ICE CSIC IEEC

Vacuum Fluctuations: Regularization & Cosmological Issues

EMILIO ELIZALDE

ICE/CSIC & IEEC, UAB, Barcelona

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Outline

- On Einstein's Cosmological Constant:
  a Historical Perspective
- Quantum Vacuum Fluctuations: the Casimir Effect
- Vacuum Fluctuations and the Equivalence Principle
- The Sign of the Vacuum Forces
- Repulsion from Higher Dimensions and BCs
- CE and Accelerated Expansion (Dark Energy):
  a Cosmo-Topological Casimir Effect?
- Gravity Equations as Equations of State

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- Led to Big Bang theory (Fred Hoyle, BBC radio's Third Programme, 18:30 GMT, 28 March 1949)

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- On a different level: Richard Dawkins, Hoyle's fallacy
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THE COSMOLOGICAL CONSTANT PROBLEM

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Even then: Has the final value real sense ?

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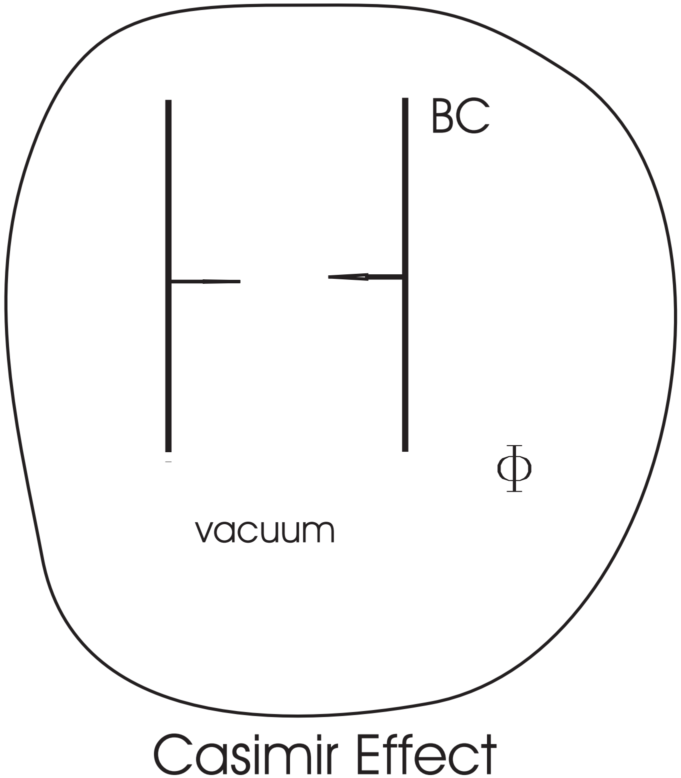

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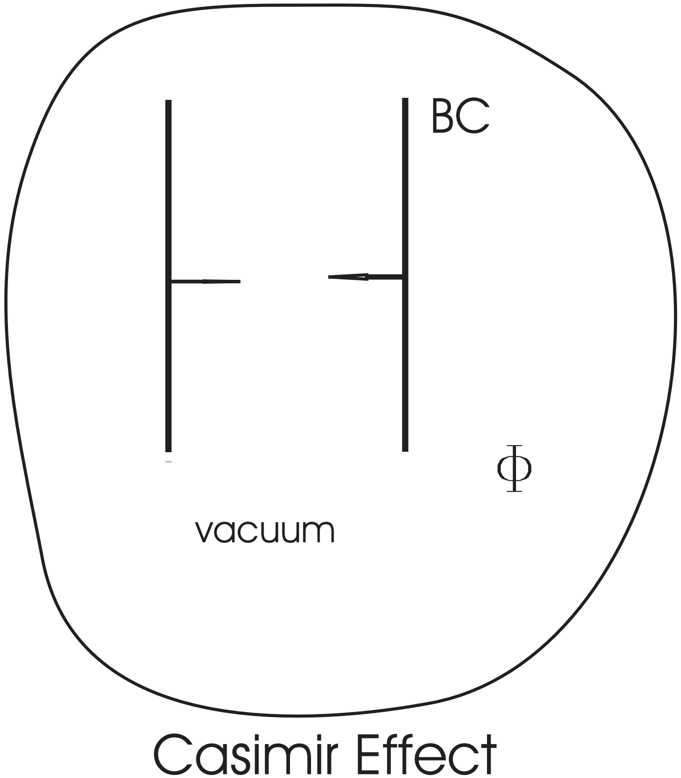

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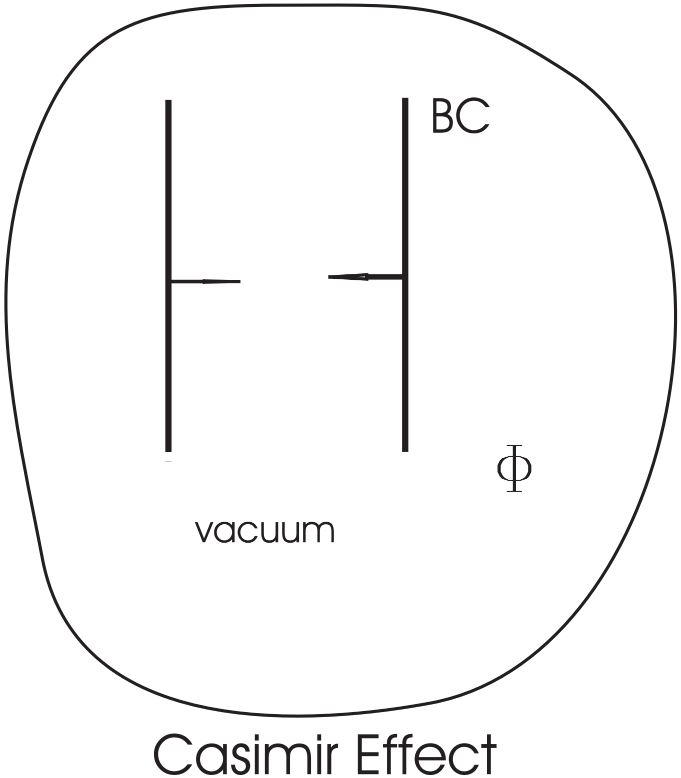

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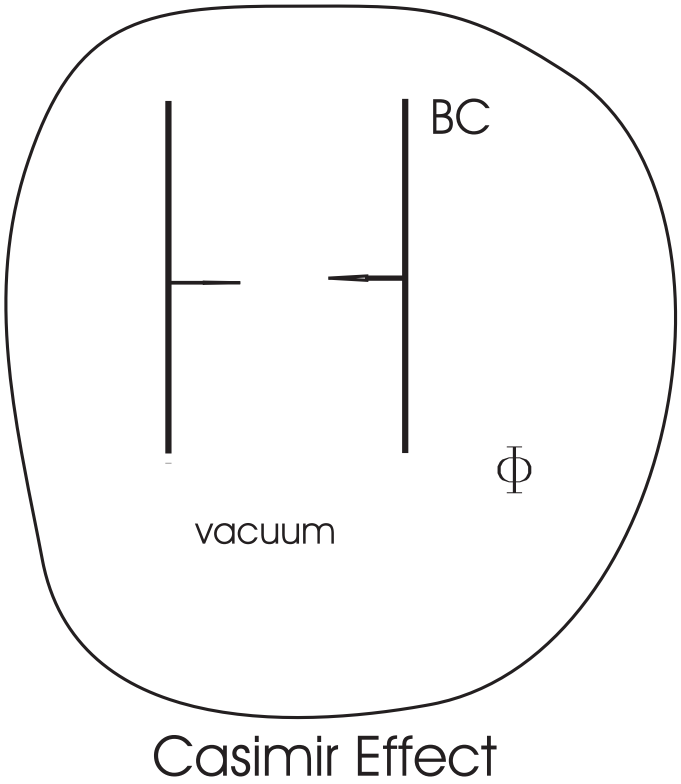

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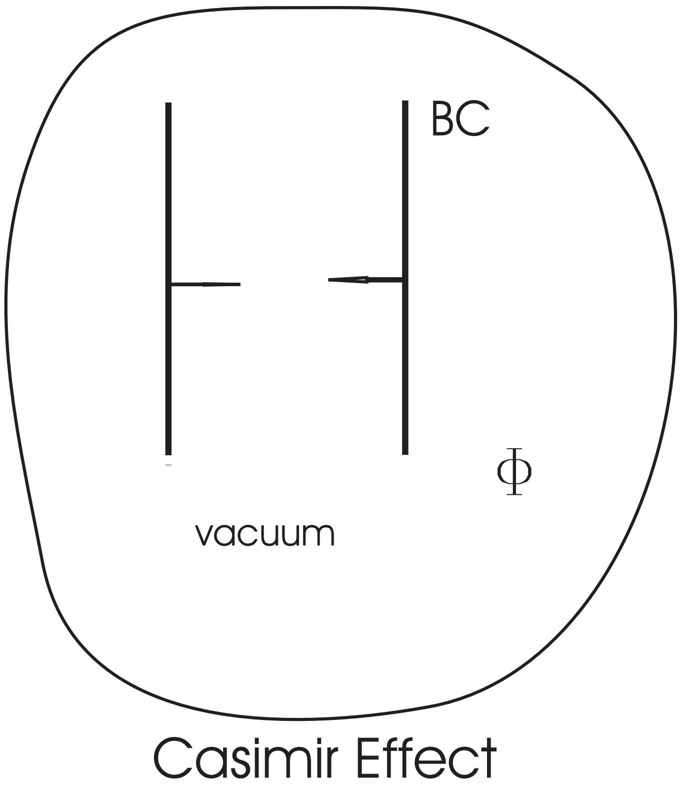

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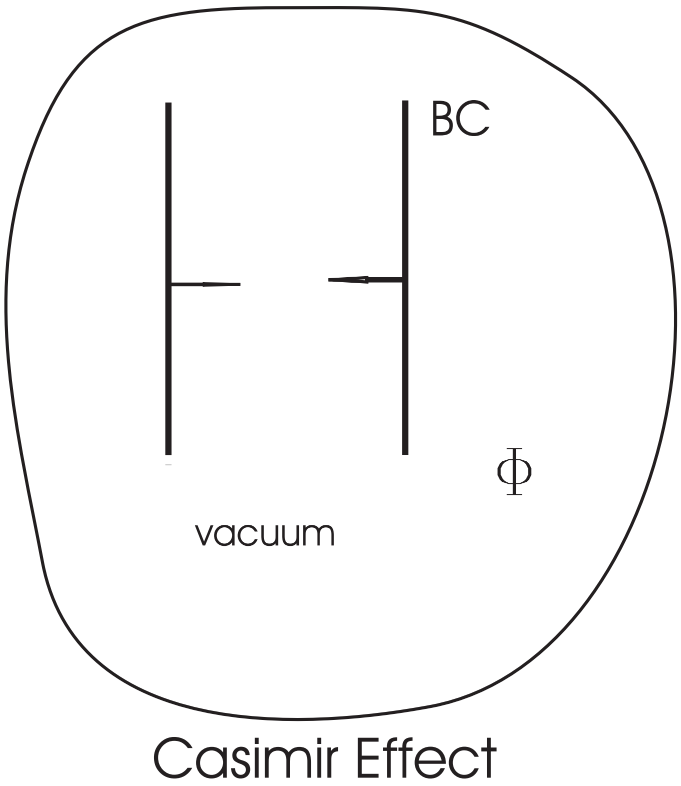

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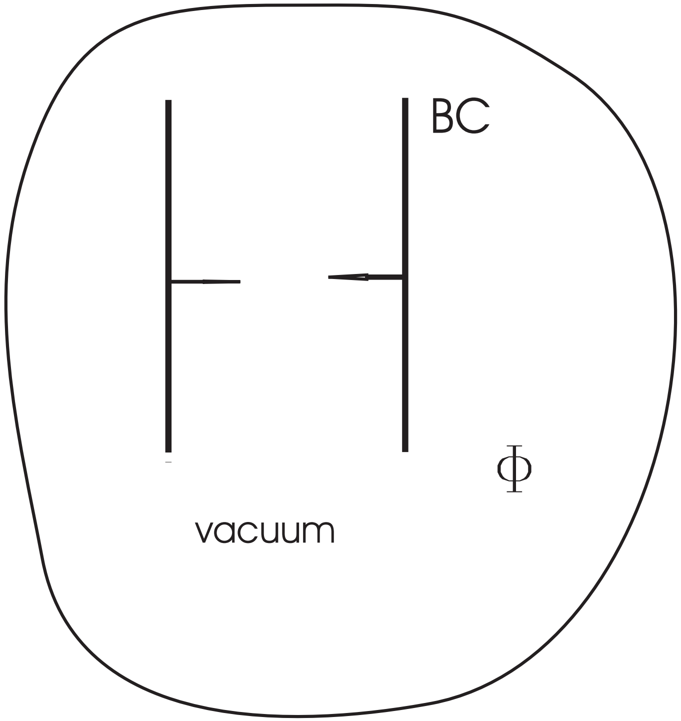

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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

The standard approach

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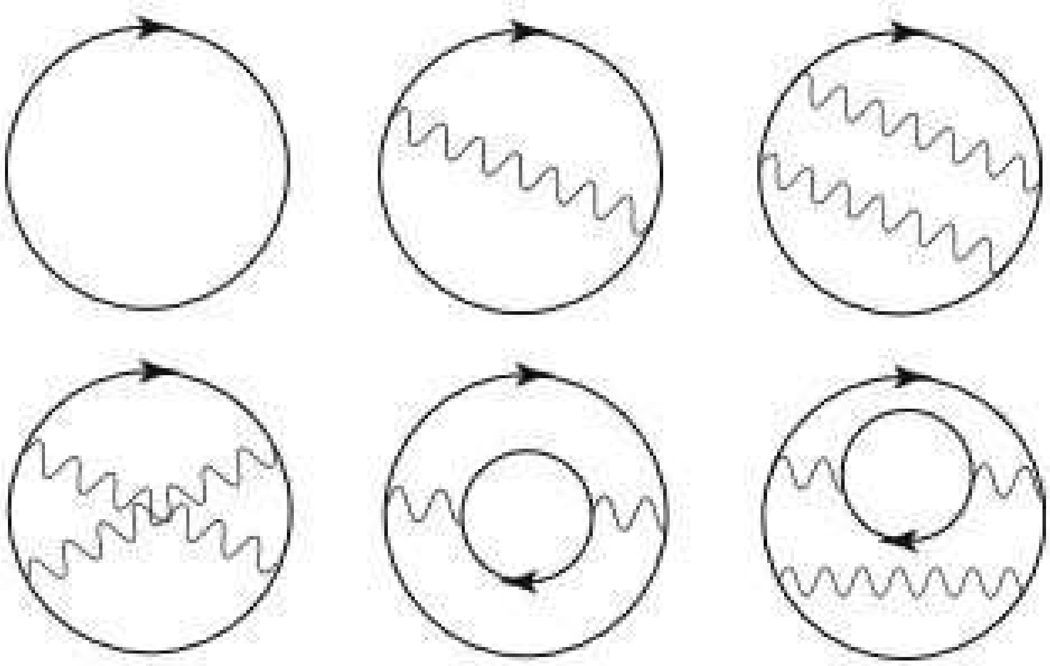

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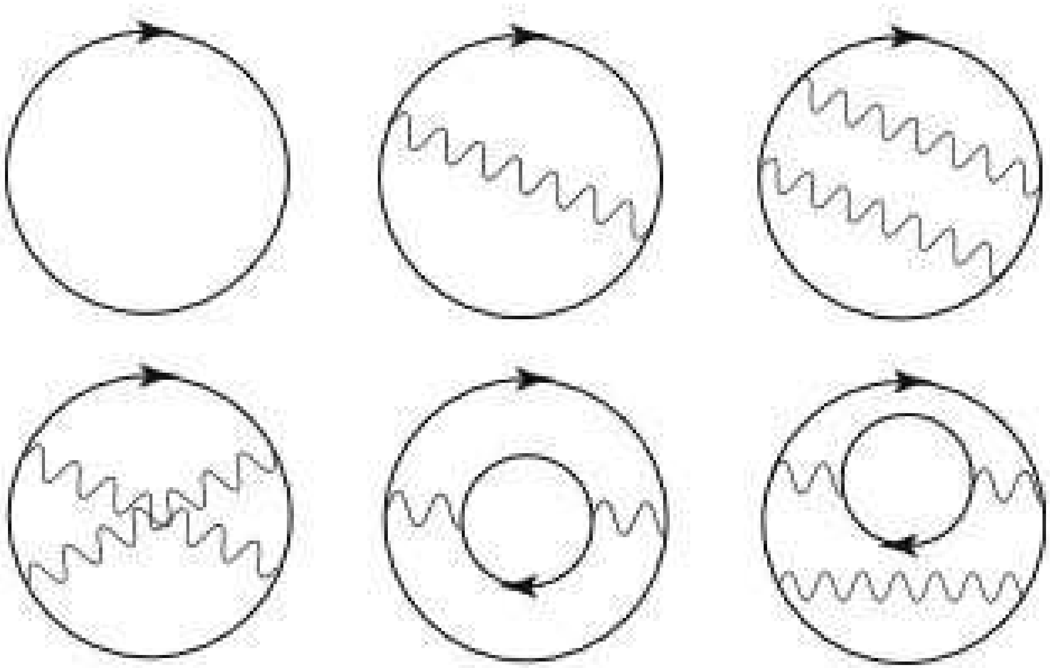

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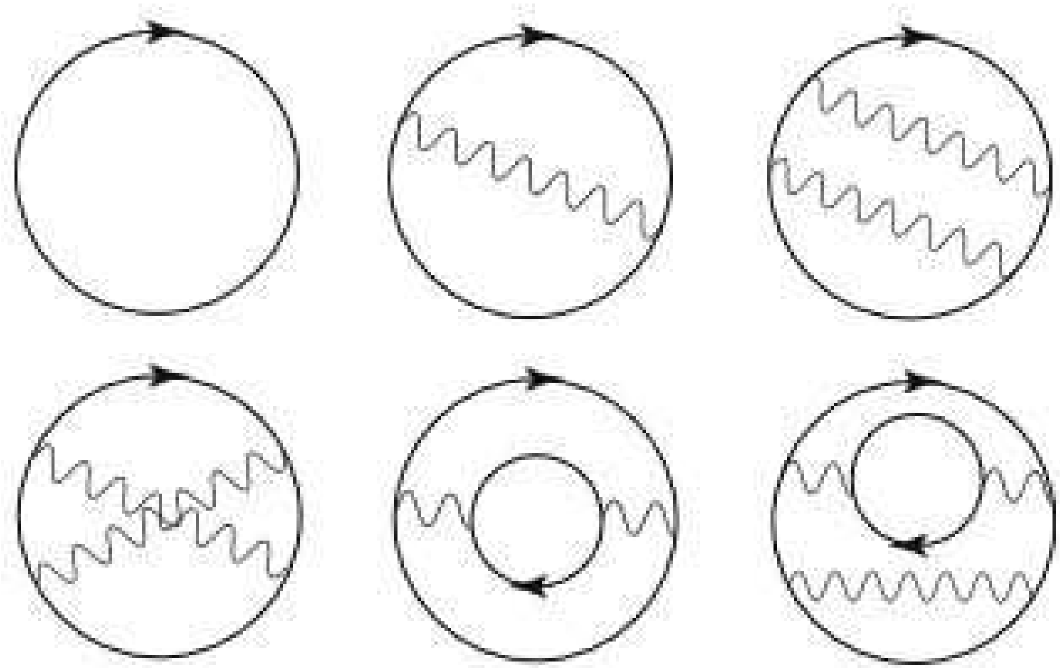

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$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon)-\mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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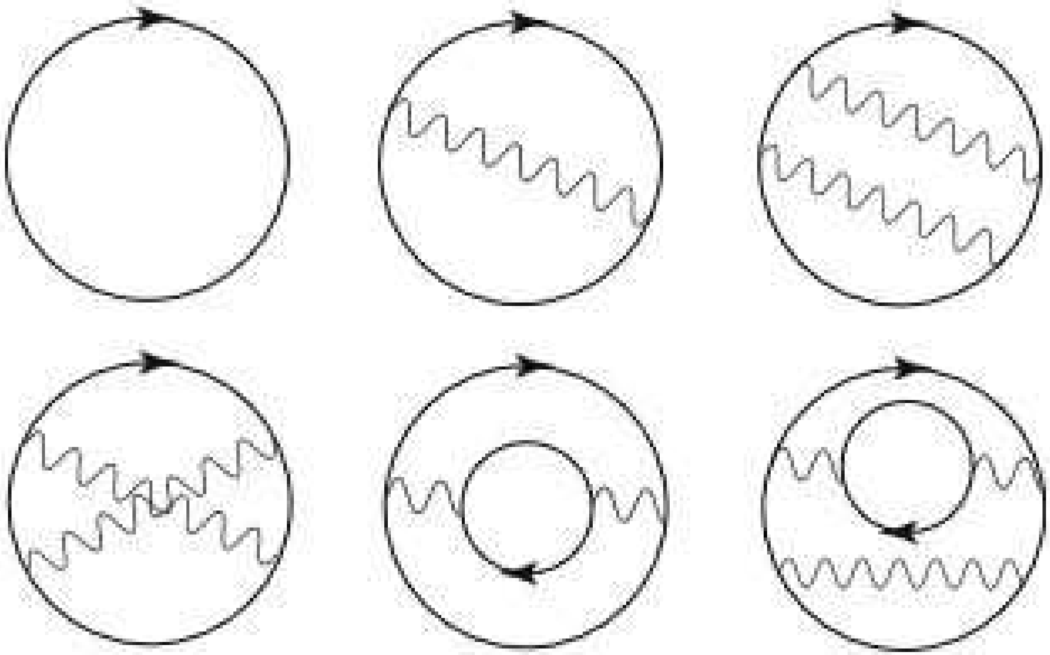

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function Trace is over spin

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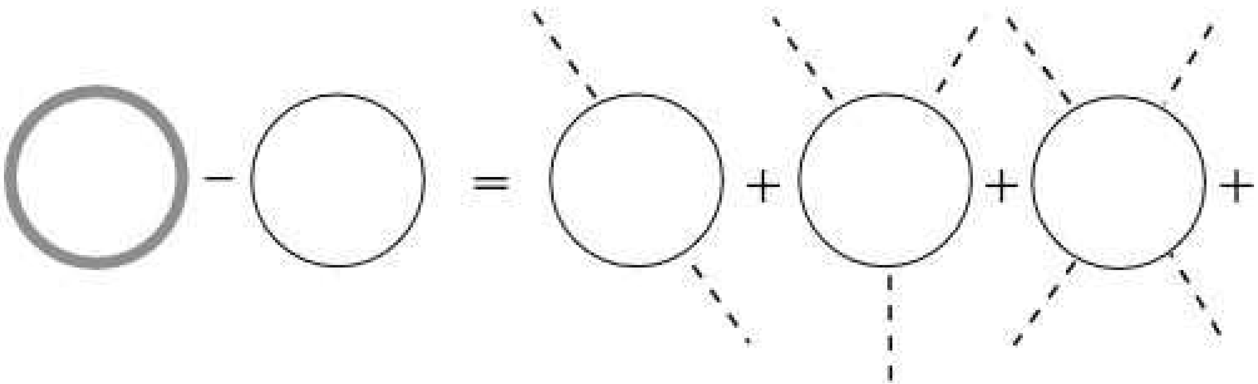

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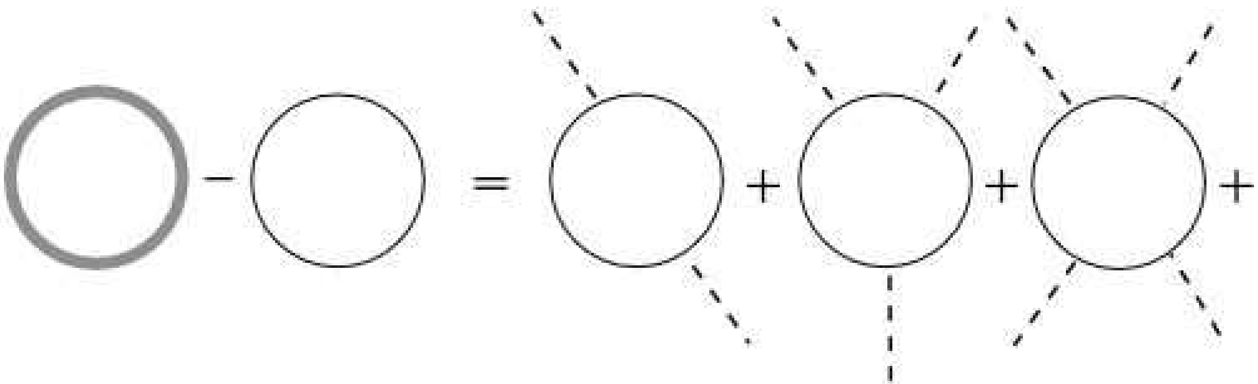

$\Longrightarrow$ "Experimental confirmation of the Casimir effect doesn't establish the reality of zero point fluct's better than say the Lamb shift does" [R Jaffe e a]

Vacuum Fluct & the Equival Principle

- The main issue: S.A. Fulling et. al., hep-th/070209

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- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

Cosmo-Topol Casimir Eff't & Alternat's

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- B. Other alternatives: (i) L Faddeev 0911.0282 (Adler '82) Newton const in E-H Lag has dim of mass $\longrightarrow$ non-renormalizability Describe gravity by vector field (as Higgs mechanism) (ii) Porto & Zee 0910.3716 Dynamical critical behavior of gravity in euIR sector and a mechanism to relax the cc. Also Shapiro+Sola, ...

More recent alternatives (a sample)

- (iii) E Mottola 1006.3567 Effective field theory approach
  – Casimir effect in flat s-t and large quantum backreaction
  are effects at the horizon scale of cosmological s-t
  – imply the cosmological VE is dynamical
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- (iv) T Padmanabhan Ad Sci Lett 2 74 09 cc problem and explaining
  DE independent issues: first find mechanism to make the cc vanish
  – new degrees of freedom, kind of 'gauge freedom'
  to absorb any λ while maintaining general covariance
  – could succeed in making gravity decouple from the bulk VE
  – emergent gravity approach: thermodynamic description is far more
  general than just Einstein theory
  – observed cc should be a relic of quantum gravitational physics and
  arise from degrees of freedom which scale as the surface area
  – numerics: $L_\Lambda / L_P \sim \exp \sqrt{2}\, \pi^4 \sim 10^{60}$ (hierarchy squared) $\sim 10^{61}$

- (v) Shao & Chen 1005.1920 no attempt at explaining the old cc prob

– an extremely small quantum correction can in fact be produced quite naturally from a massive bulk field, introducing a massive bulk fermion

– naturally as superpartner of the radion field in a SUSY theory (especially the string theory realization) of brane-world scenario

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- (vi) JA Dixon 1006.2334 CyberSUSY solves the cc problem
  – a new mechanism for SUSY breaking
  – its realization mixes elementary and composite states
  – SUSY anomalies present, generates spectrum for SUSY breaking consistent with known particles
  – no cc generated, because SUSY is not spontaneously broken ...

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem
2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds [A Flachi]

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, Nojiri, Odintsov, Ogushi, PRD67(2003)063515, hep-th/0209242 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, Odintsov, Saharian PRD79(2009)065023, 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

The Sign of the Casimir Force

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- More general results: Kenneth, Klich, PRL 97, 160401 (2006)
  a mirror pair of dielectric bodies always attract each other
  CP Bachas, J Phys A40, 9089 (2007) from a general property of Euclidean QFT 'reflection positivity' (Osterwalder - Schrader 73, 75):
  $\exists$ of positive Hilbert space and self-adjoint non-negative Hamiltonian

- E.g. $\exists$ correlation inequality: $\langle f\Theta(f)\rangle > 0$
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  - Robin BCs in general $\Leftarrow$

Casimir eff in brworl's w large extra dim

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- Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein's entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]

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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T\delta S$$

  – entropy proport to variation of the horizon area: $\delta S = \eta\, \delta \mathcal{A}$
  – local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$
  – functional dependence of S wrt energy and size of system

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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_{\Sigma} E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta \left(\eta_e A \right)$$

η_e is a function of the metric and its deriv's to a given order

$$\eta_e = \eta_e \left(g_{ab}, R_{cdef}, \nabla^{(l)} R_{pqrs} \right)$$

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T\delta S$$

– entropy proport to variation of the horizon area: $\delta S = \eta\,\delta\mathcal{A}$
– local temperature T defined as Unruh temp: $T = \hbar k/2\pi$
– functional dependence of S wrt energy and size of system

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$
\begin{aligned}
\frac{1}{8\pi G_{eff}} &= E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs} \\
&= \frac{\partial \mathbf{f}}{\partial R} = \frac{\eta_e}{2\pi}, \qquad\qquad S = \frac{A}{4\,G_{eff}}
\end{aligned}
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- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

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