ICE CSIC IEEC

On a Family of Non-local Gravity Models

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Outline

- Intro: recalling a past story
- The accelerating Universe
- Different types of models
- Several problems
- Models with non-local interactions (Deser & Woodard): motivations
- Our model of type $f(\Box^{-1}R)$

With THANKS to:

Sergey Yu. Vernov, Ying-li Zhang, Ekaterina Pozdeeva, Sergei Odintsov, Misao Sasaki, Guido Cognola, Sergio Zerbini


12+
SMITHSONIAN
WASHINGTON
26 APRIL 1920
HARLOW
SHAPLEY
VS
HEBER
CURTIS
GREAT DEBATE
МИЦ
WORLD HEAVYWEIGHT CHAMPIONSHIP
WBA / IBF / WBO / IBO
MOSCOW 2013
РОСНЕФТЬ

An Island Universe

- Apr 26, 1920: The Great Debate – Shapley vs Curtis

  Harlow Shapley – the Milky Way was the entire Universe

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- Present “island Universe": Josiah McElheny, glass artist, White Cube London, Nat Mus Reina Sofia Madrid – The Multiverse Feeney ea ’11

The accelerating Universe

Dark Energy: effects on the expansion rate of Universe, 3 approaches:
–- Standard candles: measure luminosity distance as function of redshift
–- Standard rulers: angular diam distance & expansion rate as f of redsh
–- Growth of fluctuations: generated at origin of U & amplified by inflation

Both angular diameter and luminosity distances are integrals of (inverse) expansion rate: encode effects of DE
To get competitive constraints on DE need see changes in $H(z)$ at 1% would give statistical errors in DE EoS of $O(10\%)$

–- calibrate the ruler accurately over most of the age of the universe
–- measure the ruler over much of the volume of the universe
–- make ultra-precise measurements of the ruler

On large scales or early times the perturbative treatment is valid: calculations are perfectly under control
Length scales from physics of the early universe are imprinted on the distribution of mass and radiation: they form time-independent rulers
(M White, Berkeley)

The accelerating Universe (II)

Evidence for the acceleration of the Universe expansion:

- distant supernovae

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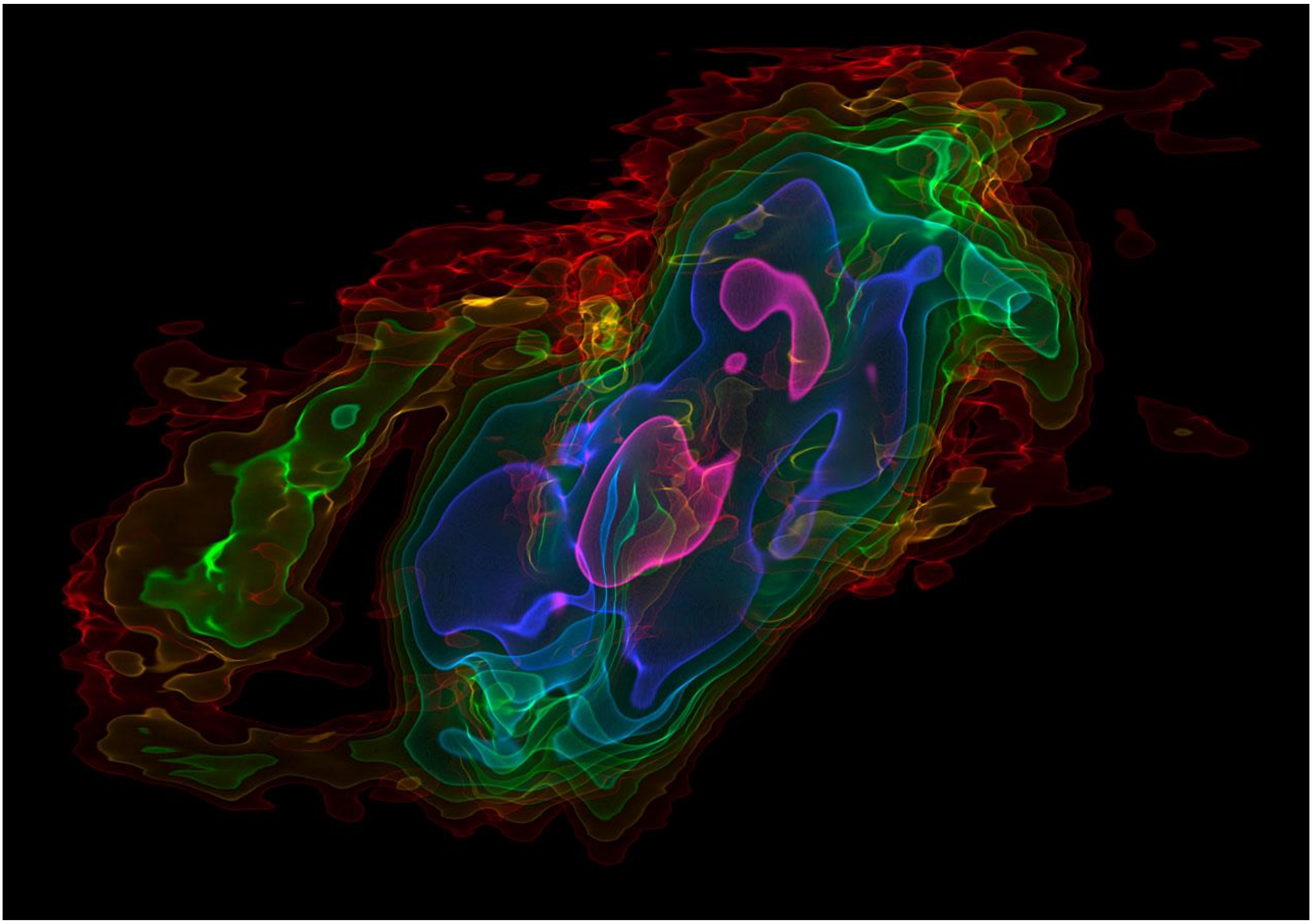
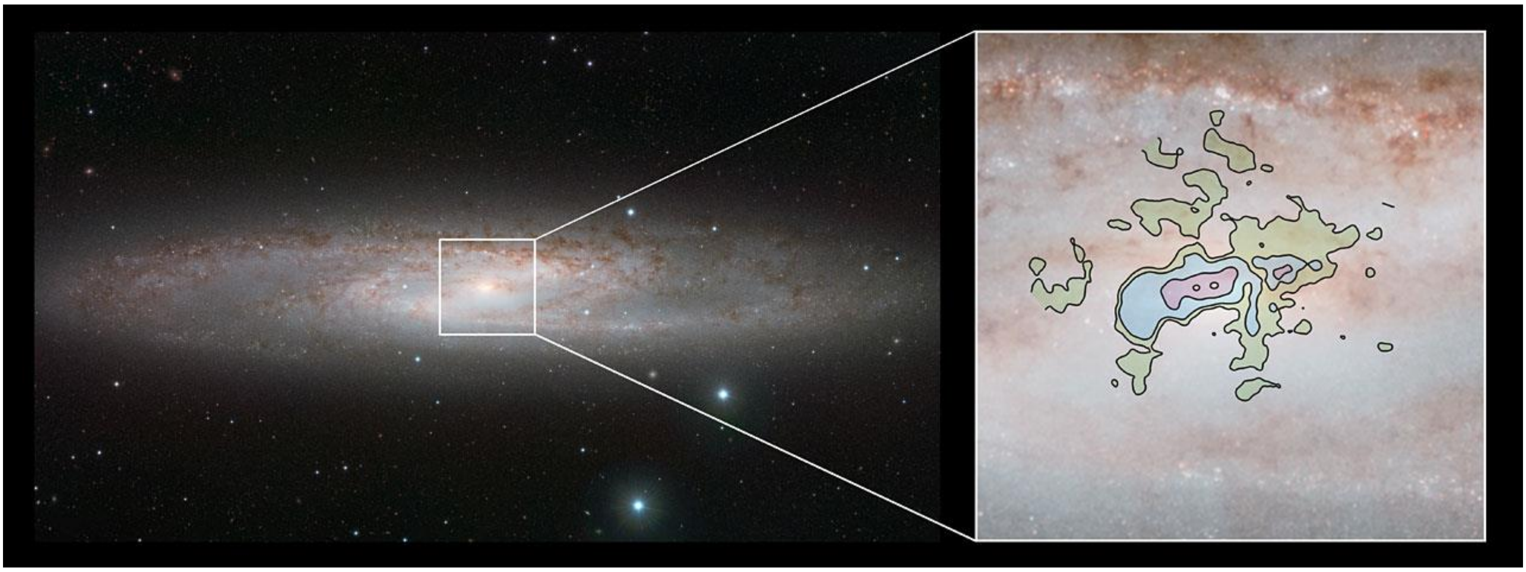
- correlations of galaxy distribs R Scranton ea ’03,

Multiple sets of evidence: no systematics affect the conclusion that $\ddot{a} > 0$, a scale factor of the Universe

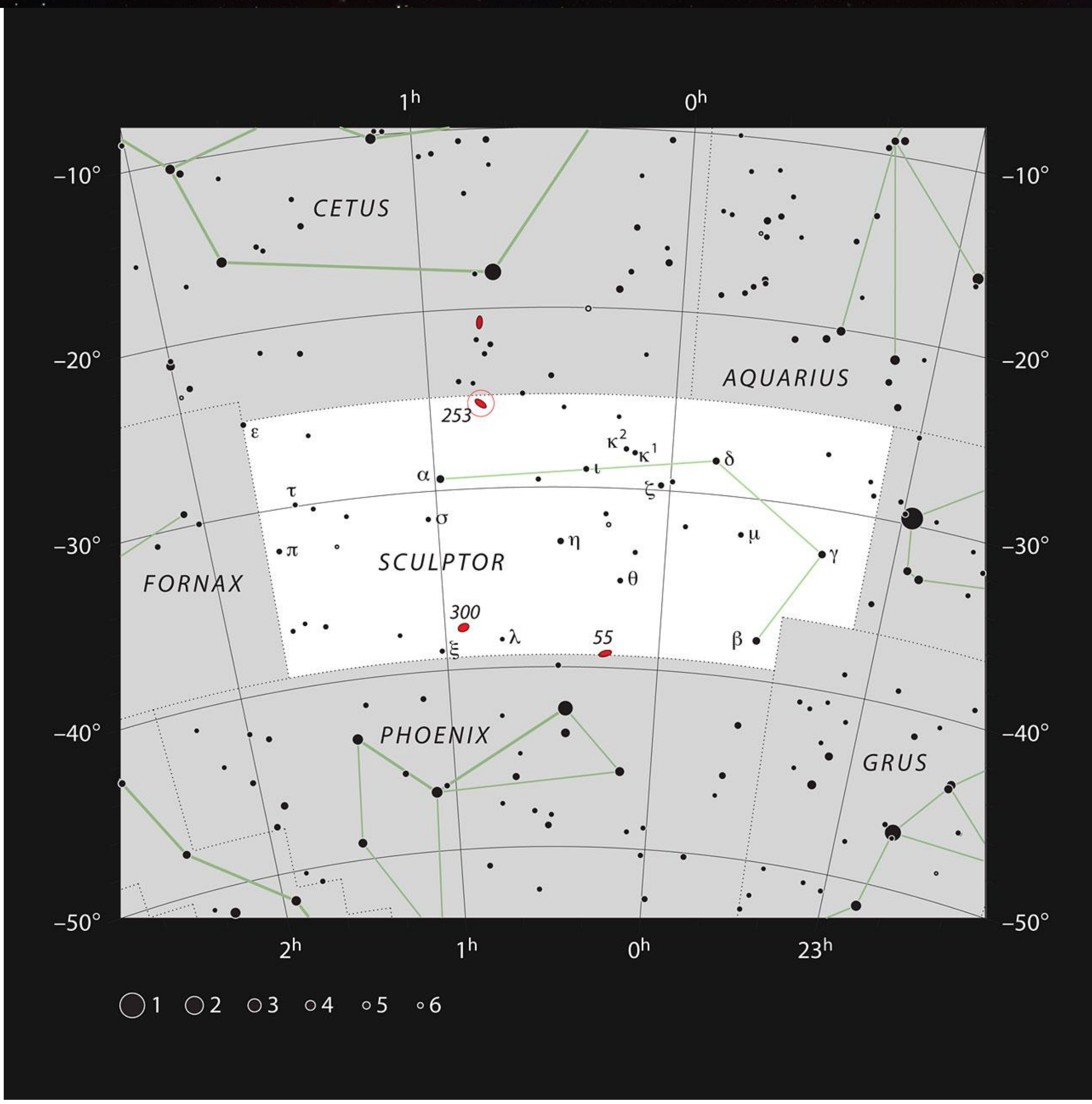
Starburst to Star Bust

ALMA on Mystery of Missing Massive Galaxies, Nature, 25 July 2013





Vigorous star formation can blast gas out of a galaxy and starve future generations of stars of the fuel they need to form and grow. Enormous outflows of molecular gas are ejected by star-forming regions in the nearby Sculptor Galaxy NGC253 (11.5MLY) They help explain the strange paucity of very massive galaxies in the Universe.



1h
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−20°
−30°
−40°
−50°
CETUS
AQUARIUS
SCULPTOR
FORNAX
PHOENIX
GRUS
253
300
55
ε
τ
π
α
σ
η
ι
κ2
κ1
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Different types of models

- In General Relativity (GR): Gravity leads to deceleration

  But pressure also influences geometry: R Tolman '32

  negative pressure can drive acceleration

  Cosmological evidence could be explained by an undiscovered substance with negative pressure, so-called dark energy

  J.A. Frieman ea '95, K. Coble ea '97, R. Caldwell ea '97, B. Ratra, P.J.E. Peebles '98, C. Wetterich '98, D. Huterer, M.S. Turner '99

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- Another possib: GR is (wrong!) not accurate enough at large scales

  S.M. Carroll, V. Duvvuri, M. Trodden, M.S. Turner '04, Capozziello '04

  GR was developed using information and intuition at Solar System scales, it is (almost) only checked there, it could need to be modified on large scales

  A. Starobinsky arrived at same conclusion, through very different arguments based on quantum corrections to ordinary GR: lead to terms of second order in R, and higher

Several problems

- Do not have simple guidelines, gedanken experiment, reasons of elegance and simplicity, as those of Einstein in constructing GR

Besides that, even if beauty is abandoned, a modification of gravity must still confront three additional problems (Park & Dodelson)

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- Another problem: modified gravity models should comply with the successes of GR in the Solar System

  These constraints already doomed one of the first most promising modified gravity models introduced to explain acceleration and still place tight constraints on many models

Models with non-local interactions

- One class of modified gravity models that overcomes most of these problems contains non-local interactions

  S. Deser and R. Woodard, Phys.Rev.Lett. 99, 111301 (2007), 0706.2151

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- At cosmological scales, $\Box^{-1}R$ grows very slowly:
  - as $(t/teq)^{1/2}$ in the radiation dominated era
  - logarithmically in the matter dominated era
  - So, at the time of Nucleosynthesis $\Box^{-1}R$ is about 10^{-6}
    and at matter-radiation equilibrium it is only order 1

  In a natural way, these terms are irrelevant at early times and begin to affect the dynamics of the Universe only after the matter-radiation transition
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- Finally, because $\Box^{-1}R$ is extremely small in the Solar System, these models easily pass local tests of gravity

Sufficient theoretical motivation

- In string theory, $R\,\Box^{-1}R$ is precisely the term generated by quantum corrections (away from the critical dimension) as first pointed out by Polyakov

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- A realistic model for acceleration, with an arbitrary function of $R\,\Box^{-1}R$, reproducing the expansion history of ΛCDM

  C. Deffayet, R. Woodard, JCAP 0908, 023 (2009), 0904.0961

  Used by S. Park & S. Dodelson, arXiv:1209.0836, to discuss structure formation in a nonlocally modified gravity

Additional considerations

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- Anyway, the very first problem still remains: a completely arbitrary free function can be chosen at will to fit the expansion history
- But a very interesting aspect of modified gravity models in general is that, even if they are constructed to reproduce the expansion history equivalent to that of a given dark energy model (such as ΛCDM), perturbations will often evolve differently than in a model with standard GR plus dark energy
- Indeed, the way to distinguish DE models from modified gravity is to measure the growth of structure in the Universe

  The deviations from dark energy models are at the 10 to 30$\%$ level and have a characteristic signature as a function of redshift, which suggests that the class of models could be tested by upcoming surveys

Our model of type $f(\Box^{-1}R)$

- Consider a nonlocal gravity which contains a function of the $\Box^{-1}$ operator, thus not assuming the existence of a new dimensional parameter in the action. We focus on the study of cosmological solutions both in Jordan and Einstein frames, including matter in the last case

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- Several models are able to reproduce observations, as quintom models, which involve two fields: a phantom and an ordinary scalar

E Elizalde, S Nojiri, S Odintsov, Phys.Rev.D70(2004)043539, hep-th/0405034

W Zhao and Y Zhang, Phys.Rev. D73 (2006) 123509, arXiv:astro-ph/0604460

H Štefančić, Phys.Rev. D71(2005)124036, arXiv:astro-ph/0504518

Our model (cont.)

- The nonlocal model we will consider is usually studied in the Jordan frame, but recently its behavior in the Einstein frame, for the case without matter, has been explored too

  K Bamba, S Nojiri, S Odintsov, M Sasaki, GRG44(2012)1321,arXiv:1104.2692

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- In the Jordan frame we obtain an exhaustive class of power-law solutions (we prove that other power-law solutions cannot exist)

- We analyze the correspondence between solutions got in different frames and prove explicitly how knowledge of power-law solutions in Jordan's frame can be used to get power-law solutions in Einstein's one

The action

- Consider a class of nonlocal gravities, with action

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}\left[R\left(1+f(\Box^{-1}R)\right)-2\Lambda\right]+\mathcal{L}_\mathrm{m}\right\}$$

where $\kappa^2 = 8\pi G = 8\pi/{M_\mathrm{Pl}}^2$, the Planck mass being $M_\mathrm{Pl} = G^{-1/2} = 1.2\times 10^{19}$ GeV, f differentiable function (characterizes nature of nonlocality), $\Box^{-1}$ inverse of d'Alembertian operator, Λ cosmological constant, and $\mathcal{L}_\mathrm{m}$ matter Lagrangian. For definiteness, we assume that matter is a perfect fluid. We use the signature $(-,+,+,+)$, g determinant of $g_{\mu\nu}$

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- Introducing two scalar fields: $\psi = \Box^{-1} R$ & Lagrange multiplier ξ

$$S_{loc} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} \left[R \left(1 + f(\psi) \right) + \xi \left(R - \Box\psi \right) - 2\Lambda \right] + \mathcal{L}_{\rm m} \right\}$$

Original non-local action is recast as a local action in the Jordan frame. Varying this action with respect to ξ and ψ, one resp gets the field eqs

$$\Box\psi = R\,, \qquad \Box\xi = f_{,\psi}(\psi) R\,,$$

where $f_{,\psi}(\psi) \equiv {\rm d}f/{\rm d}\psi$

The action (II)

- The corresponding Einstein equations are obtained by variation of the local action wrt the metric tensor

$$\frac{g_{\mu\nu}}{2}\left[R\Psi+\partial_\rho\xi\partial^\rho\psi-2(\Lambda+\Box\Psi)\right]-R_{\mu\nu}\Psi-\frac{1}{2}\left(\partial_\mu\xi\partial_\nu\psi+\partial_\mu\psi\partial_\nu\xi\right)+\nabla_\mu\partial_\nu\Psi=-\kappa^2T_{(\mathrm{m})\,\mu\nu}$$

where $\Psi\equiv 1+f(\psi)+\xi$, and $T_{(\mathrm{m})\,\mu\nu}$ energy-momentum tensor of matter sector

$$T_{(\mathrm{m})\,\mu\nu}\equiv-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\mathrm{m}}\right)}{\delta g^{\mu\nu}}$$

Note the system of equations here does not include the function ψ itself, but instead $f(\psi)$ and $f_{,\psi}(\psi)$, together with time derivatives of ψ. Also, $f(\psi)$ can only be determined up to a constant: one may add a constant to $f(\psi)$ and subtract the same constant from ξ without changing eqs

The action (II)

- The corresponding Einstein equations are obtained by variation of the local action wrt the metric tensor

$$\frac{g_{\mu\nu}}{2}\left[R\Psi + \partial_\rho\xi\partial^\rho\psi - 2(\Lambda + \Box\Psi)\right] - R_{\mu\nu}\Psi - \frac{1}{2}\left(\partial_\mu\xi\partial_\nu\psi + \partial_\mu\psi\partial_\nu\xi\right) + \nabla_\mu\partial_\nu\Psi = -\kappa^2 T_{(\mathrm{m})\,\mu\nu}$$

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- Here, we assume a spatially flat FLRW universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

and consider the case where the scalar fields $\psi(t)$ and $\xi(t)$ are functions of the cosmological time t only

The action (III)

- Thus, the system of Eqs. reduces to

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3H^2\Psi &= -\frac{1}{2}\dot{\xi}\dot{\psi} - 3H\dot{\Psi} + \Lambda + \kappa^2\rho_{\rm m} \\
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\ddot{\psi} &= -3H\dot{\psi} - 6\left(\dot{H} + 2H^2\right) \\
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dot means differentiation with respect to time, t, in the Jordan frame: $\dot{A}(t) \equiv {\rm d}A(t)/{\rm d}t$, $H = \dot{a}/a$ Hubble parameter

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- The continuity equation is

$$
\dot{\rho}_{\rm m} = -3H(P_{\rm m} + \rho_{\rm m})
$$

Adding them we obtain a 2nd order linear differential equation for Ψ

$$
\ddot{\Psi} + 5H\dot{\Psi} + \left(2\dot{H} + 6H^2\right)\Psi - 2\Lambda + \kappa^2(P_{\rm m} - \rho_{\rm m}) = 0
$$

Power-law sol's with $f(\psi)$ exp funct

- Consider the case when $f(\psi)$ exponential function

$$f(\psi) = f_0 e^{\alpha\psi}$$

f_0 and α nonzero real parameters. The motivation: (i) simplest model with power-law and de Sitter solutions (only exp or a sum of exps); (ii) better studied case among all possible functions for expanding universe sol's (with Hubble parameter $H = n/t$, with n a nonzero constant)

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- We consider matter with the EoS parameter $w_{\rm m} \equiv P_{\rm m}/\rho_{\rm m}$ being a constant but not equal to -1. For power-law solutions $H = n/t$, the continuity eq has the following general solution (ρ_0 arbitrary const)

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- Solutions with $H = n/t$. Inserting $H = n/t$

$$\psi(t) = \psi_1 t^{1-3n} - \frac{6n(2n-1)}{3n-1} \ln\left(\frac{t}{t_0}\right)$$

ψ_1, t_0 integration const's. We consider real solutions at $t > 0$, hence, $t_0 > 0$. Note these valid provided $n \neq 1/3$ and $n \neq 1/2$ (special cases, other sect)

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- Varying the nonlocal action wrt the metric $g_{\mu\nu}$, under the spatially flat FLRW metric, the independent components of the field equations are

$$3H^2 + \Delta G_{00} = \kappa^2 \rho_{\rm m} + \Lambda, \qquad -2\dot{H} - 3H^2 + \frac{1}{3a^2}\delta^{ij}\Delta G_{ij} = \kappa^2 P_{\rm m} - \Lambda$$

ΔG_{00} and ΔG_{ij} are the modifications coming from the nonlocal terms

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- We conclude that these solutions are solutions of the initial nonlocal model as well, what can be checked immediately by direct substitution

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  In fact, if we write $\psi \longrightarrow \psi + \chi$ the corresponding term is

$$\xi(\Box\psi - R) \longrightarrow \xi\big(\Box(\psi + \chi) - R\big)$$

  after integration by parts, change is $g^{\mu\nu}\partial_\mu \xi \partial_\nu \psi \longrightarrow g^{\mu\nu}\partial_\mu \xi \partial_\nu (\psi + \chi)$

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- However, imposing appropriate BC, as $\chi = 0$, to recover original form, this would-be extra dof can be eliminated. Issue of choice of correct BC should be the only non-equivalence between original form and biscalar-tensor representation. Thus, eg, Woodard ea determine the inverse d'Alembert op using the retarded Green function: they fix a solution of eq $\Box R = 0$ putting $\widetilde{t}_0 = 0$ and $\eta_0 = 0$

- A final comment. As stated above, the biscalar-tensor representation introduces two scalars, ψ and ξ, therefore, working in this way it seems that one will encounter a ghost-like behavior (Koivisto, Nojiri, Bamba, Sasaki) However, since the original nonlocal model does not introduce any new degree of freedom, the ghost-like behavior of the biscalar-tensor theory may not be physically relevant: associated terms can be cast as a boundary term of the nonlocal operators (Koivisto 2010). At classical level, a necessary way to check this physical relevance is by considering the equivalence of the solutions coming from the original nonlocal formulation and from its biscalar-tensor form

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- The Jordan and Einstein frames

  Once a modified gravity theory is recast into its scalar-tensor presentation, it immediately follows that both the Jordan frame (where the matter sector minimally couples to gravity) and the Einstein one (where the Ricci is linear but matter couples to gravity non-minimally) are available

  They are related by conformal transformation $g_{\mu\nu} = \Omega^2 g_{\mu\nu}^{(E)}$
  metric in Jordan frame is $g_{\mu\nu}$, in Einstein frame, $g_{\mu\nu}^{(E)}$

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- First, formulate diff eqn for the conformal factor under which power-law sol’s in Jordan’s correspond to other power-law sol’s in Einstein’s

Conclusions

- In GR, power-law solutions of type $H = n/t$ correspond to models with a perfect fluid whose EoS param $w_{\rm m} \equiv P_{\rm m}/\rho_{\rm m}$ is related to power-index by $w_{\rm m} = -1 + 2/(3n)$

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- In Einstein's, we obtained the power-law solutions either by solving the EoM, or by performing a conformal transformation of the sol's obtained in Jordan's. For this purpose, we extended the correspondence to include the matter sector. Using this powerful, non-trivial tool, we got the sol's when $w_\mathrm{m} = 1/3$ and $w_\mathrm{m} = 1$ (very difficult to obtain by directly solving the system), thus proving the usefulness of the method

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- The biscalar-tensor representation introduces two extra scalars. Can lead to a ghost problem. Equivalence between the initial nonlocal theory and local formulation has not been established yet The ghost-like behavior of the biscalar-tensor theory may not be a physical problem, since the associated terms can be cast as boundary terms of the nonlocal operators (would-be ghost mode might correspond to an inappropriate choice of BC). Plan to consider this important question in future work

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