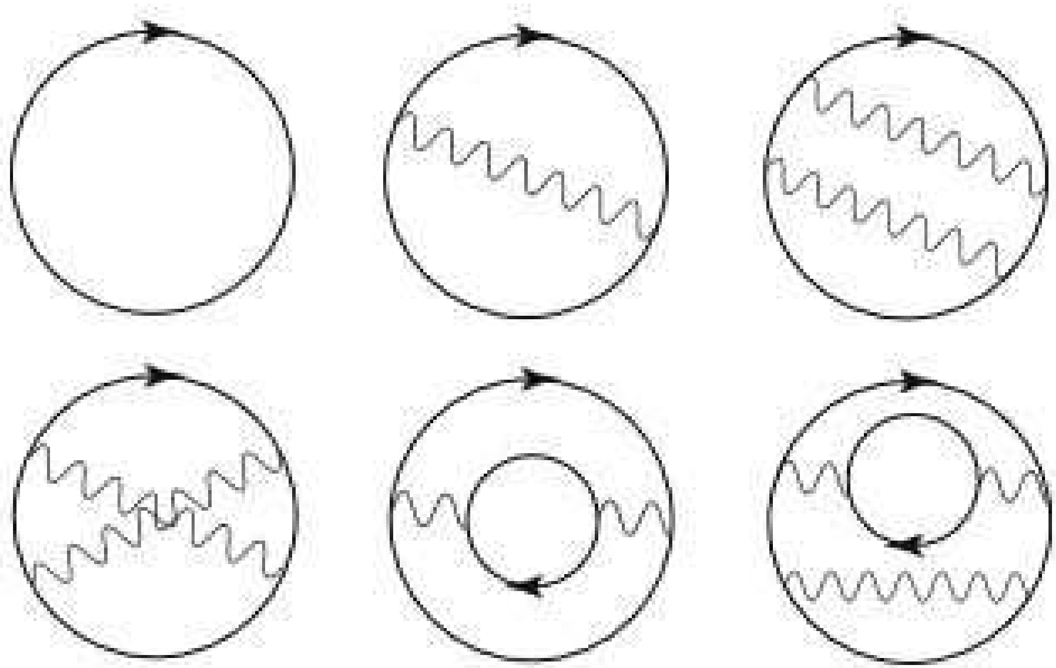

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$$\mathcal{E} = \frac{\hbar}{2\pi} \mathrm{Im} \int d\omega\omega \ \mathrm{Tr} \int d^3x \left[\mathcal{G}(x, x, \omega + i\epsilon) - \mathcal{G}_0(x, x, \omega + i\epsilon)\right]$$

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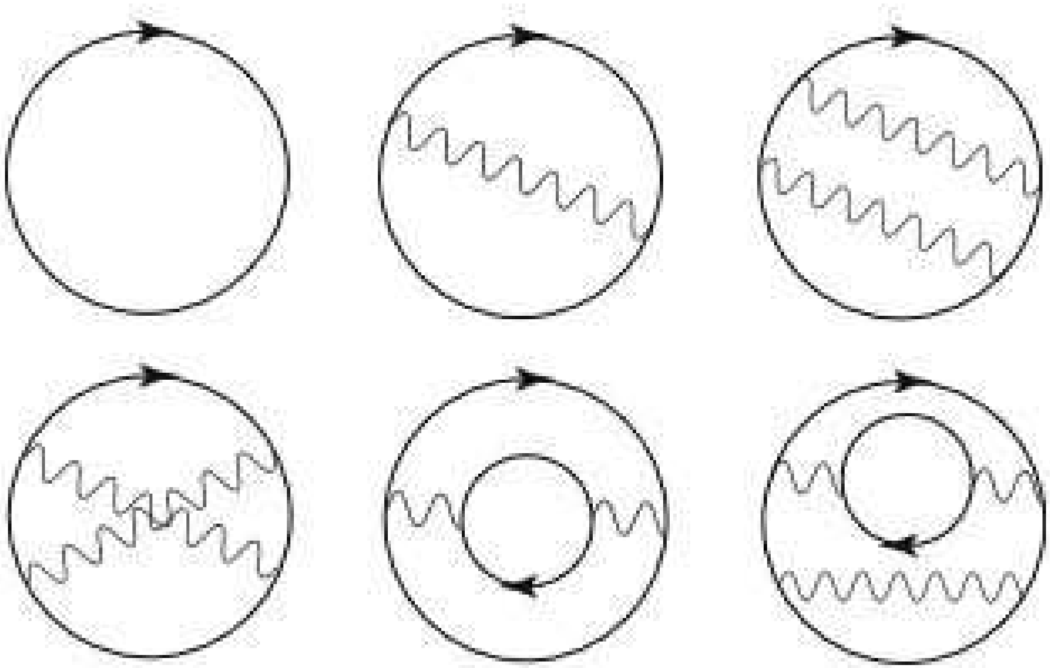

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function

Trace is over spin

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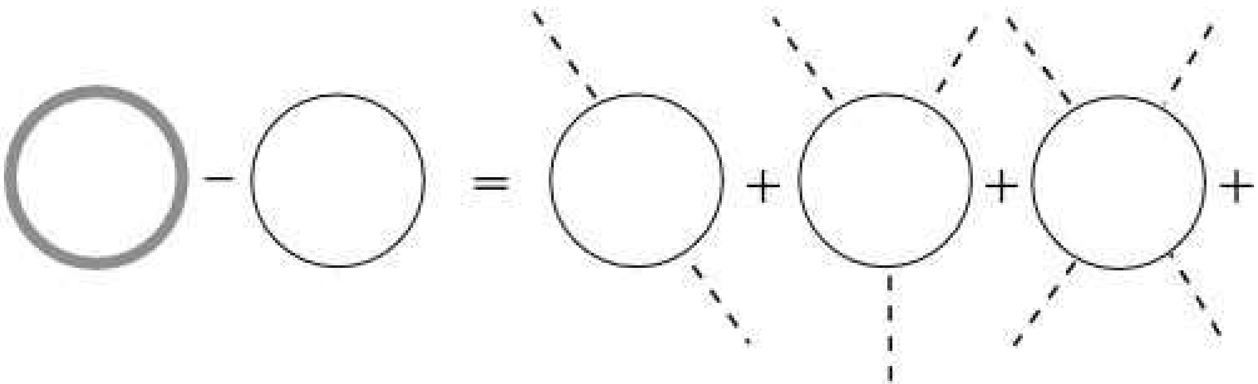

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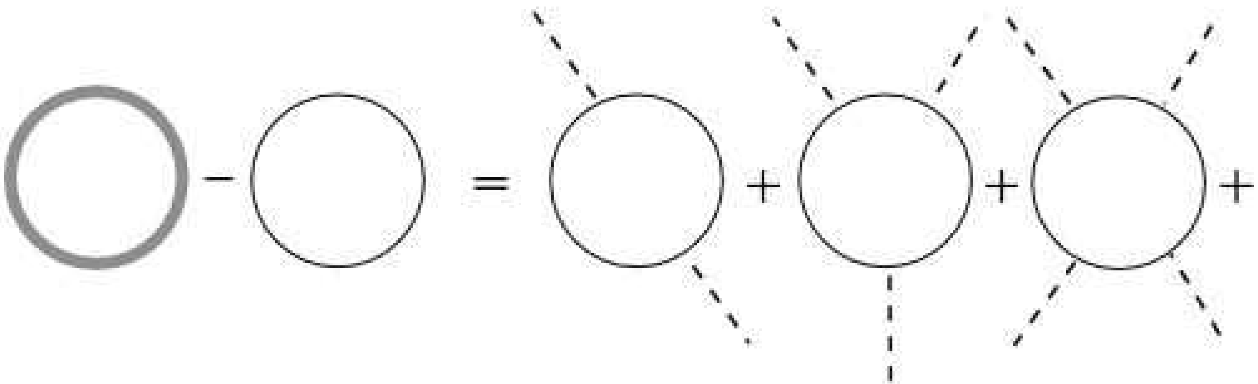

$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Ford, Vilenkin; Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

Some details of the method (1)

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \rightarrow (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$$\widetilde{\Omega}: (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

(with $\bar{t}$ the new time)

Some details of the method (2)

- Hamiltonian density

$$\widetilde{\mathcal{H}}(\bar{t},\mathbf{y}) = \frac{1}{2}\left(\widetilde{\xi}^2 + J|\nabla_{\mathbf{x}}\phi|^2\right) + \widetilde{\xi}\left(\partial_{\bar{t}}\widetilde{\phi} - \sqrt{J}\partial_t\phi\right)$$

$\widetilde{\phi}$ field, $\widetilde{\xi}$ conjugate momentum, J Jacobian: $d^3\mathbf{x} \equiv J d^3\mathbf{y}$

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- It turns out that

$$\mathcal{H}(t,\mathbf{x}) = \mathcal{E}(t,\mathbf{x}) + \xi(t,\mathbf{x}) < \partial_s \mathbf{R}(\mathcal{R}^{-1}(t,\mathbf{x})), \nabla_{\mathbf{x}}\phi(t,\mathbf{x}) >$$
$$+\frac{1}{2}\,\xi(t,\mathbf{x})\phi(t,\mathbf{x})\partial_s(\ln J)|_{\mathcal{R}^{-1}(t,\mathbf{x})}$$

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- A simple example:

Single mirror following a prescribed trajectory

$$R(\bar{t},y) = y + \epsilon g(\bar{t})$$

We explicitly get

$$\mathcal{H}(t,x) = \mathcal{E}(t,x) + \epsilon\dot{g}(t)\xi(t,x)\partial_x\phi(t,x)$$

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

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Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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Consider, instead, a partially transmitting mirror, transparent to high frequencies (math. implementation of a physical plate).

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Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)e^{-2i\omega L} \\ r(\omega)e^{2i\omega L} & s(\omega) \end{pmatrix}$$

$\Longrightarrow$ S matrix is taken to be: ($x = L$ position of the mirror)

Case of a single, partially transmitting mirror

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

RESULTS ARE REWARDING:

Results are rewarding:

In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \, \text{Re} \left[e^{-i(\omega+\omega')t} \, \widehat{\dot{g}\theta_t}(\omega + \omega') \right]$$
$$\times [|r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2] + \mathcal{O}(\epsilon^2)$$

Note this integral diverges for a perfect mirror ($r \equiv -1$, $s \equiv 0$, ideal case), but nicely converges for our partially transmitting (physical) one where $r(\omega) \to 0$, $s(\omega) \to 1$, as $\omega \to \infty$

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Energy conservation is fulfilled: the dynamical energy at any time t equals, with the opposite sign, the work performed by the reaction force up to that time t

$$\langle \hat{E}(t)\rangle = -\epsilon\int_0^t \langle \hat{F}_{Ha}(\tau)\rangle \dot{g}(\tau)d\tau$$

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- Consider the change

$$R(\bar{t}, y) = \frac{1}{L_2 - L_1} \left[L_2(\bar{t};\epsilon)(y - L_1) + L_1(\bar{t};\epsilon)(L_2 - y) \right]$$

the Hamiltonian density of the field is then

$$\mathcal{H}(t,x) = \mathcal{E}(t,x) + \sum_{j=1,2} \frac{(-1)^j \dot{L}_j(t;\epsilon)\xi(t,x)}{L_2(t;\epsilon) - L_1(t;\epsilon)} \left[\partial_x \phi(t,x)(x - \bar{L}_j(t;\epsilon)) + \frac{1}{2}\phi(t,x) \right]$$

where $\bar{L}_{\binom{1}{2}}(t;\epsilon) \equiv L_{\binom{2}{1}}(t;\epsilon)$

- In the interaction picture, while mirrors move, the full Hamiltonian is

$$\hat{H}_I(t) = -\frac{\epsilon(g_2(t) - g_1(t))}{L_2 - L_1}\left[\int dy \left(\partial_y \widehat{\widetilde{\phi}}_I(y)\right)^2 + \sum_{j=1,2} \alpha_j \left(\widehat{\widetilde{\phi}}_I(L_j)\right)^2\right]$$

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- There are proposals to detect the radiated photons: Kim, Brownell, Onofrio, PRL 96 (2006) 200402

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209v2

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmolog Imprint of the Casimir Eff't?

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two 10^{-2}mm dims, bulk vs brane Susy breaking scales
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