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# Vacuum Fluctuations, The Casimir Effect & Dark Energy

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# Outline

- On Einstein's Cosmological Constant:  
a Historical Perspective
- Quantum Vacuum Fluctuations: the Casimir Effect
- Zeta and 'Physical' Regularizations
- Vacuum Fluctuations and the Equivalence Principle
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs
- A Cosmo-Topological Casimir Effect?
- Gravitation as equilibrium spacetime thermodynamics
- The case of modified,  $f(R)$  theories
- With THANKS to:

S Carloni, G Cognola, J Haro, S Nojiri, S Odintsov,  
D Sáez-Gómez, A Saharian, P Silva, S Zerbini, ...

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- Finally, Edwin Hubble, in 1922-1924, realized Andromeda nebulae (800.000 ly) far beyond our Milky Way: nebulae are other galaxies!  
Henrietta Leavitt (*Ed Pickering's Harvard Harem*), Hertzsprung-Russell

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- **Riess, Schmidt ea '98, Perlmutter '99:** expansion is **accelerating** !

# Trying to solve these puzzles !

- The cc  $\lambda$  is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density  $\mu$  of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g} \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g} \Lambda$$

- In other words: two contributions on the same footing [Pauli 20's, Zel'dovich '68]

$$\frac{\Lambda c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar c}{2} \sum_i \omega_i$$

- For elementary particle physicists: a great embarrassment  
no way to get rid off  
Coleman, Hawking, Weinberg, Polchinski, ... '88-'89

THE COSMOLOGICAL CONSTANT PROBLEM

# Zero point energy

**QFT** vacuum to vacuum transition:  $\langle 0|H|0\rangle$

Spectrum, normal ordering (harm oscill):

$$H = \left( n + \frac{1}{2} \right) \lambda_n a_n a_n^\dagger$$

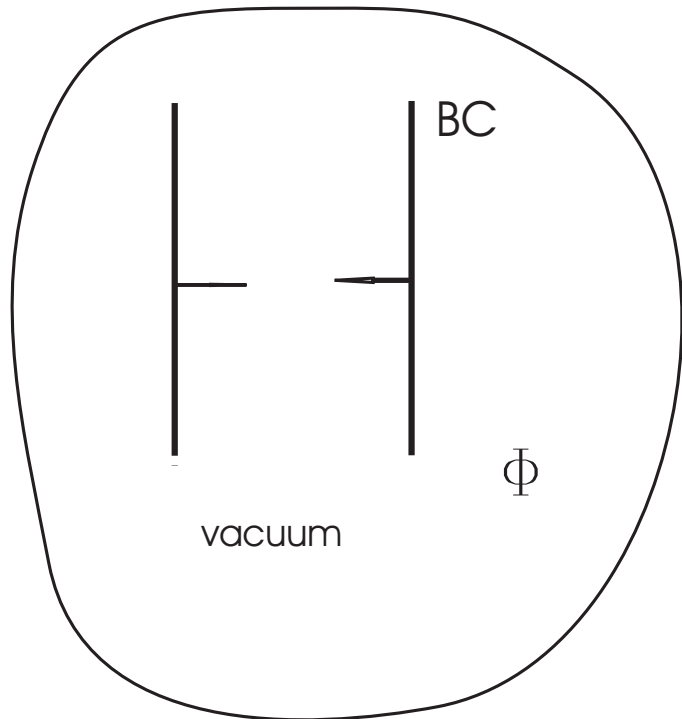
$$\langle 0|H|0\rangle = \frac{\hbar c}{2} \sum_n \lambda_n = \frac{1}{2} \text{tr } H$$

gives  $\infty$  physical meaning?

Regularization + Renormalization ( cut-off, dim,  $\zeta$  )

Even then: Has the final value real sense ?

# The Casimir Effect



Casimir Effect

- BC e.g. periodic
- $\Rightarrow$  all kind of fields
- $\Rightarrow$  curvature or topology

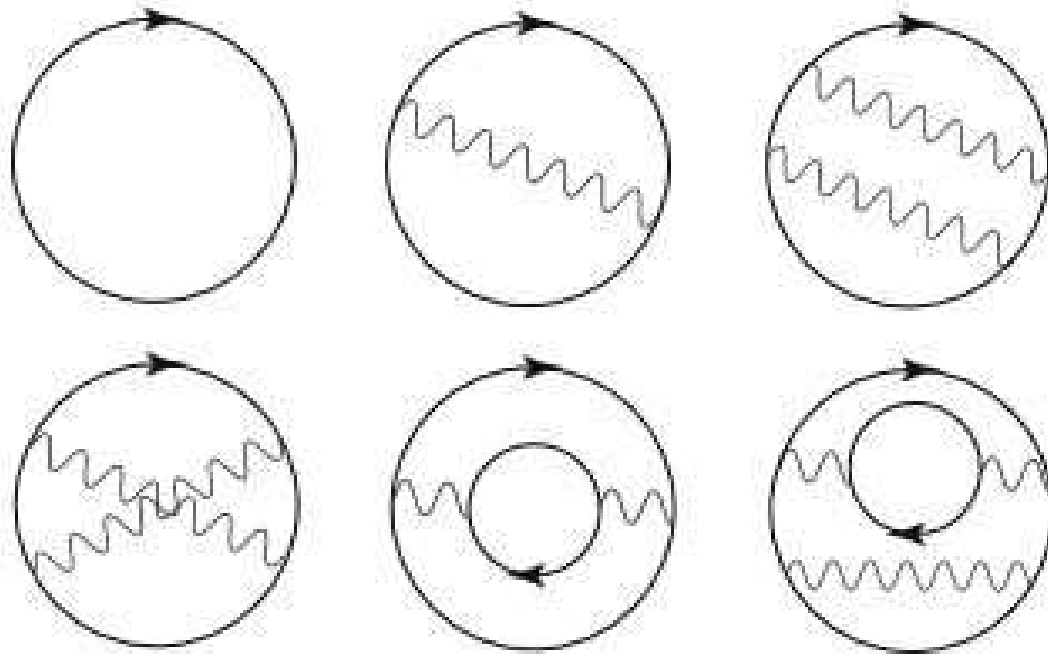
Universal process:

- Sonoluminescence (Schwinger)
- Cond. matter (wetting  $^3\text{He}$  alc.)
- Optical cavities
- Direct experim. confirmation

Van der Waals, Lifschitz theory

- Dynamical CE  $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant  $\Leftarrow$

# The standard approach



⇒ Casimir force: calculated by computing change in zero point energy of the em field

⇒ But Casimir effects can be calculated as  $S$ -matrix elements: Feynman diagrs with ext. lines

In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi} \text{Im} \int d\omega \omega \text{Tr} \int d^3x [\mathcal{G}(x, x, \omega + i\epsilon) - \mathcal{G}_0(x, x, \omega + i\epsilon)]$$

$\mathcal{G}$  full Greens function for the fluctuating field

$\mathcal{G}_0$  free Greens function

Trace is over spin

$$E_C = \langle \quad \rangle_{\text{plates}} - \langle \quad \rangle_{\text{no plates}}$$

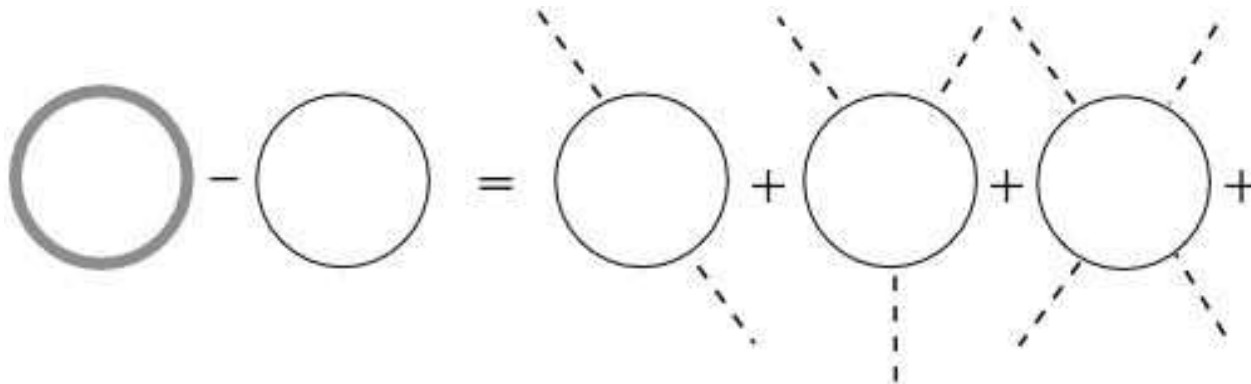
$$\frac{1}{\pi} \text{Im} \int [\mathcal{G}(x, x, \omega + i\epsilon) - \mathcal{G}_0(x, x, \omega + i\epsilon)] = \frac{d\Delta N}{d\omega}$$

change in the density of states due to the background

⇒ A restatement of the Casimir sum over shifts in zero-point energies

$$\frac{\hbar}{2} \sum (\omega - \omega_0)$$

⇒ **Lippman-Schwinger eq.** allows full Greens f,  $\mathcal{G}$ , be expanded as a series in free Green's f,  $\mathcal{G}_0$ , and the coupling to the external field



⇒ “Experimental confirmation of the Casimir effect doesn’t establish the reality of zero point fluct’s better than say the Lamb shift does” [R Jaffe e a]



# Vacuum Fluct & the Equival Principle

● The main issue:

S.A. Fulling et. al., hep-th/070209

energy **ALWAYS gravitates** therefore the energy density of the vacuum appears on the rhs of Einstein's equations:

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- Calculations done also in **Rindler coord** (uniform accel obs)

# Cosmo-Topol Casimir Eff't & Alternat's

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- B. Other alternatives: (i) L Faddeev 0911.0282 (Adler '82)  
Newton const in E-H Lag has dim of mass → non-renormalizability  
Describe gravity by vector field (as Higgs mechanism)  
(ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in euIR sector and a mechanism to relax the cc. Also Shapiro+Sola, ...

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- More general results:      **Kenneth, Klich, PRL 97, 160401 (2006)**  
a mirror pair of dielectric bodies always attract each other  
**CP Bachas, J Phys A40, 9089 (2007)** from a general property of Euclidean QFT '**reflection positivity**' (Osterwalder - Schrader 73, 75):  
 $\exists$  of positive Hilbert space and self-adjoint non-negative Hamiltonian



- E.g.  $\exists$  correlation inequality:  $\langle f\Theta(f) \rangle > 0$   
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  - periodic BCs for fermions

- E.g.  $\exists$  **correlation inequality**:  $\langle f\Theta(f) \rangle > 0$   
 $\Theta$  **reflection** with respect to a 3-dim hyperplane in  $R^4$   
 the action of  $\Theta$  on  $f$  is anti-unitary  $\Theta(cf) = c^*\Theta(f)$
- The existence of the reflection operator  $\Theta$  is a consequence of **unitarity** only, and makes no assumptions about the discrete  $C, P, T$  symmetries
- Boyer's result does not contradict the theorem, since cutting an elastic shell into two rigid hemispheres is a **mathematically singular** operation (which introduces divergent edge contributions)
- Theorem does **not** apply for
  - mirror probes in a **Fermi sea** (chemical-potential term), eg when electron-gas **fluctuations become important**
  - periodic BCs for **fermions**
  - **Robin BCs** in general  $\Leftarrow$

# Casimir eff in brworld's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime  $R^{(D_1-1,1)} \times \Sigma$ ,  $\Sigma$  compact internal space
- Most general case: constants in the BCs different for the two plates  
It is shown that Robin BCs with different coefficients are necessary to obtain repulsive Casimir forces
- Robin type BCs are an extension of Dirichlet and Neumann's  
⇒ most suitable to describe physically realistic situations
- Genuinely appear in: → vacuum effects for a confined charged scalar field in external fields [Ambjørn ea 83],  
→ spinor and gauge field theories,  
→ quantum gravity and supergravity [Luckock ea 91]  
Can be made conformally invariant, purely-Neumann conds cannot  
⇒ needed for conformally invar theories with BC, to preserve cf invar



# Gravity Eqs as Eqs of State: $f(R)$ Case

## Introductory ideas

- The formulation of **black hole entropy (Bekenstein 73)** and **Hawking BH radiation (1974)** pointed clearly towards a profound connection between gravity and **thermodynamics**
- Actually, gravity equations closely resemble those of **thermodynamics** and **hydrodynamics (Verlinde 2010, and many others)**
- **Ted Jacobson (PRL 1995)** went one step further and obtained Einstein's equations starting from **local thermodynamics (Clausius eq)** and the entropy formula for all **local Rindler horizons**
- That is, by way of **generalizing** black hole thermodynamics to space-time thermodynamics as seen by a **local observer**
- Also, this is an expression of the holographic principle (**'t Hooft 93, Susskind 95**)

# On holography:

- Maldacena's AdS/CFT correspondence (1998) is a direct manifestation of the **holographic principle**
- Actually, these are the strongest supporting evidences for the **holographic principle**: **BH physics** and the **AdS/CFT correspondence**
- Suggest that gravitation is **induced** by a QFT in lower dimensions: gravitation on the macroscopic scale could be a manifestation of the **thermodynamics of the vacuum state**
- (At least part of) the microscopic degrees of freedom can be **represented holographically**: either on the boundary of space-time or on horizons
- Recently: idea that **gravity is emergent**, rather than fundamental (**E Verlinde**; **Thanu Padmanabhan**; **W Gu**, **R-X Miao**, ... & more)
- Its **emergence** should be understood from **general principles**: independent of specific details of underlying microscopic theory

- **Verlinde:** Gravity is explained as an **entropic force** caused by changes in the information associated with the positions of material bodies
- A **relativistic generalization** of the argument leads to **Einstein's equations**
- Most important assumption: the **information** associated with a part of space obeys the **holographic principle**
- The ST description of gravity as due to exchange of closed strings **cannot be valid:** it appears that strings have to be emergent too
- But ... **Holography is also an hypothesis!**  
May be as absurd as an action at a distance (EV *dixit*)
- In this picture, there is **no place for gravitons** and hence no need to worry about quantization of gravity and related problems
- **A Kobakhidze (PRD 2011):** for microscopic QM systems the situation is **different**, results of experiments with ultracold neutrons in the gravitational field of the Earth are in **disagreement** with EV's

- **Two extra terms present:** one seems to account for the relativistic rest energy of the neutron, a **constant shift** in the neutron's energy states
- The other is a form of an **extreme suppression** of certain parts of the neutron's wavefunction:  
**would change dynamics** of the experiment, **not seen** at all!

## On Jacobson's discovery:

- It strongly suggests, in a fundamental context, that **Einstein's Eqs are to be viewed as EoS**
- EE's should probably **not** be taken as **basic** for quantizing gravity
- **C Eling, R Guedens and T Jacobson (PRL2006): extension to polynomial  $f(R)$  gravity but as non-equilibrium s-t thermodynamics**
- In fact, there is an **entropy production term**, which is a **local boost dependent quantity**

- R Brustein and M Hadad (2009, err 2010) equations of motion of generalized theories of gravity are equivalent to the thermodynamic relations ( $\delta Q = T\delta S$ ), extending previous arguments by using Noether charge entropy
- W Gu, R-X Miao and G Wu (1104.3194) focus on the origin & features of the entropy production term: a consequence of an  $O(x^3)$  ambiguity in an approximate boost Killing vector field
- Derive the entropy form: in  $f(R)$ -gravity it is a function of the “boost variant” (Lie derivative of  $f(R)$  along Killing vector at point  $p$  in s-t)
- The entropy production term is just needed for the entropy balance relation, it is not related to the equation of motion
- Leads back to EEPS08, clarifies relation between these equil thermod and Jacobson’s ea non-equil thermod for  $f(R)$ -gravity as s-t thermod
- W Guang and G Wei (1106.4941) with a new interpretation of the chemical potential show that an entropy bound is the natural result of the 1st law of thermod on the holographic screen

● **Jacobson's argument:** basic thermodynamic relation

$$\delta Q = T\delta S$$

- entropy proportional to variation of the horizon area:  $\delta S = \eta \delta \mathcal{A}$
- local temperature  $T$  defined as **Unruh temp**:  $T = \hbar k / 2\pi$
- functional dependence of  $S$  wrt energy and size of system

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- Key point in **our generalization**: the definition of the local entropy (**Iyer+Wald 93**: local boost inv, **Noether charge**)

$$S = -2\pi \int_{\Sigma} E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \quad \delta S = \delta (\eta_e A)$$

$\eta_e$  is a function of the metric and its derivatives to a given order

$$\eta_e = \eta_e \left( g_{ab}, R_{cdef}, \nabla^{(l)} R_{pqrs} \right)$$

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- **Case of  $\mathbf{f}(R)$  gravities:**  $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$



- Also the concept of an **effective Newton constant** for graviton exchange (**effective propagator**) [BGH 2007-09]

$$\begin{aligned} \frac{1}{8\pi G_{eff}} &= E_R^{pqrs} \epsilon_{pq} \epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R} (g^{pr} g^{qs} - g^{qr} g^{ps}) \epsilon_{pq} \epsilon_{rs} \\ &= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \quad S = \frac{A}{4 G_{eff}} \end{aligned}$$

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- For these theories, the different polarizations of the gravitons only enter in the definition of the **effective Newton constant through the metric itself**
- Final result, for  $\mathbf{f}(R)$  gravities: [EEPS08]  
*the local field equations can be thought of as an equation of state of equilibrium thermodynamics* (as in the GR case)

- Jacobson's argum **non-trivially extended to  $f(R)$**  gravity field eqs  
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**Bolshoie Spasiva**