ICE CSIC IEEC

Vacuum Fluctuations: Moving Boundaries & the Equivalence Principle

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Cosmology Workshop Montpellier07, October 25-26, 2007

Outline of this presentation

- Euler and the Zeta Function

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- The Casimir Effect: Theory, Experiments, Uses

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- Vacuum Fluctuations and the Equivalence Principle
- CE and Accelerated Expansion (Dark Energy)

Euler and the Zeta Function

- How did Euler discover the zeta function?
  [R Ayoub, Am Math Month 81 (1974) 1067]. The harmonic series

$$H = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots$$

has an infinite sum. Euler: what about the 'prime harmonic series'

$$PH = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \cdots$$

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- Cannot split the first into two

$$\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots\right] + \left[\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \cdots\right]$$

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- Euler considered $$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \cdots$$

- Provided s is bigger than 1, you can split it up

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- Idea: s closer to 1, first sum increases without bound. Key step is the celebrated equation (p prime)

$$\zeta(s)=\frac{1}{1-(1/2^s)}\times\frac{1}{1-(1/3^s)}\times\frac{1}{1-(1/5^s)}\times\frac{1}{1-(1/7^s)}\times\cdots$$

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- For any p prime and any $s > 1$, set $x = 1/p^s$ to give:

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- Euler multiplied together these inf sums: his inf product as single inf sum:

$$\frac{1}{p_1^{k_1 s}\ldots p_n^{k_n s}}$$

$p_1,\ldots,p_n$ primes $k_1,\ldots,k_n$ positive integers, each such combination occurs exactly once, rhs just rearrangement of $\zeta(s)$.

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- Euler's infinite product formula for $\zeta(s)$ marked the beginning of analytic number theory.

- Dirichlet modified the zeta function: primes separated into categories, depending on the remainder when divided by k:

$$L(s,\chi) = \frac{\chi(1)}{1^s} + \frac{\chi(2)}{2^s} + \frac{\chi(3)}{3^s} + \frac{\chi(4)}{4^s} + \cdots$$

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- Conditions: (i) $\chi(mn) = \chi(m)\chi(n)$ for any m, n
(ii) $\chi(n) = \chi(n+k), \ \forall n$
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- Subsequent generalizations: allow s and $\chi(n)$ to be complex. The celebrated Riemann zeta function, Hurwitz, Epstein, etc. Many results about prime numbers were proven: L-series provide a powerful tool for the study of the primes [Keith Devlin]

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Even then: Has the final value real sense ?

The Casimir Effect

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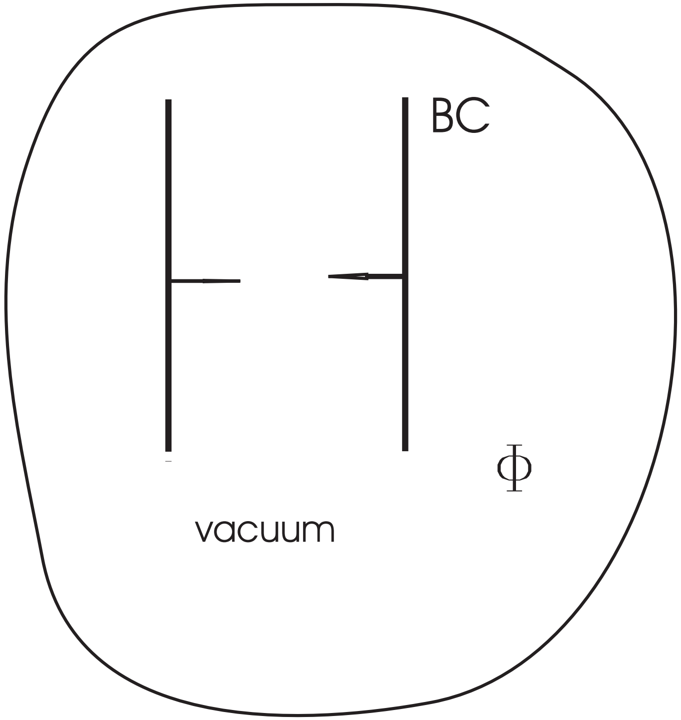
BC e.g. periodic



Casimir Effect

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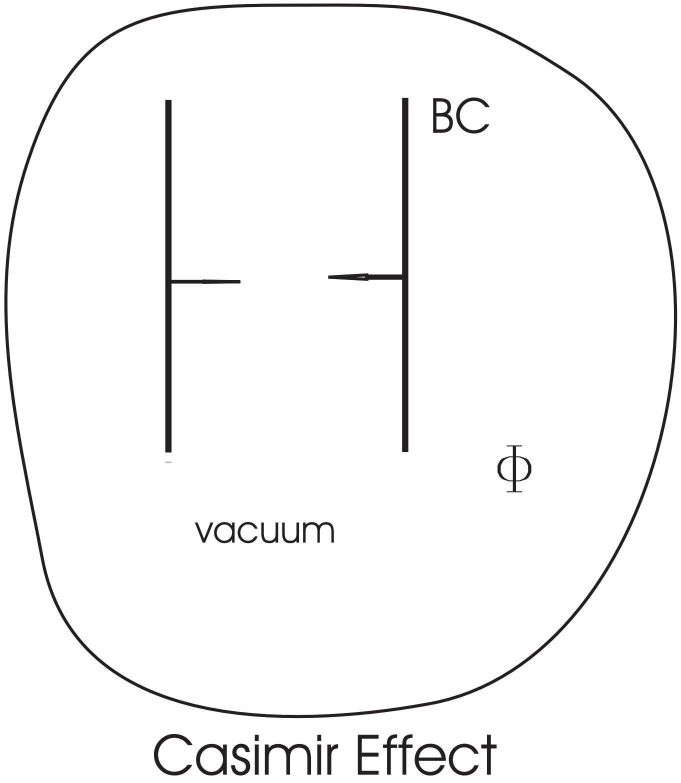

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$\Longrightarrow$ all kind of fields

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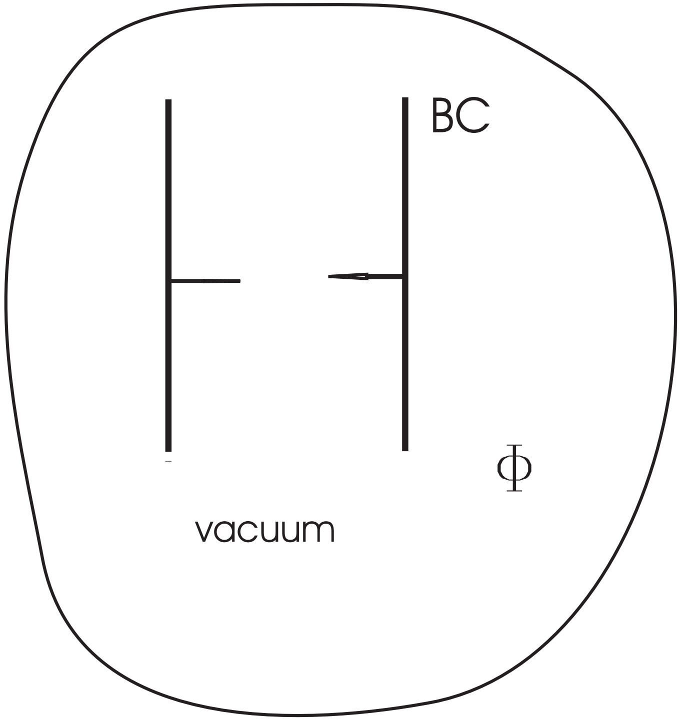

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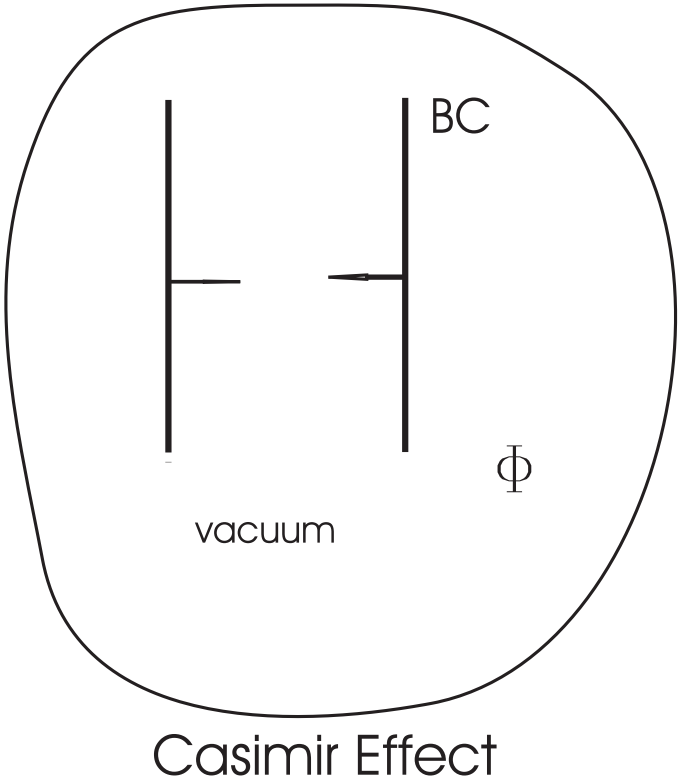

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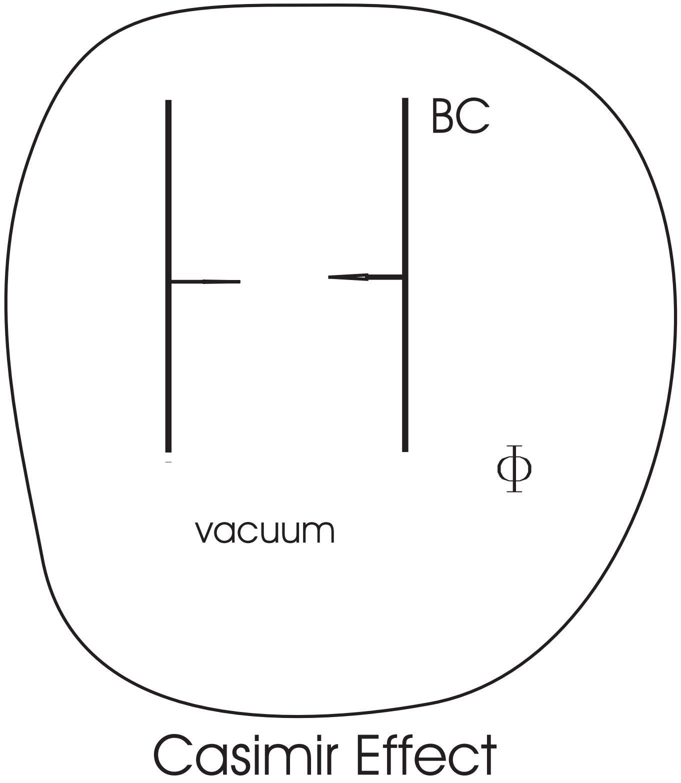
The Casimir Effect



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- Sonoluminiscence (Schwinger)
- Cond. matter (wetting ^{3}He alc.)
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- Direct experim. confirmation

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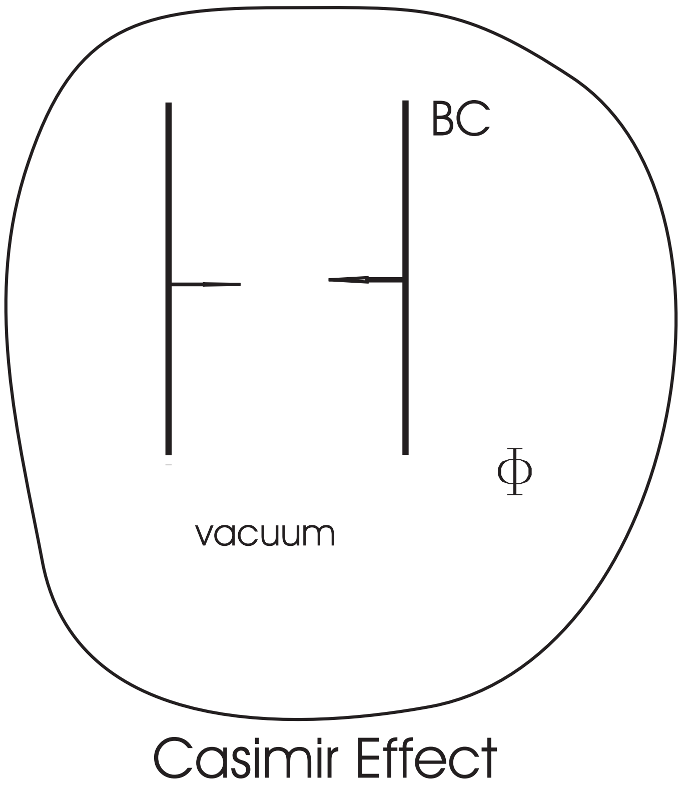

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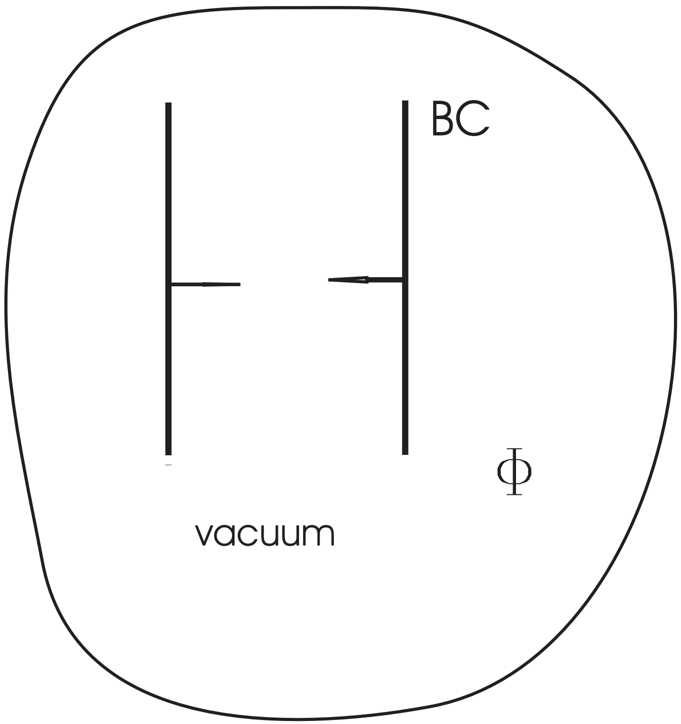

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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

The standard approach

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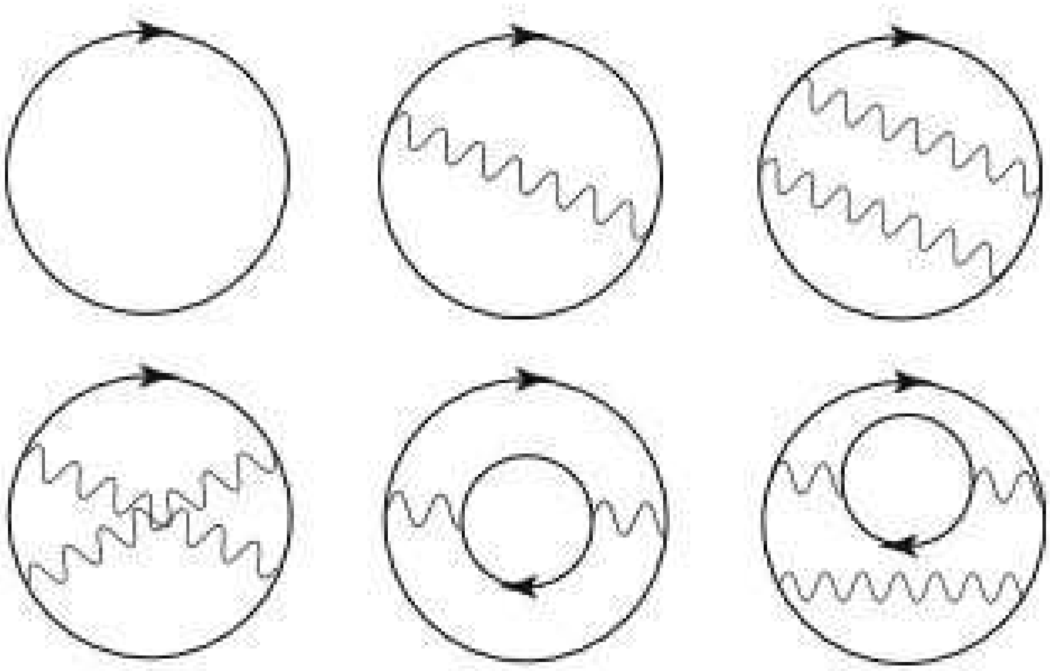

$\Longrightarrow$ Casimir force: calculated by computing change in zero point energy of the em field

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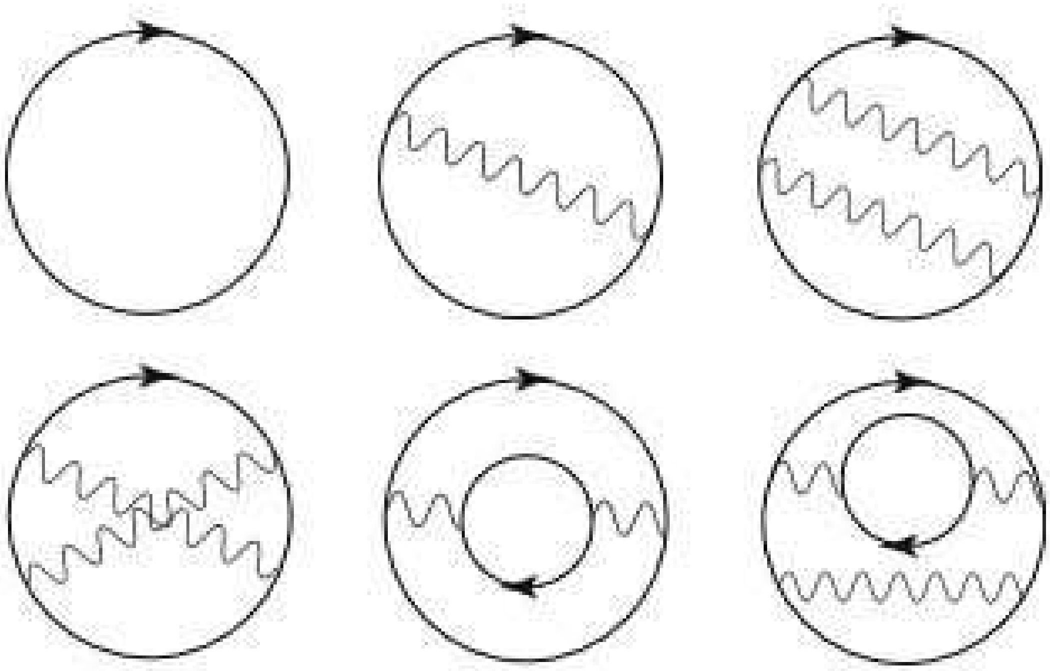

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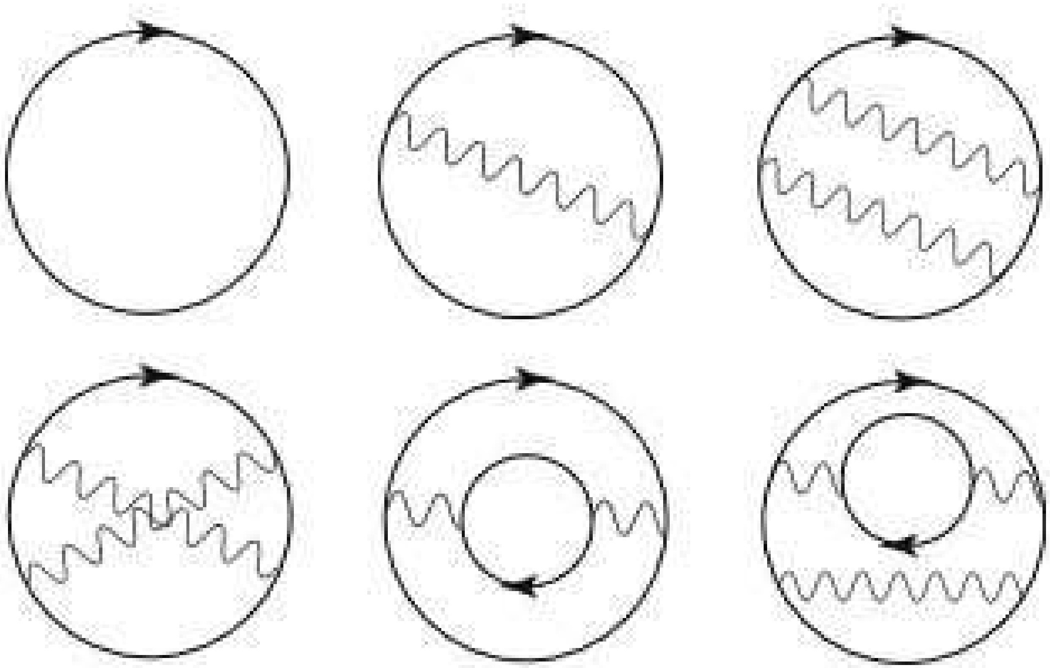

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi} \mathrm{Im} \int d\omega\omega \ \mathrm{Tr} \int d^3x \left[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)\right]$$

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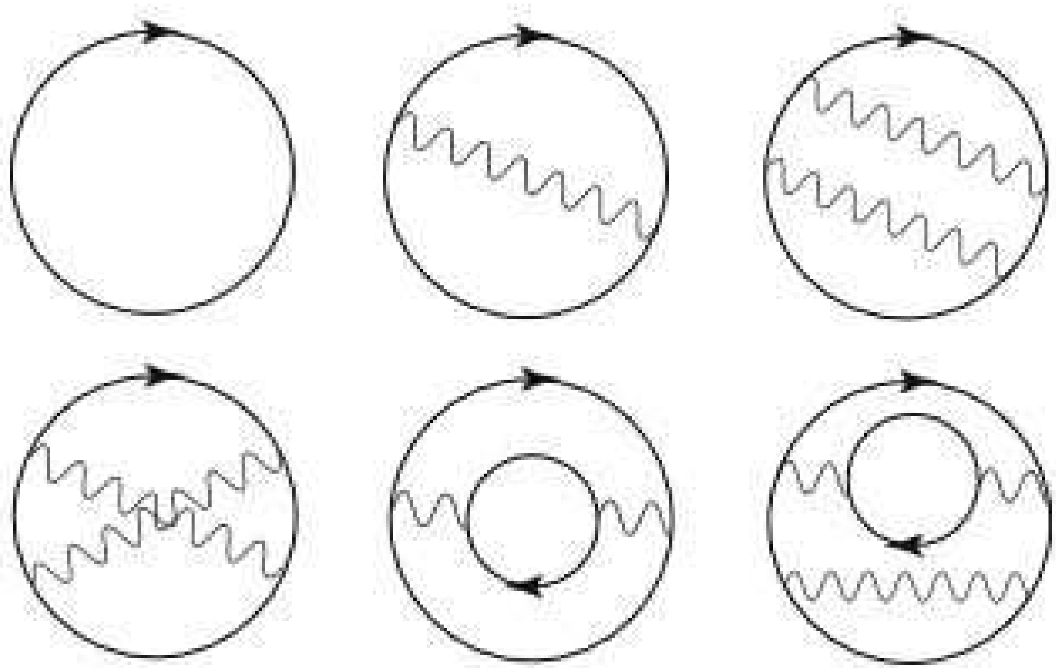

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function

Trace is over spin

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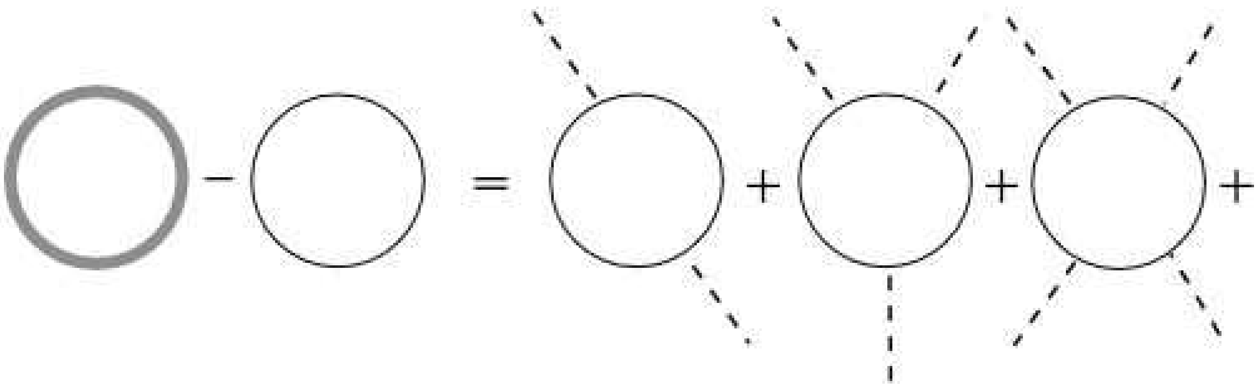

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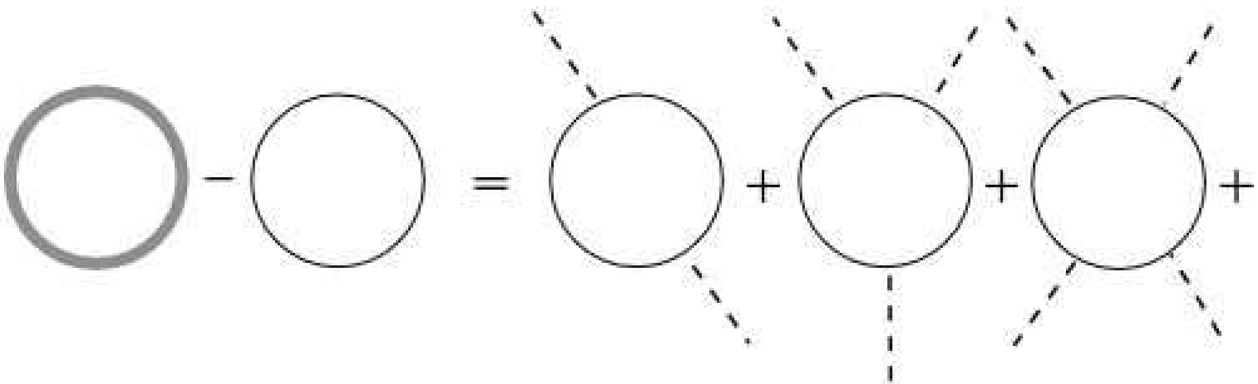

$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Ford, Vilenkin; Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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J. Haro & E.E., PRL 97 (2006); arXiv:0705.0597

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest

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- Such force is split into two parts: a dissipative force whose work equals minus the energy of the particles that remain & a reactive force vanishing when the mirrors return to rest
- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

SOME DETAILS OF THE METHOD

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

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Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

cannot be considered as the energy of the produced particles at time t

[cf. paragraph after Eq. (4.5)]

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Consider, instead, a partially transmitting mirror, transparent to high frequencies (math. implementation of a physical plate).

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Trajectory $(t, \epsilon g(t))$. When mirror at rest, scattering described by matrix

$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)\, e^{-2i\omega L} \\ r(\omega)\, e^{2i\omega L} & s(\omega) \end{pmatrix}$$

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

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In our Hamiltonian approach

$$\langle \hat{F}_{Ha}(t) \rangle = -\frac{\epsilon}{2\pi^2} \int_0^\infty \int_0^\infty \frac{d\omega d\omega' \omega\omega'}{\omega + \omega'} \text{Re}\left[e^{-i(\omega+\omega')t}\, \widehat{\dot{g}\theta_t}(\omega + \omega') \right]$$
$$\times \left[|r(\omega) + r^*(\omega')|^2 + |s(\omega) - s^*(\omega')|^2 \right] + \mathcal{O}(\epsilon^2)$$

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209v2

  energy ALWAYS gravitates, therefore the energy density of the vacuum, more precisely, the vacuum expectation value of the stress-energy tensor $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

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- Boundary divergences resolved by understanding renormalization process

- Casimir stress tensor between pair of parallel perfectly conducting plates, at distance a, transverse dimensions $L >> a$

[Brown and Maclay, Phys. Rev. 184 (1969) 1272]

$$\langle T_{\mu\nu} \rangle = \frac{\mathcal{E}_{\mathcal{C}}}{a} \, \text{diag}(1, -1, -1, 3)$$

third spatial direction is normal to plates, $\mathcal{E}_{\mathcal{C}}$ Casimir energy per unit area

$$\mathcal{E}_{\mathcal{C}} = -\frac{\pi \hbar c}{720 a^3}$$

Outside the plates, $\langle T_{\mu\nu} \rangle = 0$

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- Gravitational interaction of the Casimir apparatus: use gravitational definition of the energy-momentum tensor as variation of matter part of action:

$$\delta W_m = \frac{1}{2} \int \sqrt{-g}\, \delta g_{\mu\nu}\, T^{\mu\nu} \qquad (*)$$

Following Schwinger (note the factor 2 in the definition), for a weak field Fulling et al define: $g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$

- Two ways to proceed. Gauge-invariant procedure:
  energy-momentum tensor of the phys sys must be conserved, so include a physical mechanism holding the plates apart against the Casimir force
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CC PROBLEM

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

  $\Longrightarrow$ kind of cosmological Casimir effect

Cosmolog Imprint of the Casimir Eff't?

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