ICE CSIC IEEC

# Casimir Effects, Riemann, Einstein, and Cosmology

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# Outline of this presentation

- Einstein's Cosmological Constant

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# Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cosmological constant

- For cosmologists and general relativists: a great mistake

  why was it put there, in the first place? (Einstein)

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc $\Lambda$ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density $\mu$ of the vacuum

$$L_{cc} = \int d^4x \sqrt{-g}\, \mu^4 = \frac{1}{8\pi G} \int d^4x \sqrt{-g}\, \lambda$$

In other words: two contributions on the same footing (Zel'dovich, 68)

$$\frac{\Lambda\, c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar\, c}{2} \sum_i \omega_i$$

# Einstein Eqs, FLRW Sol, Hubble Const

Einstein Equations (1915-17): $G_{\mu\nu} - \lambda g_{\mu\nu} = -(8\pi G/c^4) T_{\mu\nu}$

Geometry = Energy-Matter

$G_{\mu\nu}$ linear combination of the metric $g_{\mu\nu}$ and 1st & 2nd derivatives

$T_{\mu\nu}$ energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

Schwarzschild solution (1916) $r$, $\theta$, $\varphi$ *comoving co*

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

Friedmann-Lemaître-Robertson-Walker (1935-36) sol (A. Friedmann 1922)

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

*most general family: homogeneous and isotropic,* $k$ *parameter* $\pm 1, 0$

One field eq looks like Newtonian eq for the gravit pot: $\nabla^2 \phi = 4\pi G \left(\rho + 3p/c^2\right)$

density & pressure contribute to the gravit pot $\lambda = 8\pi G \rho_{vac}, \quad p_{vac} = -\rho_{vac} c^2$

From the FRW metric and Einstein Eqs, an "equation of motion" of the universe

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

# From GR to Cosmology

With the definitions:

$$\Omega_m \equiv \frac{8\pi G}{3H^2}\rho_m, \qquad \Omega_\lambda \equiv \frac{\lambda}{3H^2}, \qquad \Omega_k \equiv -\frac{k}{H^2}$$

The equation of motion becomes

$$\left(\frac{da}{dt}\right)^2 = H_0^2\left[\frac{\Omega_m^{(0)}}{a} + a^2\Omega_\lambda^{(0)} + \Omega_k^{(0)}\right]$$

*(the superscript (o) represents quantities measured at the present time)*

In other terms, Friedmann equation in Cosmology:

$$\frac{\dot{a}^2}{a^2} = H_0^2\left[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_{NR}\left(\frac{a_0}{a}\right)^3 + \Omega_k\left(\frac{a_0}{a}\right)^2 + \Omega_\lambda\right]$$

$\Omega_R$ relativistic matter ($p_R = \frac{1}{3}\rho_R;\ \ \rho_R \propto a^{-4}$) Mach's princ

$\Omega_{NR}$ nonrelativistic matter ($p_{NR} = 0;\ \ \rho_{NR} \propto a^{-3}$)

$\Omega_\lambda$ cosmological constant ($p_\lambda = -\rho_\lambda;\ \ \rho_\lambda = \text{const}$)

$\Omega = \Omega_R + \Omega_{NR} + \Omega_\lambda$ total energy density (cosmic triangle)

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QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

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Even then: Has the final value real sense ?

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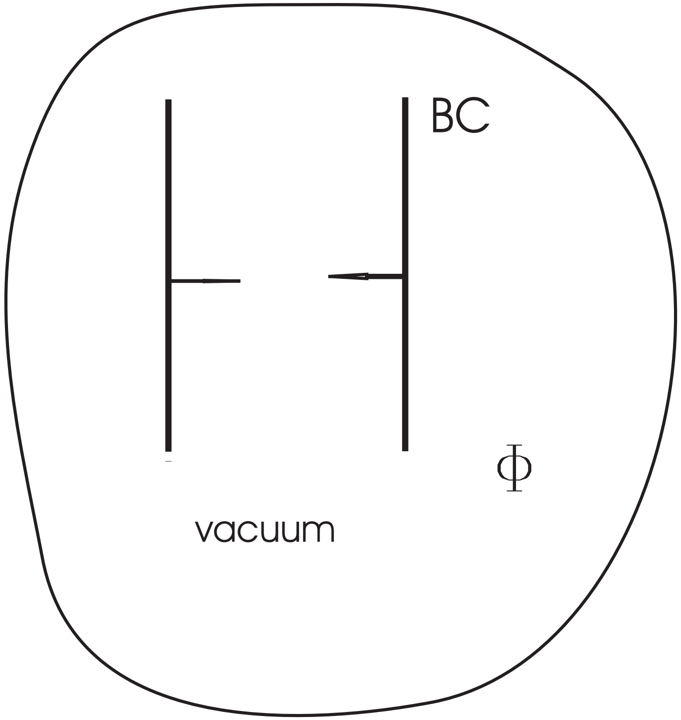
BC e.g. periodic

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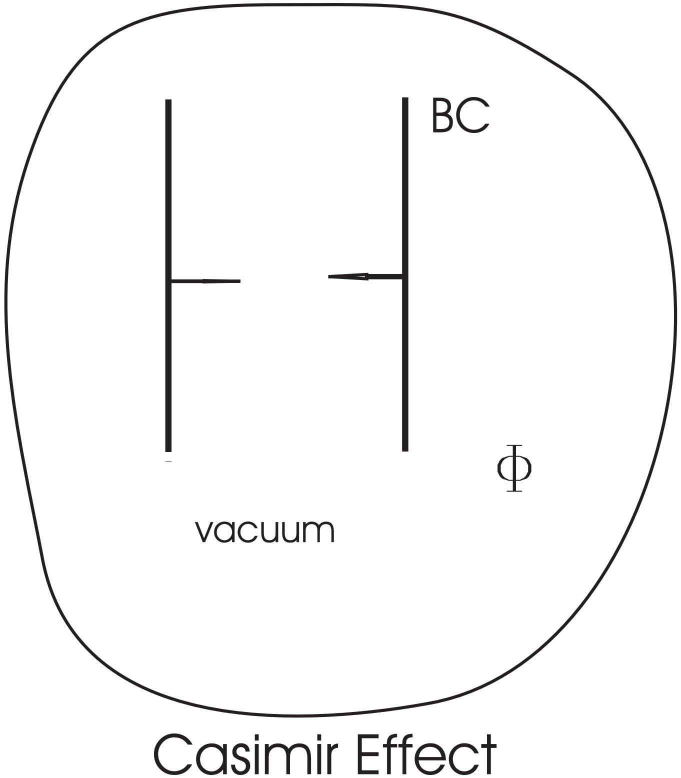


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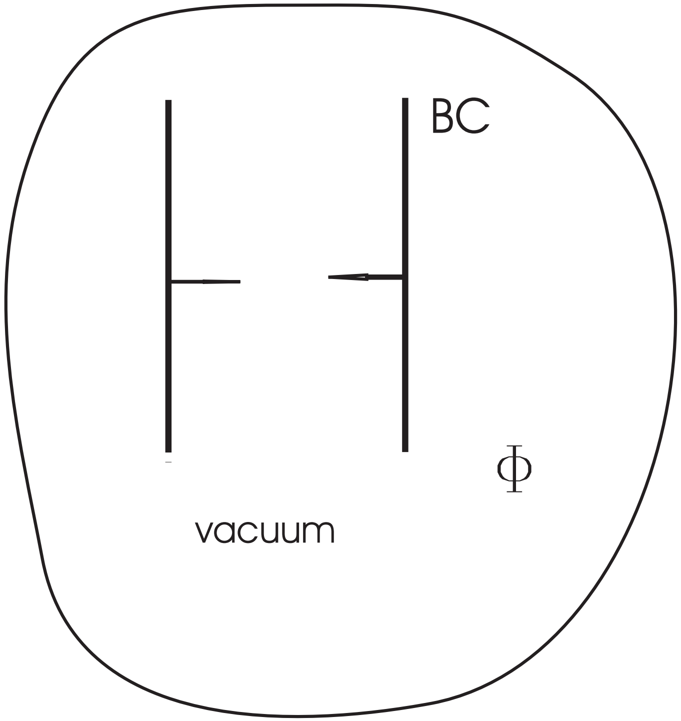

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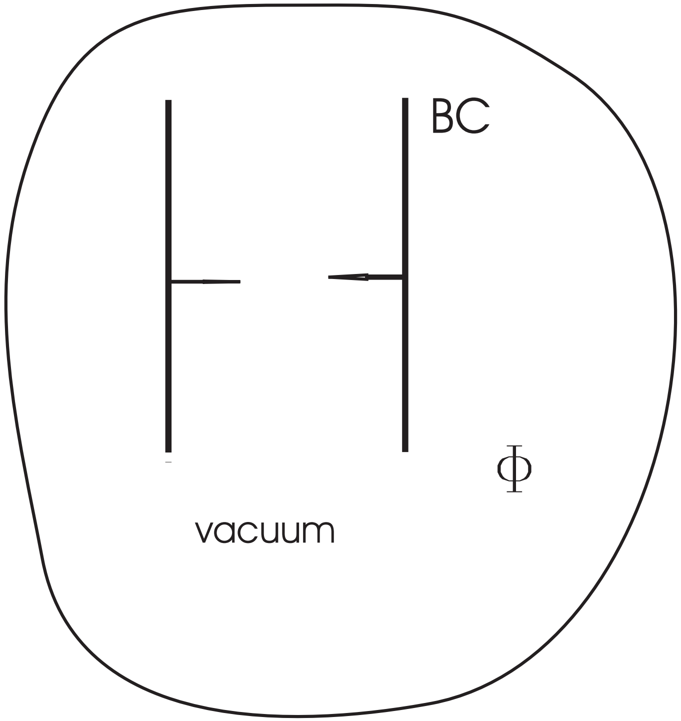


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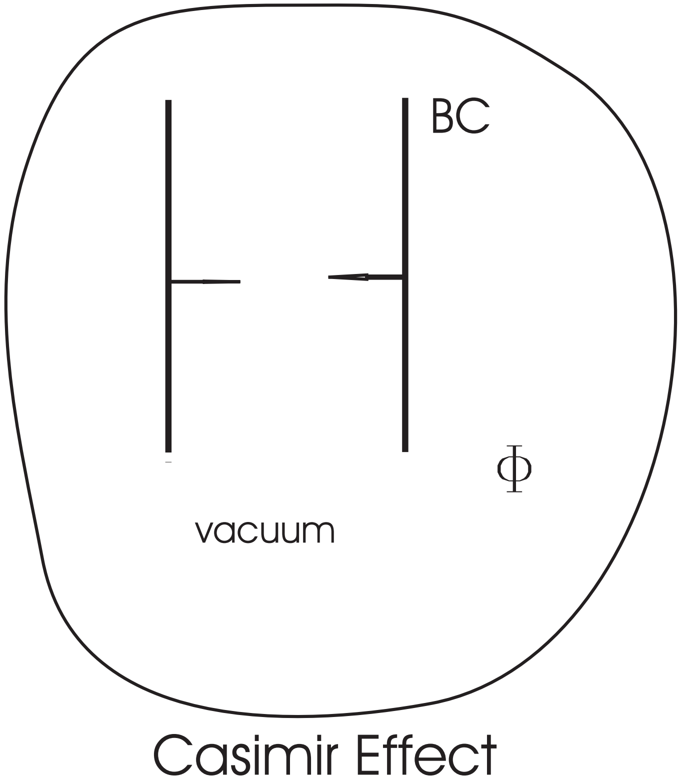


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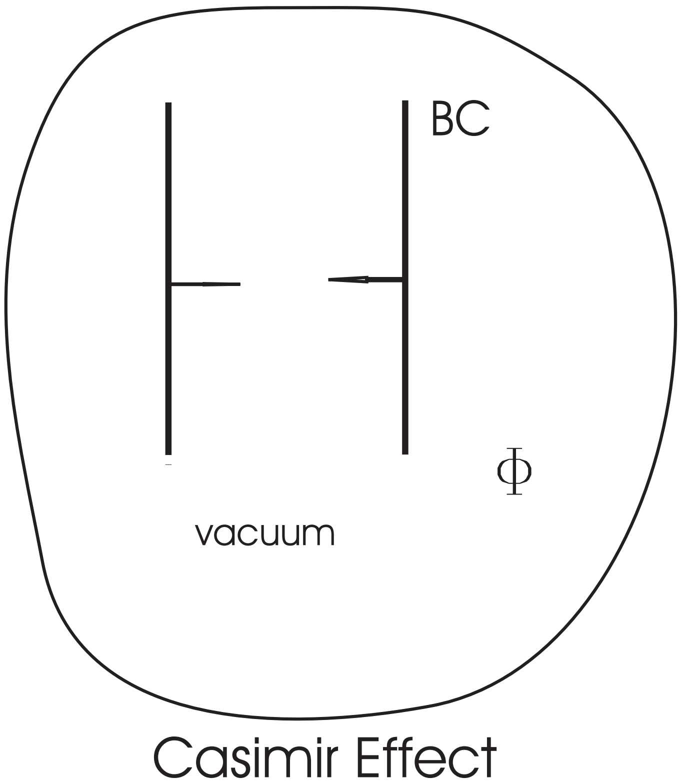


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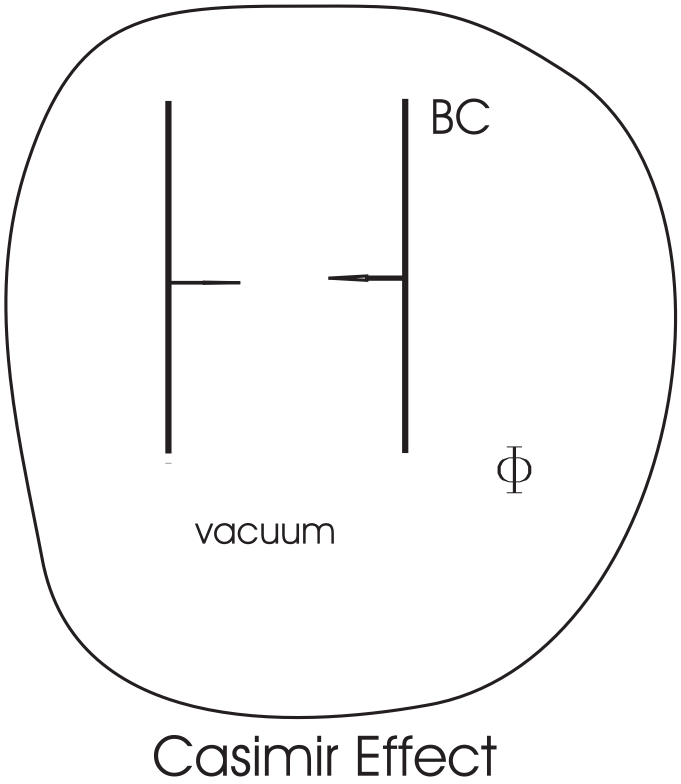


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- Dynamical CE $\Longleftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Longleftarrow$

# The standard approach

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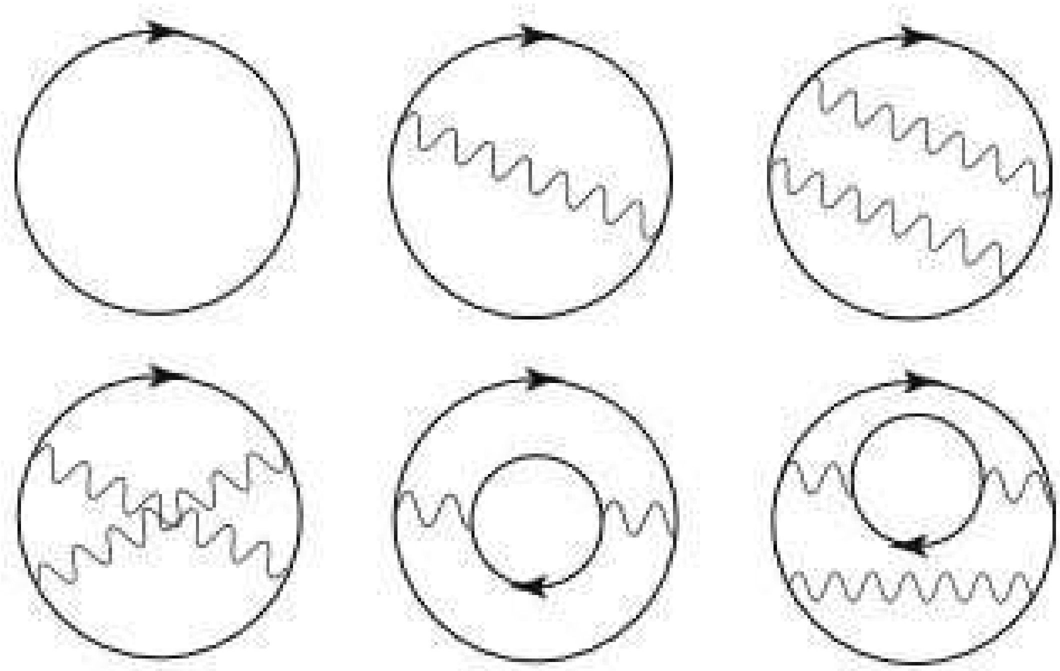

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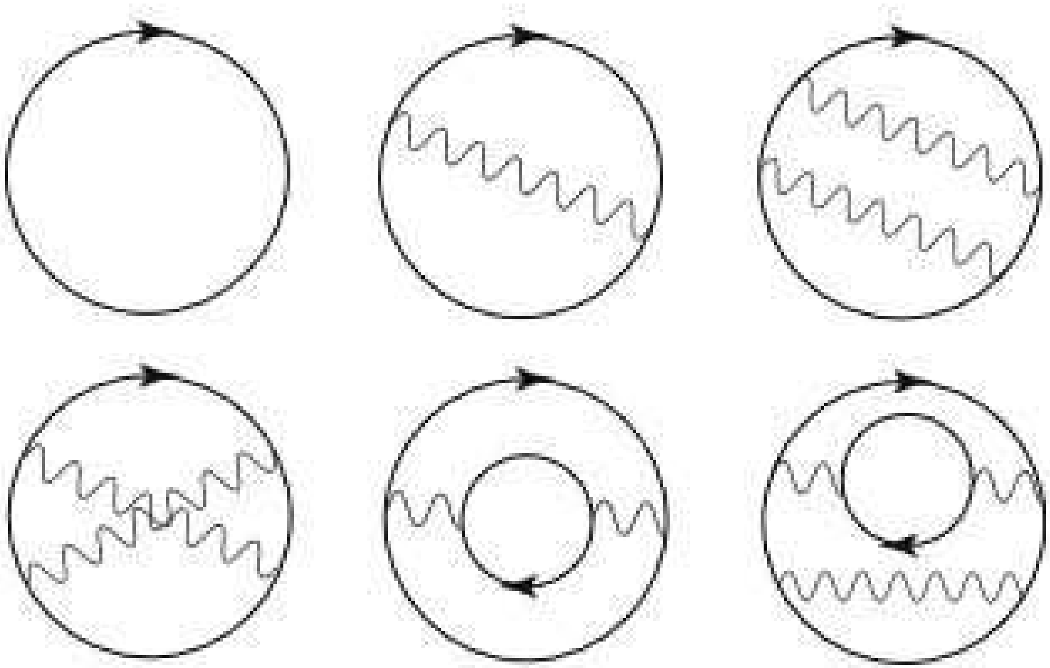

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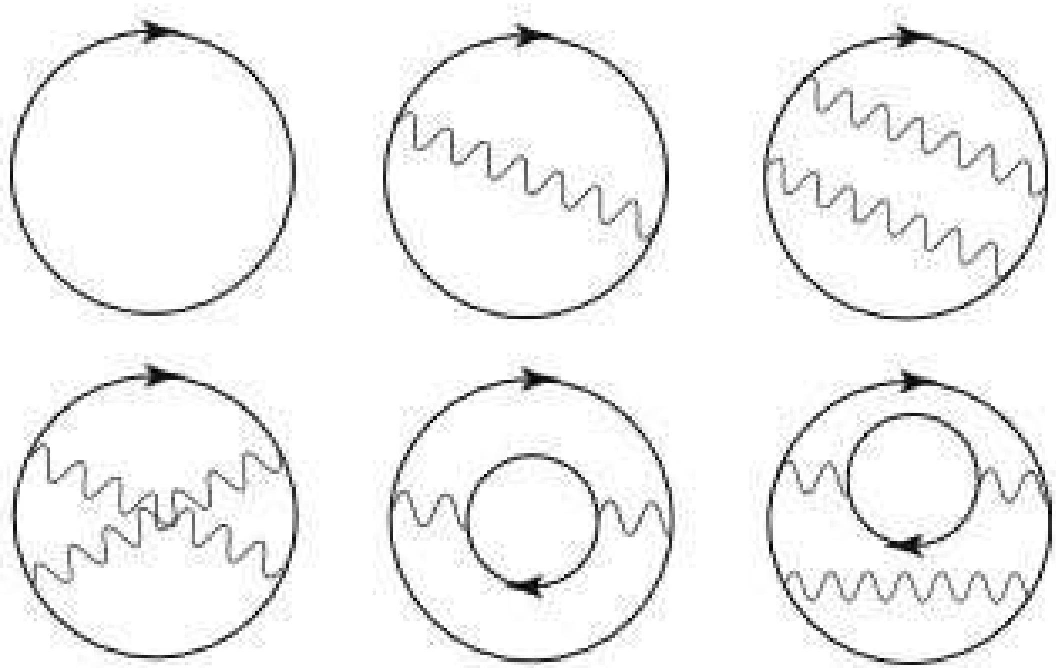

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi} \mathrm{Im} \int d\omega \omega \; \mathrm{Tr} \int d^3x \left[ \mathcal{G}(x, x, \omega + i\epsilon) - \mathcal{G}_0(x, x, \omega + i\epsilon) \right]$$

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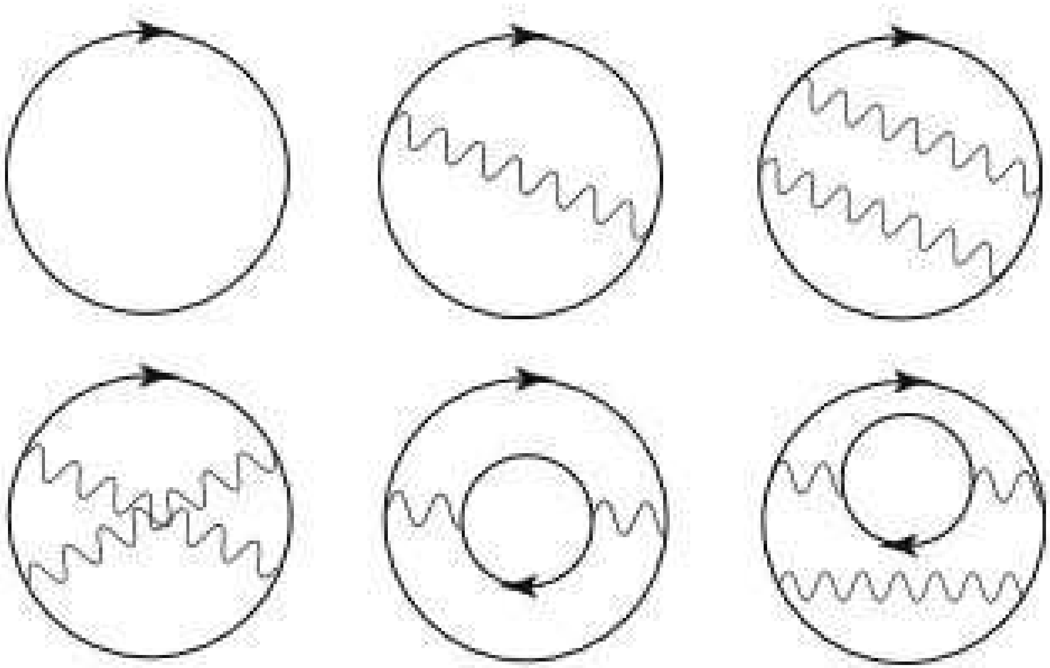

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function Trace is over spin

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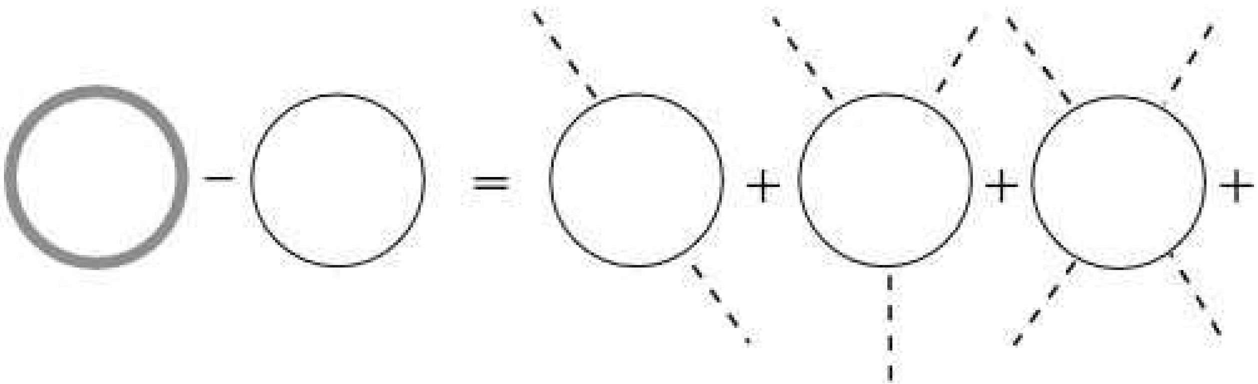

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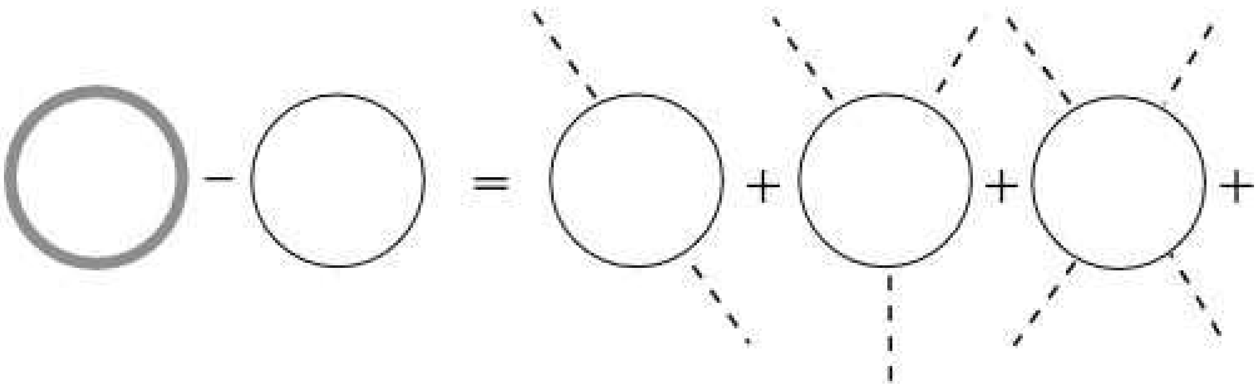


$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

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Moore; Razavy, Terning; Johnston, Sarkar; Dodonov et al; Plunien et al; Barton, Eberlein, Calogeracos; Ford, Vilenkin; Jaeckel, Reynaud, Lambrecht; Brevik, Milton et al; Dalvit, Maia-Neto et al; Law; Parentani, ...

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- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

# SOME DETAILS OF THE METHOD

- Hamiltonian method for neutral Klein-Gordon field in a cavity $\Omega_t$, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform $\Omega_t$ into a fixed domain $\widetilde{\Omega}$

$$\widetilde{\Omega}: \ (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

(with $\bar{t}$ the new time)

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$$S(\omega) = \begin{pmatrix} s(\omega) & r(\omega)\, e^{-2i\omega L} \\ r(\omega)\, e^{2i\omega L} & s(\omega) \end{pmatrix}$$

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

# Results are rewarding:

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

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- The main issue: S.A. Fulling et. al., hep-th/070209v2

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

# CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

# Cosmolog Imprint of the Casimir Eff't?

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
  * C.P. Burgess et al., hep-th/0606020 & 0510123: Susy Large Extra Dims (SLED), two $10^{-2}$mm dims, bulk vs brane Susy breaking scales
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  - (c) supergraviton theories (discret dims, deconstr)

# The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem

2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, S Nojiri, SD Odintsov, S Ogushi, Phys Rev D67 (2003) 063515 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, SD Odintsov, AA Saharian 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T\delta S$$

  – entropy proport to variation of the horizon area: $\delta S = \eta\, \delta \mathcal{A}$
  – local temperature $T$ defined as Unruh temp: $T = \hbar k/2\pi$
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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

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$\eta_e$ is a function of the metric and its deriv's to a given order

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\begin{aligned}\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\ &= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad\qquad S = \frac{A}{4\,G_{eff}}\end{aligned}$$

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- Final result, for $\mathbf{f}(R)$ gravities:
  *the local field equations can be thought of as an equation of state of equilibrium thermodynamics* (as in the GR case)

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
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- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
  T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
  C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]

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Happy Birthday, Iver! Long and Pleasant Life!